# PRELIMINARY STUDY ON NUMERICAL ANALYSIS AND MEMBRANE THEORY OF CABLE NETS USED AS PREVENTION FOR COLLAPSE OF CEILINGS

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**ABSTRACT**: This paper reports the mechanism of cable nets used as prevention for collapse of ceilings. In this paper, the mechanism of cable nets is described by two methods. First is numerical analysis in hybrid formulation, and second is funicular net formulation derived by membrane theory. The comparison of the results obtained by two methods shows that the formula in this paper ensures the safe design of cable nets.

**Key Words**: *Ceilings, Cable nets, Funicular net formulation, Hybrid formulation*

## INTRODUCTION

This paper reports the mechanism of cable nets used as prevention for collapse of ceilings. A lot of investigations about collapse of ceilings have been reported, and most of them are in the case of earthquakes, and **Figure 1 (a)** shows an example. Seismic strengthening has usually been done in order to prevent these accidents, but cable nets system as shown in **Figure 2** is more effective solution. It has three merits as shown below.

1. Seismic strengthening needs work above the ceilings. This work has difficulty to be done and sometimes dangerous, but cable nets system can be installed with work only under the ceilings.
2. Cable nets system doesn't affect existing equipment.
3. Some collapses are due to leak of water, dew condensation and moisture. Ceiling boards are generally made of gypsum, so deteriorate easily with these factors, and fall out from the screws. **Figure 1 (b)** shows an example of this type of collapse. The problem is that seismic strengthening can't prevent these accidents, but cable nets system can do.

**(a)** Concert hall **(b)** Indoor poor

**Figure 1**. Collapse of ceilings

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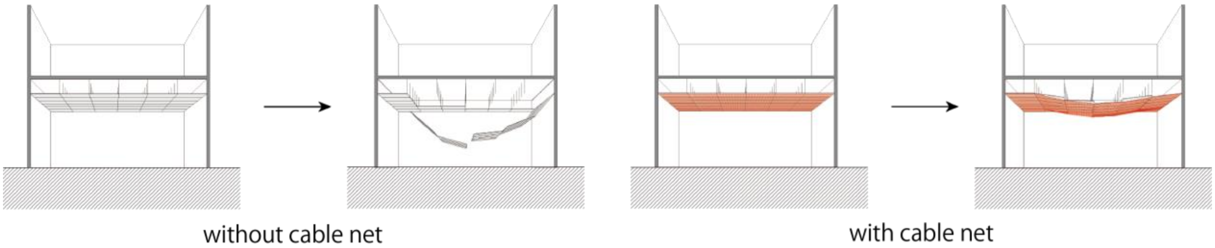


**Figure 2**. Cable nets used as prevention for collapse of ceilings

## NUMERICAL ANALYSIS IN HYBRID FORMULATION

Hybrid formulation is the method of numerical analysis of cable nets as link mechanism. The potential energy function of a link mechanism is derived by Hellinger-Reissner principle in the continuum mechanics, and expressed as Eq. (1). The unknowns of this function are nodal displacement $\{D\}$ and element force $\{p\}$.

$$\Pi = -\sum_{i=1}^{f} F_i \cdot D_i + \sum_{k=1}^{m} \left[ p_k \cdot d_k(D_i) - \sum_{k=1}^{m} \frac{L_k}{2E_k A_k} p_k^2 \right] \tag{1}$$

where $f$: nodal degree of freedom, $F_i$: nodal external force, $D_i$: nodal displacement, $m$: number of element, $p_k$: element force, $d_k(D_i)$: elongation of element, $A_k$: element cross section, $E_k$: Young's modulus, $L_k$: initial element length

From the stationary condition of this function, partial differentiations of this potential energy function with respect to the nodal displacement and element force equal to 0 ($\partial\Pi/D_i = 0, \partial\Pi/p_k = 0$), and it leads to nonlinear simultaneous equations. These equations are expressed as Eq. (2).

$$\begin{aligned}
f_1(D_1,\cdots,D_f,p_1,\cdots,p_k) &= \sum_{i=1}^{m} \left[ \frac{\partial d_i}{\partial D_1} p_i \right] - F_1 = 0 \\
&\vdots \\
f_f(D_1,\cdots,D_f,p_1,\cdots,p_k) &= \sum_{i=1}^{m} \left[ \frac{\partial d_i}{\partial D_f} p_i \right] - F_f = 0 \\
f_{f+1}(D_1,\cdots,D_f,p_1,\cdots,p_k) &= d_1(D_i) - \frac{L_1}{E_1 A_1} p_1 = 0 \\
&\vdots \\
f_{f+m}(D_1,\cdots,D_f,p_1,\cdots,p_k) &= d_m(D_i) - \frac{L_m}{E_m A_m} p_m = 0
\end{aligned} \tag{2}$$

In this paper, these equations are solved by means of Newton-Raphson method, and nodal displacement $\{D\}$ and element force $\{p\}$ can be obtained.

## FUNICULAR NET FORMULATION DERIVED BY MEMBRANE THEORY

Funicular net formulation is the approximation of rectangle uniform tension membrane with deadweight derived by membrane theory, and can be expressed as Eq. (3). **Figure 3** shows the shape of funicular net ( $2a$ =20, $2b$=10, $w(0,0)$=1).

$$w(x,y) = \frac{16 q_n a^2}{(N - N_{ini}) \pi^3} \sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{(n-1)/2}}{n^3} \left[ 1 - \frac{\cosh(n\pi y/2a)}{\cosh(n\pi b/2a)} \right] \cos\frac{n\pi x}{2a} \qquad (3)$$

where $w(x,y)$: sag, $q_n$: vertical distributed force, $N$: tension per unit width, $N_{ini}$: initial tension per unit width, $2a$: length of the long side of the rectangle, $2b$: length of the short side of the rectangle

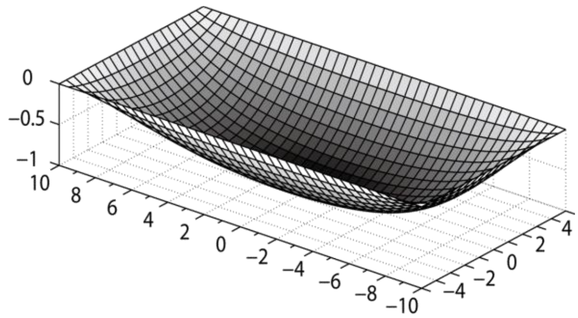


**Figure 3**. Shape of funicular net ( $2a$ =20, $2b$=10, $w(0,0)$=1)

In the case that the form of the net is a square, Eq. (3) lead to Eq. (4).

$$N = 0.3\frac{pa^2}{w} \qquad (4)$$

where $N$: tension per unit width, $2a$: length of the each side of the square, $p$: vertical distributed force, $w$: sag of the middle point

Element force can be calculated from multiplying tension per unit width by length of the each side of the square, and dividing it by number of element. In this paper, Eq. (5) which takes in a safety factor is considered.

$$N = 0.5\frac{pa^2}{w} \qquad (5)$$

## COMPARISON HYBRID FORMULATION WITH FUNICULAR NET FORMULATION

This section reports comparison hybrid formulation with funicular net formulation. The subject of this comparison is the maximum element force. **Table 1** shows parameters, and these parameters are defined with an assumption that the net used in this system is polyethylene net whose mesh is 100mm and the ceiling is gypsum board including substrate material (154N/㎡). **Figure 4** shows the method of comparison in the case that the form of the net is a rectangle. Eq. (4) and Eq. (5) can be used only in the case that the form of the net is a square, so in the case of the rectangle, the aspect ratio of the form in hybrid formulation varies with the plane area being kept constant, and the aspect ratio in funicular net formulation is fixed to 1:1 with the same plane area. The calculation in hybrid formulation contains the assumption that the nodes on the edge of the net are pines, and the sag used in Eq. (4) and Eq. (5) ( $w$) refers to the calculation in hybrid formulation. **Table 2** shows the sag used in the case that the form of the net is a square.

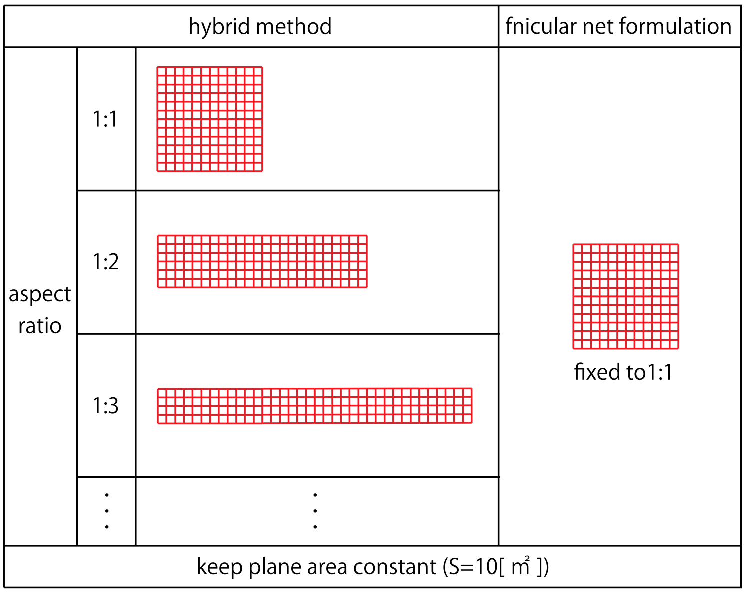


**Figure 4**. The method of comparison in the case that the form of the net is a rectangle

**Table 1**. Parameters of the calculation

| mesh | vertical distributed force | diameter of the net | Young's modulus |
|---|---|---|---|
| 100[mm] | 154[N/㎡] | 2.4[mm] | 5635[N/㎟] |

**Table 2**. Sag used in the case that the form of the net is a square

| length of the each side of the square[m] | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|
| sag[cm] | 1.1 | 2.8 | 4.6 | 6.7 | 9.0 | 11.5 |

**Figure 5** shows the result of square, and **Figure 6** shows the result of rectangle. Maximum element force of Eq. (4) always fall under that of hybrid formulation, but Eq. (5) always exceed hybrid formulation both in the result of square and rectangle. Furthermore, it can be seen from **Figure 6** that maximum element force of short side in hybrid formulation always exceed that of long side.

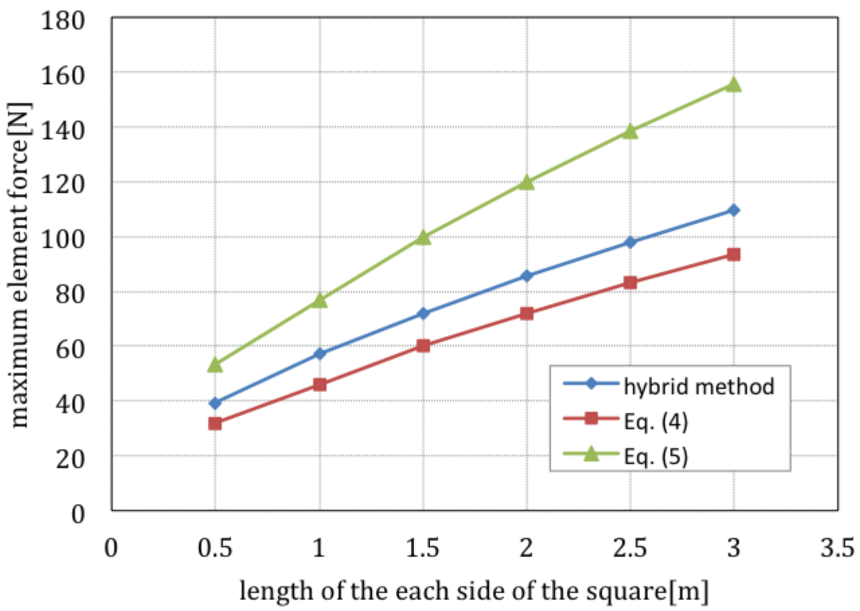


**Figure 5**. The result of comparison in the case that the form of the net is a square

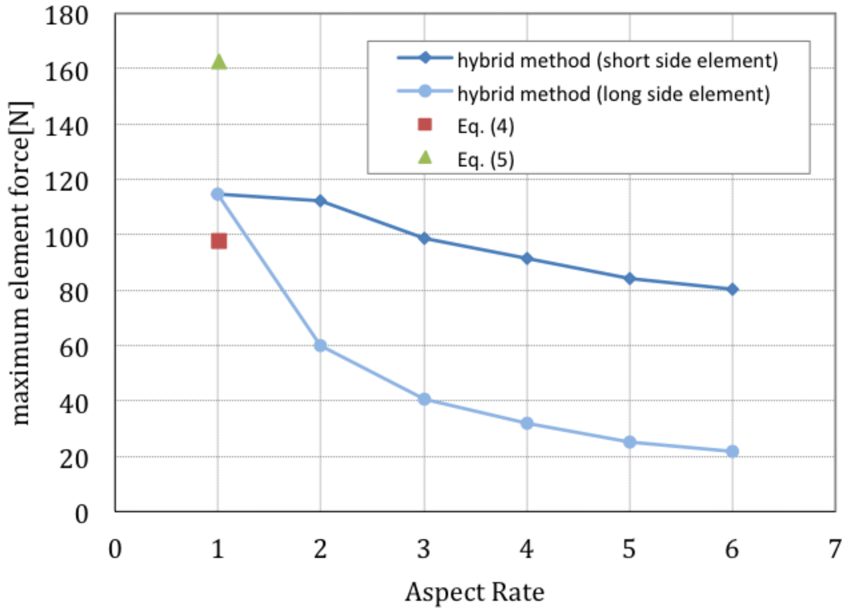


**Figure 6**. The result of comparison in the case that the form of the net is a rectangle

## CONCLUSIONS

This paper reports about cable nets used as prevention for collapse of ceilings. In introduction, the merits of cable net are mentioned, and next two sections are about the mechanism of cable nets. The former is the numerical analysis in hybrid formulation, and the latter is funicular net formulation derived by membrane theory. Finally, maximum element forces calculated by both methods are compared, and it shows that the maximum element force of Eq. (5) always exceed that of Hybrid formulation. This result ensures the safe design of cable nets by using Eq. (5).


## ACKNOWLEDGMENT

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## REFERENCES


Architectual Institute of Japan. (2015.1) *Guidelines for safety measures against accidental fall of ceilings and other non-structural components*, Architectual Institute of Japan, Tokyo, Japan.

Ozawa, Y. and Kawaguchi, K. (2010.3) “Fundamental study of shapes and mechanism of safety nets for the failure of non-structural components.” *Journal of Structural Engineering*, Vol. 56B, 517-520.

Magara, E., Kunita, J. and Kawamata, S. (1974.4) “Analysis of cable nets in mixed formulation: Part1: Property of instable frameworks and analysis of link mechanisms.” *Journal of Architectual Institute of Japan*, Vol. 218, 37-48.

Magara, E., Kunita, J. and Kawamata, S. (1974.6) “Analysis of cable nets in mixed formulation: Part2: Rigoros solution of the geometrically nonlinear problems.” *Journal of Architectual Institute of Japan*, Vol. 220, 35-45.