PRELIMINARY STUDY ON MPS METHOD FOR SIMULATION OF WATER INFLOW INTO UNDER GROUND SPACE

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ABSTRACT: Water inflow should be carefully evaluated when the new application of existing underground space to emergency shelter is considered. In this report, fundamental theory of MPS to calculate incompressible water flows is reviewed with numerical examples of a MPS model. A simple model to calculate Dambreak problem with polygon wall boundary, to verify the incompressibility of the calculation result, is proposed.

Key Words: *MPS, Water inflow, Fluid analysis, Shelter*

INTRODUCTION

Generally, underground space is thought to be exposed to less seismic impact than buildings mostly constructed on the ground. For the preparation of shelter space in emergency, and thinking earthquake is typical disaster to be prepared, it is important to notice that there are a lot of available under ground spaces in Japan. On the other hand, flood due to heavy rain, high-tide or Tsunami is also devastating and rather frequent disaster in Japan. Therefore, when we convert the existing underground facilities to emergency shelters, it is very important to know how water flows into the facilities and how we can control it. In Japan, so far, the most of evacuation facilities for tsunami have been set at a high place, such as Tsunami evacuation tower or evacuation places set on hills. However, it is apparently very difficult to follow quick evacuation for those who cannot evacuate by themselves such as elders or patients staying in hospitals. From such a point of view, even for flood, evacuation to underground space can be a reasonable alternative because it is much easier for them. For examining safety of underground space during flood, Kanai[1] applied Pond Model numerical simulation for Shibuya underground malls , as an example. Although pond model is a good simple method to see the general progress of the flow, it cannot tell the dynamic physical impact of water. This paper introduces Moving Particle Simulation (MPS) method as a Computational Fluid Dynamics (CFD) solution to know more details and impacts than Pond Model.
This paper reports a fundamental theory of the explicit MPS to solve Navier-Stokes eq. with polygon boundary. And then its problem of modeling is described, and a solution is proposed. The results of the program presented in this paper is compared against that of existing one.

FUNDAMENTAL THEORY OF MPS FLUID ANALYSIS

MPS method was developed as a solving procedure for incompressible flow in 1996 by Koshizuka et al[2]. Space domain for calculation is discretized as moving particles on Laglange's coordinate system, and each particle interacts with its infinite neighbor particles. These interactions are considered as interaction models of each differential operators such as gradient, divergence and Laplacian in the

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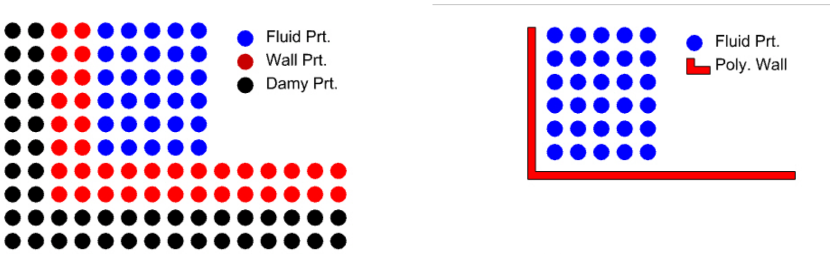
formulation. It can be adaptable for not only fluid dynamics but other problems by solving Navier-Stokes equations.

1.Discretization

In usual MPS method, space domain as fluid and wall boundary as solid are discretized in particles as indicated in Fig3 (a). For calculating pressure of wall boundary, with the same algorithm for fluid, it needs to be prepared as 3 or 4 layers of wall particles[2].It usually causes high calculation load and difficulties for expressions of thin wall. Thus, polygonal wall boundary[3] is used in this report indicated in Fig1 (b).



(a) Particle wall (b) Plygon wall

Figure 1 Discretization

2. The weight function and the particle number density

Particle interaction in MPS method uses the weight function that can be expressed as

$$w(|r_{ij}|) = \begin{cases} {r_e}/{|r_{ij}|} - 1 & (0 < |r_{ij}| < r_e) \\ 0 & (r_e < |r_{ij}|) \end{cases} \tag{1}$$

where $|r_{ij}|$ is the distance between particle *i* and *j*, r_e is effective radius.

The particle number density of particle *i* is calculated as

$$n_i = \sum_{j \neq i} w(|r_{ij}|) \tag{2}$$

To calculate average with weight for each interaction models, the particle number density of a particle which has enough neighbor particles in its effective radius is calculated before computing and it is labeled as n^0.

3. Wall weight function

For calculating the distance between a particle and polygons in a 2-dimentional space, Eq(3) is used.

$$|r_{iw}| = \frac{a_4(a_2 r_A + a_1 r_B) + a_3(a_2 r_C + a_1 r_D)}{(a_1 + a_2) + (a_3 + a_4)} \tag{3}$$

where $|r_{iw}|$ is the distance between particle *i* and neighbor wall polygon, r_A to r_D are distances from lattice points A to D to each closest polygons, respectively. a_1,a_2(a_3,a_4) are the internal division ratio of the particle *i* on an x direction (or y direction) lattice edges (figure 1). The normalized normal vector **n** of closest polygon from a particle is calculated as

$$\mathbf{n} = {\boldsymbol{a}}/{|\boldsymbol{a}|} \tag{4}$$

$$\mathbf{a} = \begin{pmatrix} (r_B + r_D - r_A - r_C) \big/ 2(a_1 + a_2) \\ (r_C + r_D - r_A - r_B) \big/ 2(a_3 + a_4) \end{pmatrix}$$

For using Eq(3) and Eq(4), the domain space is required to be divided into lattices. The distances between all lattice points and every polygon wall are calculated. Then a database of minimum distances from lattice points to the nearest polygon walls is established before the computation starts. During computing, r_A to r_D of Eq(3) and Eq(4) are calculated from this database.

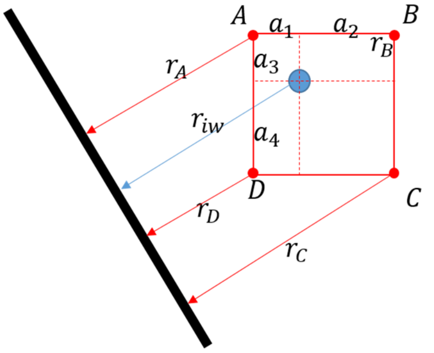


Figure 2 Parameters in Eq(3)

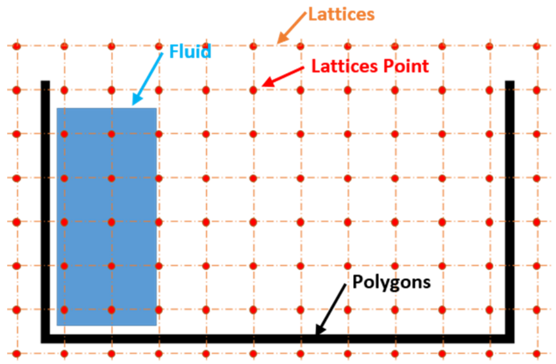
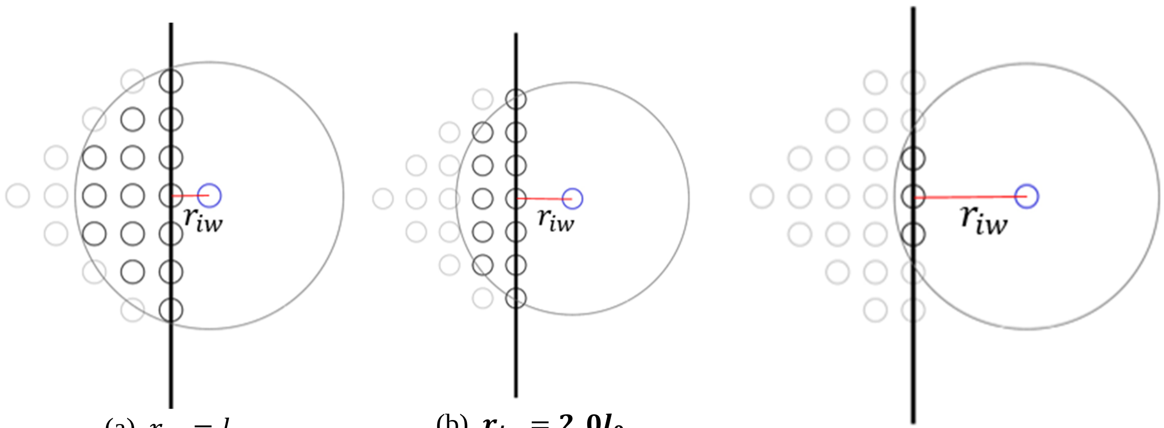


Figure 3 Domain space divided into lattices

The contribution of particle number density from wall (Z value) is calculated with this $|r_{iw}|$ using another database. To establish this database, Z value of some particles, which is put on discretized distances (as $0.1l_0, 0.2l_0, \cdots$) from a polygon, are calculated with pseud wall particles. An example of which effective radius is $4.5l_0$ is indicated in Fig3. During computation, the contribution of a particle number density of a particle is calculated with linear interpolation of this database.



(a) $r_{iw} = l_0$ (b) $\boldsymbol{r_{iw} = 2.0l_0}$ (c) $r_{iw} = 3.0l_0$

Figure 4 calcuration of Z value

4. Interaction model to solve Gradient

In polygon-wall-boundary approach, the contribution from fluid particles and wall boundaries are separately calculated as

$$\begin{gathered} \langle\nabla\phi\rangle_i = \langle\nabla\phi\rangle_{if} + \langle\nabla\phi\rangle_{iw} \\ \langle\nabla\phi\rangle_{if} = \frac{d}{n^0}\sum_{j\in\Omega_{fluid}}\left(\frac{(\phi_i - \phi_j)(\boldsymbol{r}_j - \boldsymbol{r}_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}\right) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \\ \langle\nabla\phi\rangle_{iw} = -\frac{\rho}{\Delta t^2}\frac{\boldsymbol{r}_{iw}}{|\boldsymbol{r}_{iw}|}(l_0 - |\boldsymbol{r}_{iw}|) \end{gathered} \tag{5}$$

where $\langle\nabla\phi\rangle_i$ is the gradient of scalar function ϕ at a point of particle *i*, $\langle\nabla\phi\rangle_{if}$ $and\langle\nabla\phi\rangle_{iw}$ are interactions from fluid and wall particles respectively, *d* is the dimension of the domain space (2 or 3), ϕ_i $and\phi_j$ are value of scalar function on particle *i and, j*, $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ are position vectors of particle *i and j*, respectively. ρ is density of a particle, Δt is increment of time and l_0 is distance between particles in the initial condition.

5. Interaction model to solve Laplacian

A Laplacian of a vector function $\boldsymbol{u}$ of particle *i* at a point is calculated as

$$\begin{gathered}\langle\nabla^2\boldsymbol{u}\rangle_i = \langle\nabla^2\boldsymbol{u}\rangle_{if} + \langle\nabla^2\boldsymbol{u}\rangle_{iw} \\ \langle\nabla^2\boldsymbol{u}\rangle_{if} = \frac{2d}{n^0\lambda}\sum_{j\in\Omega_{fluid}}(\boldsymbol{u}_j - \boldsymbol{u}_i)w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \\ \langle\nabla^2\boldsymbol{u}\rangle_{iw} = \frac{2d}{n^0\lambda}(\boldsymbol{u}_j - \boldsymbol{u}_i)Z(|r_{iw}|)\end{gathered} \tag{6}$$

where λ is a coefficient calculated for each particle, which has enough neighbor particles in its effective radius, given as

$$\lambda = \frac{\sum w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)} \tag{7}$$

5. Algorithm for the fluid analysis

The governing equations for incompressible flows, the mass conservation and Navier-Stokes equation, can be written as

$$\frac{D\rho}{Dt} = 0 \tag{8}$$

$$\frac{D\boldsymbol{v}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{v} + \boldsymbol{g} \tag{9}$$

Fig5 shows the flow chart used in this report. Firstly, initial conditions are defined. This includes (1) dividing the space domain for analysis into lattices and making database of minimum distances between each lattice point and polygon walls, (2) setting initial position of the fluid particles and (3) setting other parameters such as acceleration of gravity g, distance between particles l_0 and so on.

At each step, e.g. step k, velocity and position vector $\boldsymbol{u}^{\boldsymbol{k}}, \boldsymbol{r}^{\boldsymbol{k}}$ are updated using gravity and viscosity terms as

$$\begin{gathered}\boldsymbol{u}_i^* = \boldsymbol{u}_i^k + \Delta\mathrm{t}[\nu\langle\nabla^2\boldsymbol{u}\rangle_i + \boldsymbol{g}] \\ \boldsymbol{r}_i^* = \boldsymbol{r}_i^k + \Delta\mathrm{t}\boldsymbol{u}_i^*\end{gathered} \tag{10}$$

where $\langle\nabla^2\boldsymbol{u}\rangle_i$ is calculated with Eq(6). The subscript $\langle\ \ \rangle^*$ is for “temporary” because simply updated position of fluid particles by Eq(10) does not satisfy Eq(8). $\boldsymbol{u}_i^*$ and $\boldsymbol{r}_i^*$ should be modified with pressure gradient term to satisfy Eq(8). The pressure is calculated as

$$P_i^{k+1} = c^2\frac{\rho}{n^0}(n_i^* - n^0) \tag{11}$$

where P_i is the pressure at the point of particle *i*, *c* is the temporary sound velocity, 22m/s is used in this report, n_i^* is the particle number density calculated at the point of particle i with temporary particle positions. With this pressure, pressure gradient terms are calculated with Eq(5) and $\boldsymbol{u}_i^*$ and $\boldsymbol{r}_i^*$ are modified as

$$
\begin{aligned}
\boldsymbol{u}_i^{k+1} &= \boldsymbol{u}_i^* - \Delta \mathrm{t}\frac{1}{\rho}\langle \nabla \mathrm{P}\rangle_i \\
\boldsymbol{r}_i^{k+1} &= \boldsymbol{r}_i^* - \Delta t^2\frac{1}{\rho}\langle \nabla \mathrm{P}\rangle_i
\end{aligned}
\tag{12}
$$

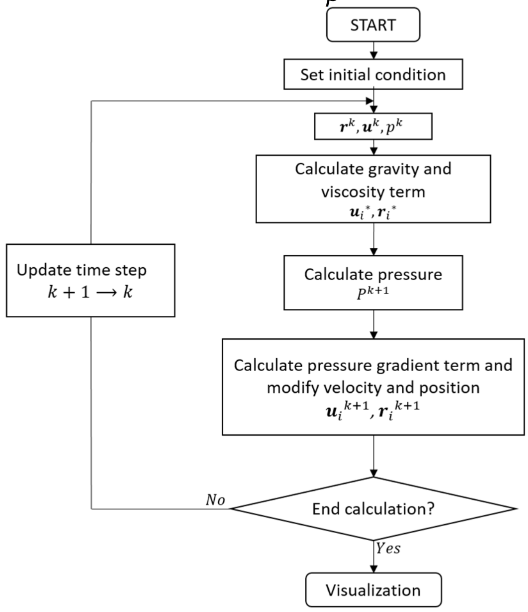
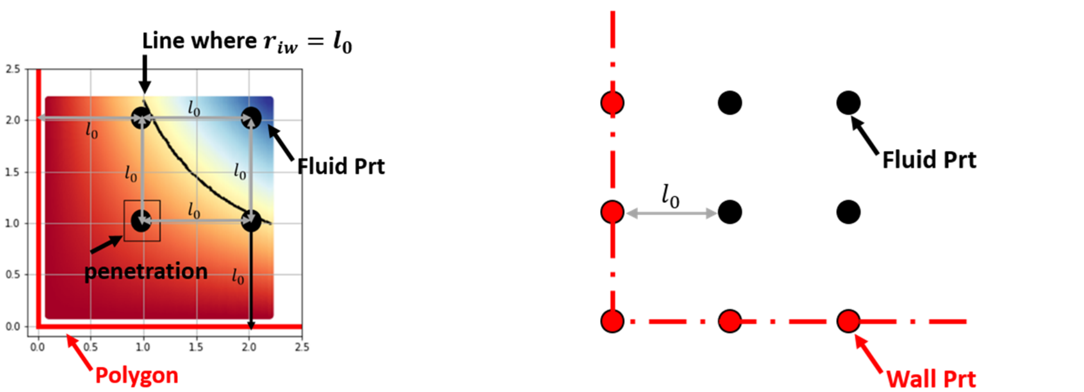


Figure 5 Flow chart of MPS analysis

6. Modeling feature in edge of Polygon wall

Modeling examples around wall edge with walls expressed as particles and polygons are shown in Fig6 (a) and (b) for each. Usually, both fluid particles and wall particles are put at an even interval l_0 for each as shown in Fig6 (a). In Fig6 (b), r_{iw} values calculated with Eq(3) of each domains are shown with contour color, and a line is put on the domains of which r_{iw} value is equal to l_0. The line where $r_{iw} = l_0$ become rounded by approximate calculation. Thus, if fluid particles are put on same positions at initial condition shown in Fig6 (a), r_{iw} value of fluid particle put on the edge is less than l_0, which means penetration of fluid particle at initial condition. This penetration causes high pressure at the position of penetrated particle and accelerates its neighbor particles at initial condition. Thus, this special phenomenon should be paid an attention in making a model around wall edge.



(b) condition by meaning of polygon modeling (b) condition by meaning of wall particle modeling

Figure 6 A feature of polygon wall modeling

RESULTS AND COMPARISONS OF CULCULATIONS

Dambreak problem is often used as a bench mark to verify the results obtained by newly proposed method through the comparison with well-examined results of conventional programs or experiments[2]. The Model of dambreak problem is shown in Fig7 and two types of an edge model are shown in Fig8. All fluid particles are put at even distance in model A as indicated in Fig8 (a), while a part of edge particles are removed in model B as indicated in Fig8 (b).

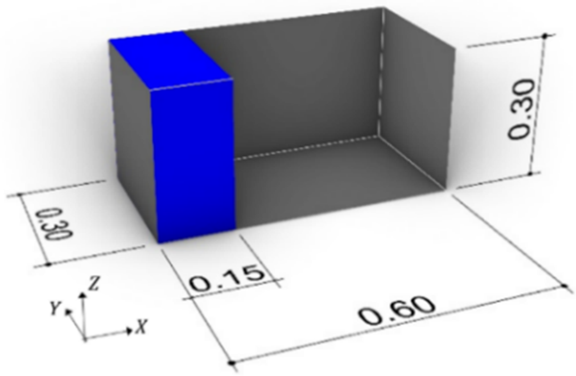


Figure 7 Dambreak model

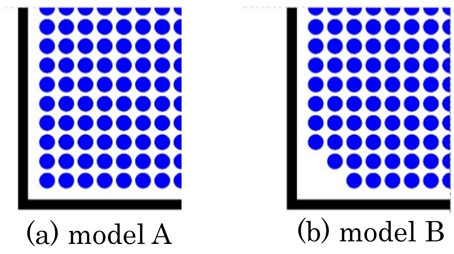


Figure 8 Modeling of edge

Pressure value of each particle at initial conditions of both model A and B are shown in Fig9 and the results of simulations are shown in Fig10. In the model A, very high pressure is generated by edge particles, thus, as shown in Fig10 (a), model A causes divergence of calculation. On the other hand, there are no particle with such a high pressure observed in the model B, and the calculation is successfully done as shown in Fig10 (b).

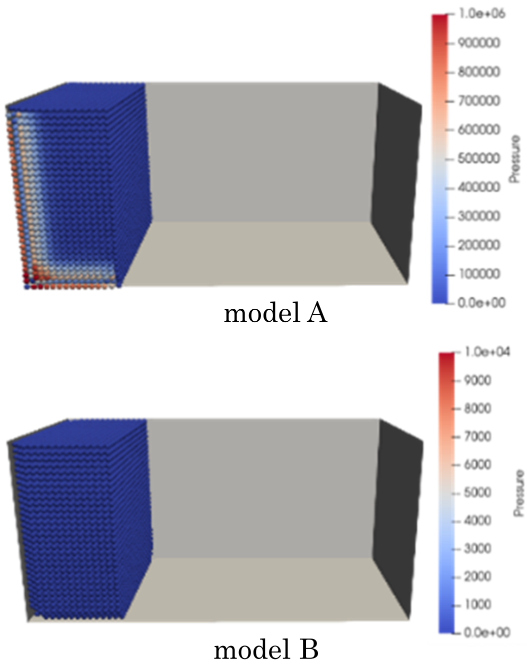


Figure 9 Pressure at initial condition

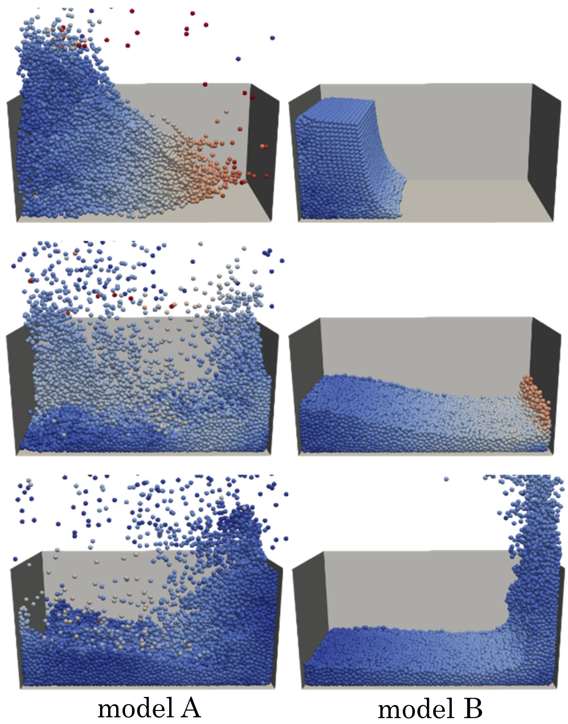


Figure 10 Results of Damberak model

As a bench mark problem, the time histories of the leading-edge, whose horizontal velocity is depends on its incompressibility, are compared. The time vs. leading-edge relationship of the program of this report with the result of Koshizuka's one[2] is shown in Fig11. The result of this report is indicated in orange line, which closely stays with Koshizuka's results and slightly closer to experimental results. Validity of efficient removal of edge particles, proposed here, is verified in this example.

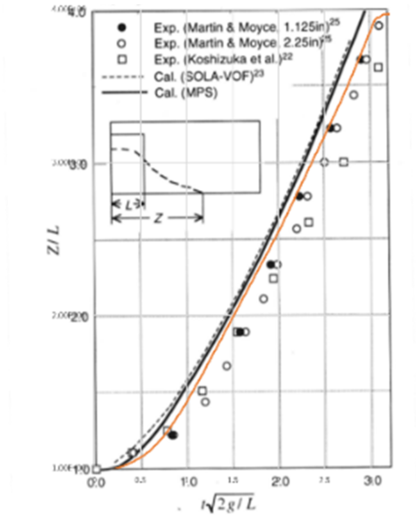


Figure 11 Time-Leading edge relationship[2]

CONCLUSIONS

As explained in the Section2, the space domain is discretized into moving particle, and intersection models for each particle about differential operators as gradient, divergence and Laplacian are prepared in MPS method. The theory to solve Navier-Stokes equation with these intersection models was also reviewed in Section 2.

In the numerical example in the Section3, a modeling of which edge particles are efficiently removed is proposed and verified by solving bench mark problem and being compared with other reasonable results.

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