ULTIMATE LATERAL RESISTANCE FOR CLOSED-SPACED GROUPED PILES BASED ON ACTIVE PILE LENGTH

Mary Roxanne AGLIPAY[1], Kazuo KONAGAI[2] and Takashi KIYOTA[3]

ABSTRACT: Active pile length, L_a, is the effective length along a long and flexible pile that undergoes significant lateral deformation. This is characterized by the relative stiffness of the pile to stiffness of the soil. Considering the soil-pile interaction mechanisms of a foundation system, this parameter can be related to the mobilization of the soil in the passive region as pile deforms due to lateral loads especially during occurrence of non-linear scenario. Hence, this active pile length can be a key parameter in developing solutions for laterally loaded pile which is deemed useful in dealing with more complex systems i.e. closed-spaced grouped piles commonly used in engineering practice. A simplified method based on the active pile length in determining the ultimate lateral pile resistance of closed-spaced grouped piles embedded in sand is presented in this paper for a more practical approach in the structural and seismic design and assessment of such foundation system.

Key Words: Active pile length, grouped piles, ultimate lateral pile resistance, soil-pile interaction, equivalent single pile

INTRODUCTION

Deep foundations are normally used to support important structures built in weak soils. The loads are transferred from these structures to deep and stronger stratum through piles. In common engineering practice, the piles used are often in groups. These grouped piles are susceptible to external lateral loads such as seismic loads. The lateral resistance of piles in response to these demand loads is generally governed by the soil-pile interaction. This is for the reason that the movement of the grouped piles is dependent on the movement of their side soils. Hence, the deformation of the side soils is relative to the pile and conversely, the deformation of the pile is relative to that of their side soils.

The deformation of laterally loaded piles that are long and flexible do not occur completely over their entire length but is significant in the upper region near the ground surface (Konagai 2003). This deformation diminishes along the pile as the level reaches greater depths and is at zero at the toe of the pile. The pile is considered to be active only at the portion of significant deformation, thus the term “active pile length”, L_a. In this region, the pile behaves effectively as a cantilever beam with fixity set at the negligible deformation. The cut-off points describing the negligible deformation have been set by Wang and Liao (1987) and Velez (1983) at 0.3% and 5% of pile head displacement, respectively. In this study, similar with Konagai (2003), the negligible deformation is defined to be at the level where the lateral deformation is 3% of the maximum pile head deformation. This parameter is considered to be reflective of the soil-pile interaction as this is characterized by the ratio of the pile stiffness to the surrounding soil stiffness.

[1] Graduate Student, Institute of Industrial Science, University of Tokyo
[2] Professor, Graduate School of Urban Innovation, Yokohama National University
[3] Associate Professor, Institute of Industrial Science, University of Tokyo

In the event of nonlinear scenario like occurrences of large seismic excitations, the soil in the passive region is mobilized, where a wedge is eventually formed and pushed up along this active pile length. The side soil resistance is represented by the soil wedge (Aglipay, 2016). Hence, the active pile length can be related to the ultimate lateral pile resistance.

Advanced technology have paved the way to high computing powers facilitating researches on soil-pile interactions with complex soil-pile configuration (Elgamal et. al., 2009; Lu et al., 2006; Wang et al., 2014, etc.). However, there is the demand from practicing engineers for simple and fast solution notwithstanding the need for reliability, especially when dealing projects that need immediate attention. Therefore, a simplified expression using L_a as a key parameter to describe the ultimate lateral resistance of closed-spaced grouped pile embedded in sand is presented for more practical approach in the seismic design and assessment of piles.

CLOSED-SPACED GROUPED PILES

Piles used as deep foundations are often in groups. This study focuses on the closed-spaced grouped piles in which it can be treated as equivalent single pile. According to Bogard and Matlock (1983), the stress formation and deformation around the piles within the group is directly influenced by the spacing in between or among piles. When pile groups are induced with lateral loads, normal and shear stresses and strains are generated in the passive region and diminishes radially outward the pile vicinity. Because of the close space in between and among piles, an overlapping happens before the stresses and strains can completely diminish out. A development of plastic zones happen around the piles within the group, thus, the stronger effect among piles that allows them to act as a unit (see **Figure 1**). In terms of the spacing-to-diameter ratio, s/d_p, closed-spaced grouped piles are defined as s/d_p<20 based on the study of Konagai (2003) comparing the static pile head stiffness of rigorous solution of grouped piles and treating the grouped pile as equivalent single piles. Therefore, the s/d_p considered in this study are 1.5, 2.5 and 4.5 to ensure a closed-grouped pile system.

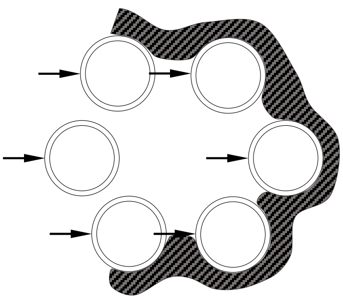

Figure 1. Schematic illustration of the patterns of stress and deformation around laterally loaded grouped piles around laterally loaded grouped

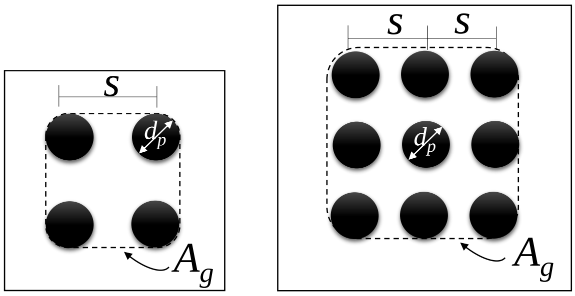

Figure 2. Equivalent single beam analogy idealization

The idealization for the equivalent single beam analogy for grouped piles consisting of the composite number of piles, n_p, and the soil entrapped among these piles as illustrated in **Figure 2**. Given this idealization, equivalent single beam parameters such as the cross-sectional area, A_g, and the grouped pile stiffness, EI_g, are defined by Equation (1) and (2) respectively.

$$A_g = \pi R_0^{\ 2} \tag{1}$$

$$EI_g = n_p EI_p \tag{2}$$

The broken lines in **Figure 2** circumscribing the outermost piles in the group determines its cross section, A_g. This cross-sectional area is a square with the sides equal to the length running until the edges of the outermost piles. From this cross-sectional area, the equivalent radius, R_0 is derived.

The stiffness of the grouped piles, EI_g, is defined by the product of the number of piles, n_p, and the stiffness of the individual piles, EI_p with the assumption that pile elements within a horizontal slice of soil deforms but keep their spacing constant and the entrapped soil moves with them. It is noted that to consider the entire cross-sectional area in calculating the bending stiffness of the grouped pile would mean an overestimation of the stiffness of the soil entrapped in the pile.

These parameters for the equivalent single beam analogy are used in the simplified expression for the closed-spaced piles based on the analysis from the results of rigorous solution using the finite element method (FEM).

NUMERICAL ANALYSIS

The simulation of the response of laterally loaded closed grouped piles in three-dimension (3D) were performed using the ABAQUS v6.13. The additional complexity in the analysis of laterally loaded piles as they come in group is easily handled by the ABAQUS v6.13, a commercial finite element analysis (FEA) software (Dassault Systemes Simulia, 2013a). The soil-pile system includes a closed-spaced end bearing pile embedded in a homogeneous sandy soil (considered as Toyoura sand) subjected to a lateral load. In this soil-pile system, the elasto-plastic behavior of the soil is modeled using hypoplastic model of von Wolffersdorff (1996) while the piles are modeled with elastic case. The following sections provide the description of the geometrical configuration of the soil-pile system and discussions on the models used for the pile and the soil in the system.

The results from this rigorous solution are used and analyzed to arrive at a simplified method in determining the ultimate lateral resistance of closed-spaced grouped piles in sands using the active pile length, L_a, as the key parameter.

Soil-pile system configuration

The programs based on FEM can rigorously model any soil-pile configurations. However, the computing time and memory requirement also increases with complexity. Thus, only the half mesh of the soil-pile system is modelled in view of the symmetry and non-uniform meshing is implemented (**Figure 3** and **Figure 4**). This soil-pile system is modelled with 3D solid deformable body. The maroon elements represent the soil medium, while the green elements represent the pile. The soil models for the 2x2 pile and 3x3 grouped piles are dimensioned as 0.70mx0.30m and 1.1mx0.45m respectively. The depth of the soil medium is 1.45m while the actual length of the pile, L_p, is 1.5m.

The boundary planes in the soil-pile system are designated as follows: (1) bottom (XY plane), (2) side (ZY plane), (3) back (ZX plane) and (4) plane of symmetry. The bottom of the soil medium is considered as a hard stratum and the pile as an end bearing type. Thus, the bottom surface of the soil and the pile is considered fixed, where it is restrained at all degrees of freedom. The sides of the soil medium is restrained at the x-axis while the back is restrained at the y-axis. Lastly, the plane of symmetry is enforced with symmetric boundary conditions, where the translations are restrained at the y-axis and rotations at z and x-axes. Slipping and gapping are implemented in the model with the assignment of the contact surfaces of piles and soil with the models inherent in the ABAQUS. The angle of internal friction of the joint element is 25^o (Wakai, 1999)

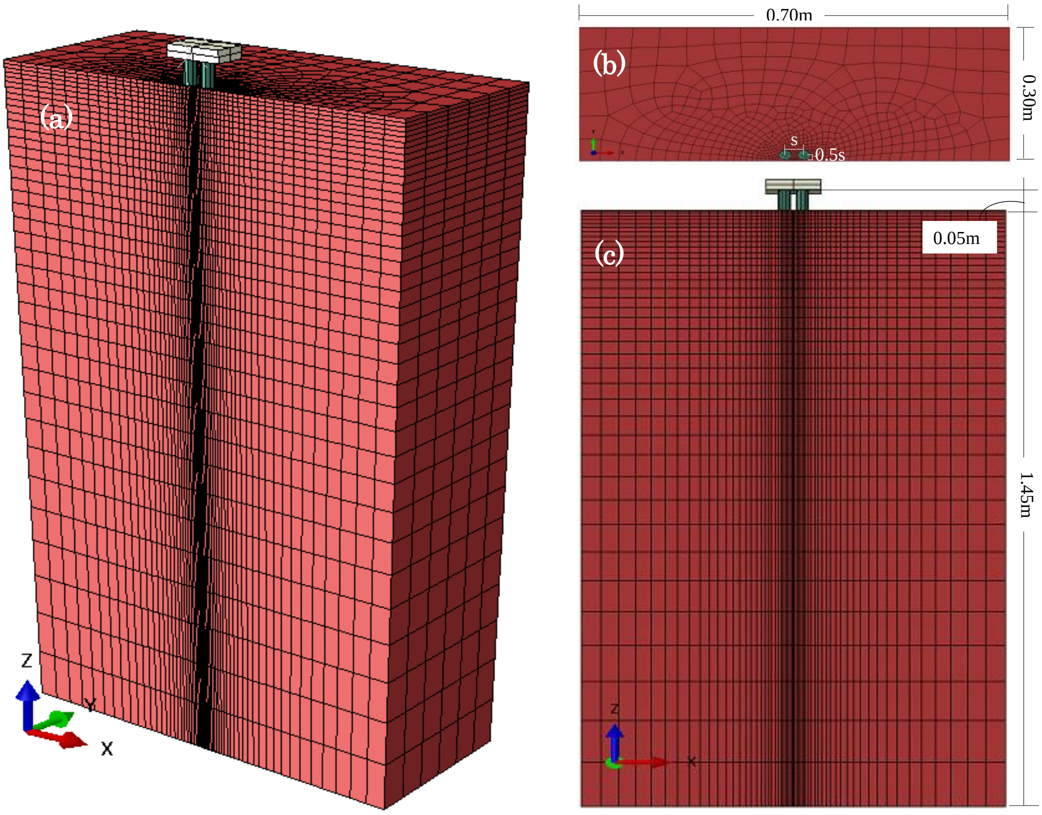

Figure 3. Soil-pile configuration for 2x2 grouped piles. (a) 3D Perspective View, (b) Plan View. Note: Pile cap not shown and (c) Cross-sectional view.

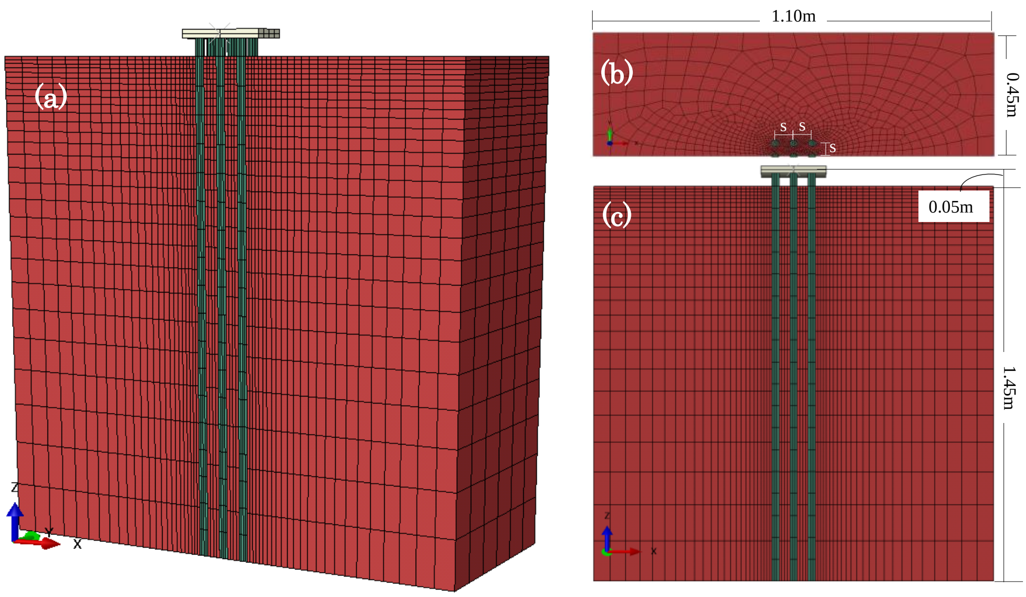

Figure 4. Soil-pile configuration for 3x3 grouped piles. (a) 3D Perspective View, (b) Plan View. Note: Pile cap not shown and (c) Cross-sectional view.

Pile modeling

The grouped pile is modelled considering a fixed head condition. A 20-node quadratic brick element is used for these piles. In this study, the piles considered are in elastic material which is defined by the following parameters: (1) Young's modulus, E_p and (2) Poisson's ratio, v.

Soil modeling

In the soil-pile system, a homogeneous Toyoura sand is considered as the soil medium. A user-defined constitutive model is implemented in the Abaqus v6.13 to model the mechanical behaviour of the granular soil, particularly of Toyoura sand. This model is based on the Abaqus UMAT (User Material) (Dassault Systemes Simulia, 2013b) code from the soilmodels.info (Gudehus et al., 2008) with minor code alteration to be installed and run with the FEA program. The code is based on formulation of the basic model of hypo-plasticity model for granular materials (von Wolffersdorff, 1996) and small-strain extension (Niemunis and Herle, 1997) suitable for cyclic loading cases. In this study, only the basic model is utilized.

This model is rooted from the elasto-plasticity theory models of the hypoplastic Drucker-Prager model (Drucker and Prager, 1952) with implementation of the yield criterion of the Matsuoka-Nakai failure surface (Matsuoka and Nakai, 1977). Detailed formulation can be found in the paper of von Wolffersdorff (1996).

In summary, there are eight parameters required for the basic hypoplastic model (**Table 1)**. Herle and Gudehus (1999) have performed laboratory tests for various types of dry clean sand material to derive these parameters. The parameters for Toyoura sand are re-calibrated and compared with conventional drained compression triaxial test. The soil parameters for Toyoura sand summarized in **Table 1** are used:

Table 1. Soil parameters of Toyoura sand

Angle of internal friction at critical state, φ_c	30
Granular stiffness, $h_s[GPa]$	2.6
Exponential material constant, n	0.35
Reference minimum characteristic void ratio, e_{d0}	0.61
Reference characteristic void ratio at critical state, e_{c0}	0.98
Reference maximum characteristic void ratio, e_{i0}	1.1
Parameter for controlling peak friction angle based on relative density, α_h	0.18
Parameter for controlling dependence of stiffness on the relative density, β_h	1.1

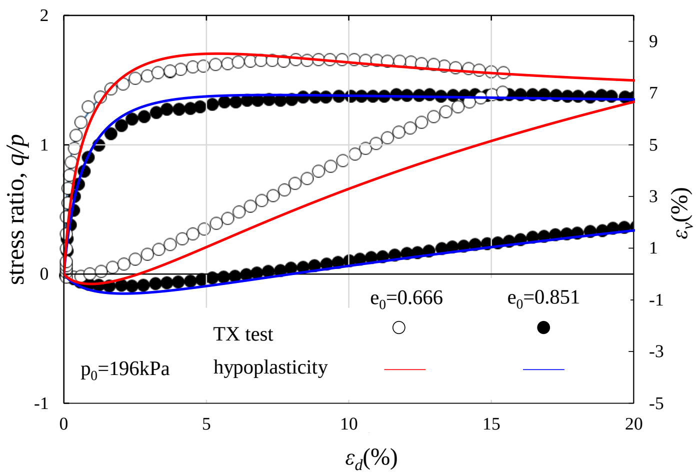

Figure 5. Comparison between experimental and numerical result for the stress-strain relationship of Toyoura sand (TX test results after Kyokawa, 2011)

Summary of cases considered

A number of static pushover tests for single end bearing pile embedded in a homogeneous Toyoura sand were simulated in this study. The static pushover test was conducted using a displacement control at pile head. A lateral displacement is applied at the pile head until it reaches the final load of 0.5m. A total of 10 cases are considered for the closed-spaced grouped piles. **Table 2** summarizes the different geometric configuration and material properties of the 2x2 and 3x3 grouped piles used in the simulation study. Corresponding equivalent single beam parameters such as R_0 and EI_g that will be used in the post-analysis of rigorous results are included. These piles are embedded in sands having initial void ratios, e_0=0.73, 0.80 and 0.90.

Table 2. Soil parameters of Toyoura sand

Grouped Pile	s/d_p	R_0 (mm)	EI_g(x10^9 mm^4)	e_0
2x2	1.5	28.21	2.16	0.73
2x2	2.5	39.49	2.16	0.73
2x2	4.5	62.06	2.16	0.73
3x3	2.5	67.70	4.85	0.73
2x2	1.5	28.21	2.16	0.80
2x2	2.5	39.49	2.16	0.80
3x3	2.5	67.70	4.85	0.80
2x2	1.5	28.21	2.16	0.90
2x2	2.5	39.49	2.16	0.90
3x3	2.5	67.70	4.85	0.90

ACTIVE PILE LENGTH

The 3% of maximum pile head lateral displacement, u_y, definition is implemented in the analysis of results of the pile deformation to obtain the active pile length. The active pile length at the passive edge of the lead piles is of interest since it directs the formation of the soil wedge (see **Figure 6**).

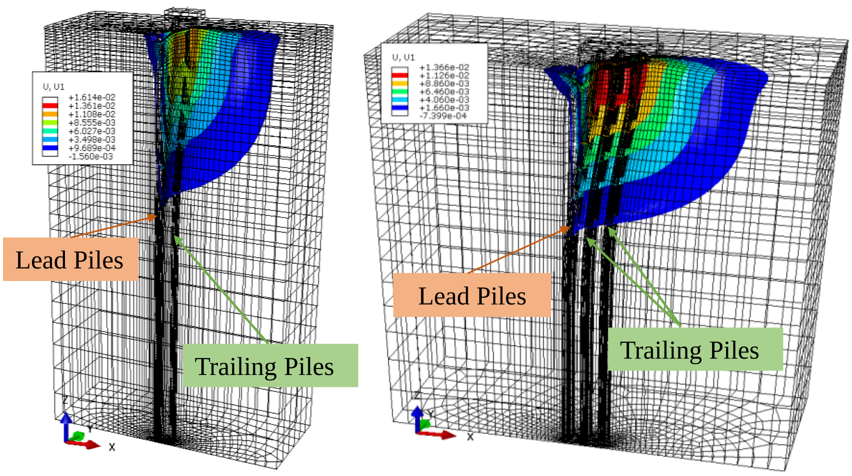

Figure 6. Soil lateral displacement (U1) distribution at the passive region for 2x2 and 3x3 grouped piles

Because of the elasto-plastic nature of the surrounding soil, the active pile length actually increases with increasing pile head displacement then becomes constant when large displacements are reached. In these progressive formation of the active pile length, two stages are highlighted that are necessary in the determination of the ultimate lateral pile resistance: the initial stage and the ultimate stage. These are discussed in the following sections.

Characteristic length, L_c

The characteristic length is the ratio of the relative stiffness of the piles to the surrounding soil stiffness. This is expressed by Konagai formula given in Equation (3), where a more rational representative of the soil stiffness is taken into account in the presence of the shear modulus.

$$L_c = \sqrt[4]{\frac{EI_g}{G_{\max}}} \tag{3}$$

The stiffness of the grouped pile treating the closed grouped pile as equivalent single pile is given by Equation (2). For the G_{max}, this can be easily derived in the site through the PS logging or other similar methods. Considering the small strain stiffness of the Toyoura sand that was used in the model, the empirical formula (Gu et al., 2013) is given in Equation (4) which is fitted based on the series of tests on Toyoura sand using resonant column (RC) apparatus with a torsional shear function and installed with bender elements.

$$G_{\max} = 95.5(\mathrm{MPa})(\sigma'/P_a)^{0.41}\ ((2.17 - e_0)^2/(1 + e_0)) \tag{4}$$

where σ': the effective vertical stress, P_a=reference atmospheric pressure, 98kPa and e_0: initial void ratio of the sand. Similar trend can be seen with the discussion of Archer and Heymann (2015) plotting the shear stiffness versus depth for different relative densities of sand.

Initial stage, L_o

The characteristic length from Equation (3) is derived and plotted in the x-axis as seen in **Figure 7**. There exists a linear relationship between the active pile length at the initial stage, L_0 and the characteristic length, L_c. The proportional factor is equal to 6.16. This is described by Equation (5) below:

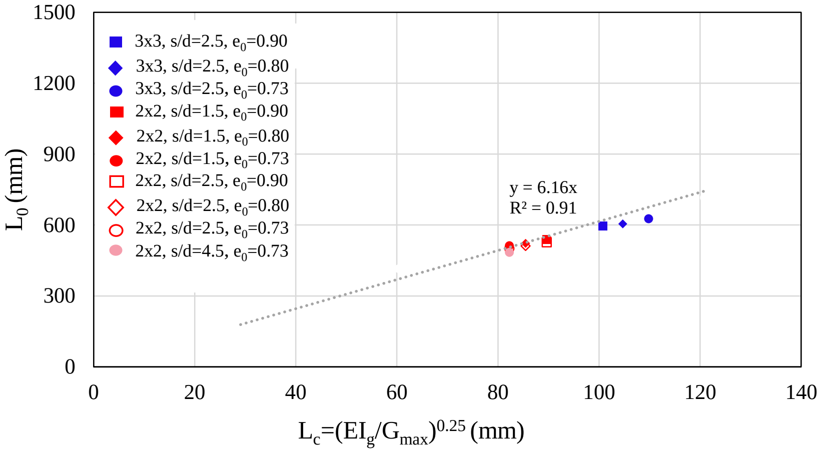

Figure 7. Relationship of L_c and L_0 using G=G_{max}

$$L_0 = 6.16L_c \tag{5}$$

Therefore, with the given pile stiffness and the shear modulus, the initial active pile length can be easily determined. The active pile length formed at the initial stage is crucial in order to describe the active pile length at the progressive stage due to increase in pile head deformations, and more importantly at the ultimate stage.

Ultimate stage, L_{au}

The progressive active pile length is normalized with the initial active pile length, L_a/L_0, knowing that the active pile length at the initial stage can be a determining factor in describing the active pile length at the ultimate stage. The average shear strain is derived by normalizing the pile head displacement with its corresponding active pile length, u_y/L_a. These parameters, L_a/L_0 and u_y/L_a for various cases in each soil type, e_0=0.73, e_0=0.80 and e_0=0.90 are plotted in the y- and x-axes respectively as seen in **Figure 8, 9** and **10**.

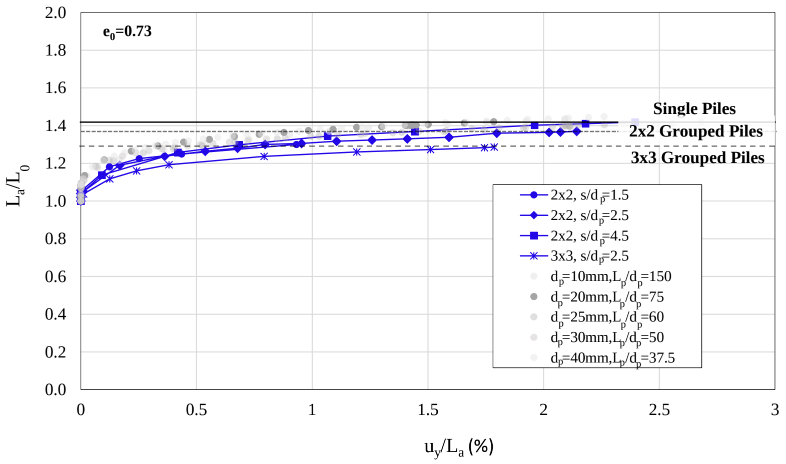

Figure 8. Relationship of L_a/L_0 with average shear strain for grouped piles embedded in soil with e_0=0.73

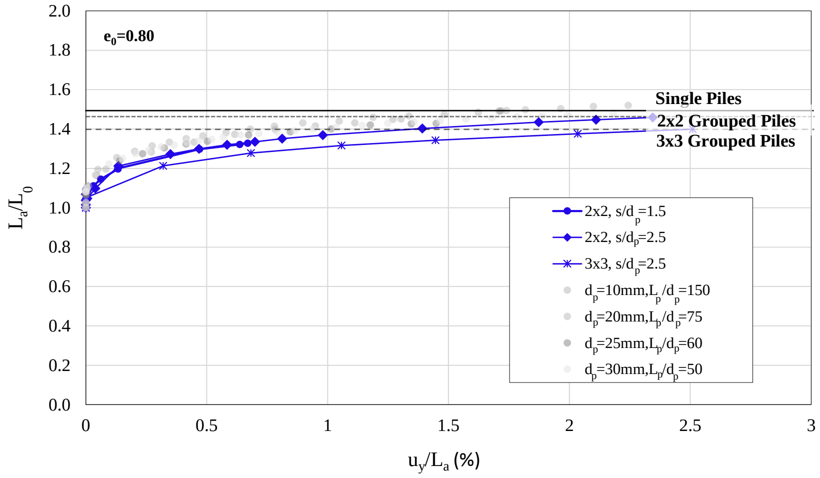

Figure 9. Relationship of L_a/L_0 with average shear strain for grouped piles embedded in soil with e_0=0.80

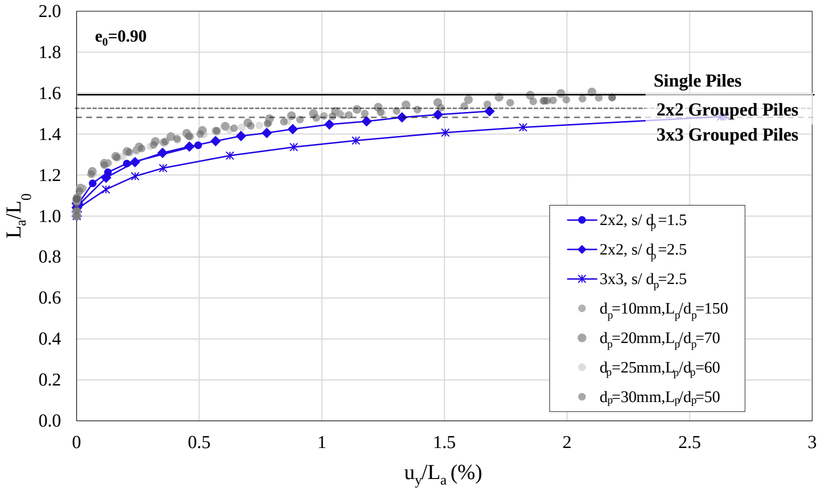

Figure 10. Relationship of L_a/L_0 with average shear strain for grouped piles embedded in soil with e_0=0.90

Generally, the plots come close to the trend of that of the single fixed head pile and the active pile length at the ultimate stage is reached when the average shear strain is at 2%.

For the same grouped pile configuration such as in the case of 2x2 grouped pile, the data points lie in a unique curve despite variations of s/d_p. However, it can be noted that the L_a/L_0 ratio decreases with the increase of number of piles in a group. The L_a/L_0 values for the grouped piles are normalized with that of the single piles to see the departure from the single piles. This is summarized in **Table 3** in accordance with the number of piles in a grouped pile. This is observed to be constant regardless of soil type. It can be seen that a reduction factor of 0.96 and 0.92 is applied to the L_a/L_0 of the single piles. Therefore, the difference from the relationship established with that of the single pile is just 4% and 8% for the 2x2 and 3x3 grouped pile respectively, which is practically small.

Table 3. L_a/L_0 values normalized with L_a/L_0 (single piles)

No. of Piles	Reduction Factor for Grouped Piles at the Ultimate Stage (x L_a/L_0)$_{\text{single piles}}$
1	1.00
4	0.96
9	0.92

Hence, the equation to derive the active pile length at the ultimate stage is to be similar to that of the single piles, to which derivation can be seen in the work of Aglipay (2016). This is dependent on the relative density of the surrounding soil given by Equation (6)

$$L_{au} = L_0\,(1.05e_0 + 0.655) \tag{6}$$

The term ($1.05e_0$+0.655) multiplied on the active pile length at the initial stage captures the elasto-plastic behavior of the surrounding soil, where there is degradation of the surrounding soil stiffness. Hence, the active pile length at the ultimate stage, L_{au}, can be determined by application of such correction factor. This correction factor must be applied with care especially to the cases where larger number of piles are used since it is still unknown if the departure from that of the single pile is also comparatively small.

APPLICATION OF L_a FOR THE ULTIMATE LATERAL PILE RESISTANCE

The numerical simulations show that when the lateral load is applied, a soil wedge is progressively formed at the passive region. The overall behaviour of the response of the grouped pile with the application pile head loading is described with the black line seen in **Figure 11**. The sole pile resistance of grouped pile is based on simulation of the grouped piles without the surrounding soil and with length equal to the active pile length at the ultimate stage. The pile resistance is plotted in **Figure 11** represented by the blue line. Then the side soil reaction is derived as the difference of overall behaviour and the pile resistance. The constant line that appears at the larger displacement for the side soil reaction curve is the ultimate side soil reaction or the ultimate lateral pile resistance for all cases of closed grouped piles.

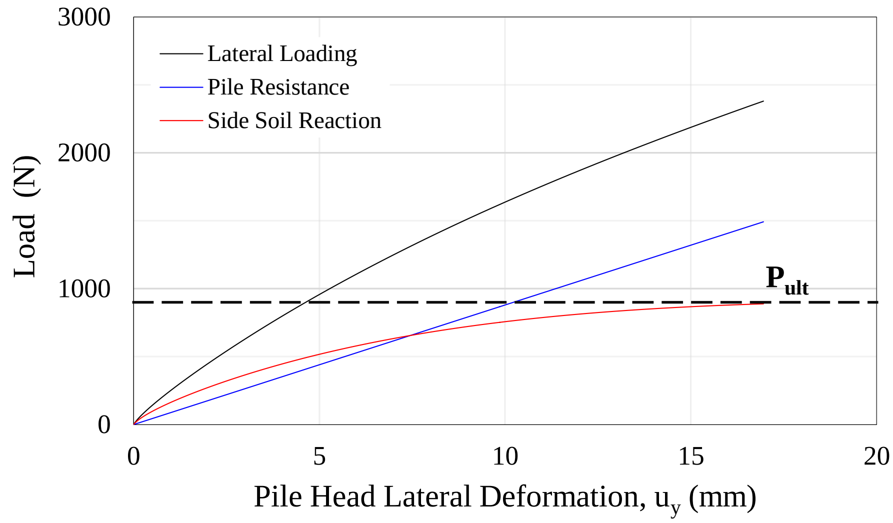

Figure 11. Load deformation curves for grouped piles

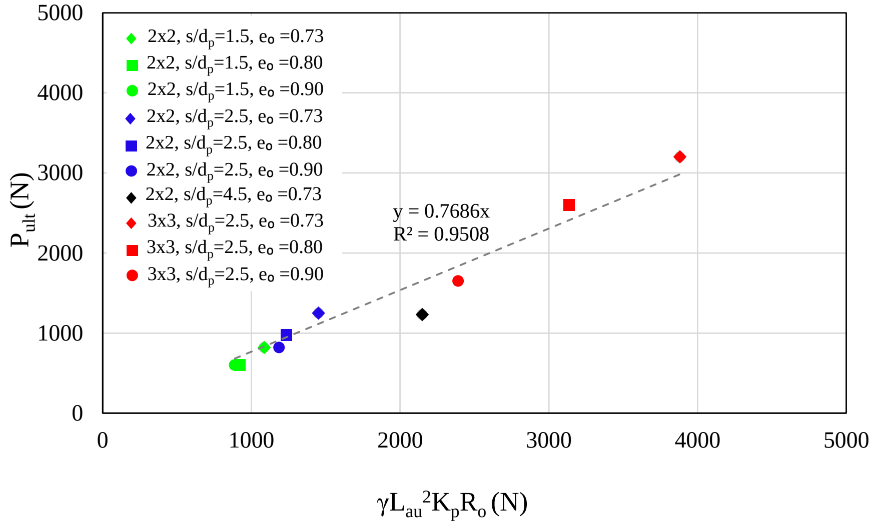

Figure 12. Relationship of ultimate lateral pile resistance with active pile length and other soil parameters

The wedge formed at the passive region is indicative of the side soil reaction. The force representation of the soil wedge can be defined by the weight of the volume of the extent of this soil wedge. The extent of this soil wedge is represented by the following parameters: the active pile length, L_{au}, Rankine passive earth coefficient, K_p. and the outer pile radius, R_0 and multiplied with the unit weight, γ. The list of the values of the soil parameters used is summarized in **Table 4**.

Table 4. Soil parameters of Toyoura sand (TS)

Initial void ratio, e_o	K_p	γ (kN/m^3)
0.73	4.81	14.90
0.80	3.69	14.44
0.90	3.10	13.68

These simple parameters are plotted in the x-axis with the corresponding ultimate lateral pile resistance for all the cases as shown in **Figure 12**. It can be seen that there is a linear relationship with high correlation between these terms. Thus, a simplified expression can describe the ultimate lateral pile resistance given by Equation (7).

$$P_{ult} = 0.77 K_p \gamma_d L_{au}^2 R_0 \tag{7}$$

From **Figure 12**, it can be seen that the ultimate lateral pile resistance increases greatly with increase of number of piles in a group. Also, the increase in spacing also contributes in the increase of the ultimate lateral pile resistance because of the coverage of the soil entrapped within the pile group represented by R_0. However, such contribution due to the spacing is limited under the condition that there is an overlapping of plastic zones among the piles in the group and is still considered as closed-spaced grouped pile. Else, the mechanism would be different and falls under the widely-spaced grouped piles. This is imminently visible with the use of s/d_p = 4.5 for the 2x2 grouped pile (given by the black square in **Figure 12**) as it starts to depart slightly from the linear trend.

CONCLUSIONS

Grouped piles behave as equivalent single piles where the spacing to diameter ratio, s/d_p, is less than 20. In determining the ultimate lateral pile resistance of this closed-spaced grouped piles, active pile length is established to be a key parameter. The simplified method entails undergoing the following process:

(1) Determination of the active pile length at the initial stage (Equation (5)).
The relative pile stiffness to the surrounding soil stiffness is the predominant driving parameter to describe the lateral deformation along the length of the grouped piles. Given these known parameters (EI_p and G_{max}), the initial active pile length can be easily derived.

(2) Determination of the active pile length at the ultimate stage (Equation (6)).
Correspondingly, the active pile length at the ultimate stage is derived by applying some correction factor to account for the elasto-plastic behavior of the soil. Careful attention must be made when using large number of piles.

(3) Use of the active pile length at the ultimate stage with other soil parameters (Equation (7)).
A high correlation is seen with the ultimate lateral pile resistance and the parameters representative of the weight of this soil wedge. The ultimate lateral pile resistance can be expressed with just simple parameters such as L_{au}, γ, K_p and R_0. Also, it is noted that while the same number of piles gives the same pile group stiffness and correspondingly almost the same active pile length, the coverage of the soil entrapped in within the piles, indicative of the equivalent R_0, gives the additional lateral pile capacity.

This idea can be extended to more complicated scenario i.e. non-homogeneous soil for a more practical approach in the structural and seismic design and assessment of such foundation system.

ACKNOWLEDGMENT

The author would like to thank the Japanese government (Monbukagakusho: MEXT) for making this study possible through their financial support.

REFERENCES

Aglipay, M.R. (2016). “Ultimate lateral pile resistance characterization using active pile length”. *PhD Thesis*. University of Tokyo.

Archer, A., Heymann, G. (2015). “Using small - strain stiffness to predict the load - settlement behaviour of shallow foundations on sand.” *Journal of the South African Institution of Civil Engineering.* Vol.57, No.2, pp. 28–35.

Bogard, D., Matlock, H. (1983). “Procedures for analysis of laterally loaded pile groups in soft clay.” in: Wright, S.G. (Ed.), *Geotechnical Practice in Offshore Engineering. American Society of Civil Engineers*, New York, pp. 499–535.

Dassault Systemes Simulia. (2013a). “Abaqus 6.13 Online Documentation” [WWW Document]. *Dassault Systèmes*. URL http://129.97.46.200:2080/v6.13/ (accessed 1.1.14).

Drucker, D.C., Prager, W. (1952). “Soil Mechanics and Plastic-Analysis or Limit Design.” *Q. Appl. Math.*

Elgamal, A, Lu J, Yang Z, Shantz T. (2009). “Scenario-focused three-dimensional computational modeling in geomechanics.” *Proc. 4th International Young Geotechnical Engineers' Conference*, ISSMGE.

Gu, X., Yang, J., Huang, M. (2013). “Laboratory measurements of small strain properties of dry sands by bender element.” *Soils Found*. Vol. 53, No. 5, 735–745.

Gudehus, G., Amorosi, A., Gens, A., Herle, I., Kolymbas, D., Mašín, D., Muir Wood, D., Niemunis, A., Nova, R., Pastor, M., Tamagnini, C., Viggiani, G. (2008). “The soilmodels.info project.” *Int. J. Numer. Anal. Methods Geomech*. 32, 1571–1572.

Konagai, K., Yin, Y. and Murono, Y. (2003). “Single beam analogy for describing soil-pile group interaction.” *Soil Dynamics and Earthquake Engineering*, Vol. 23, pp. 213-221.

Kyokawa, H. (2011). “Elastoplastic constitutive model for saturated and unsaturated soil considering the deposited structure and anisotropy”. *PhD Thesis*. Nagoya Institute of Technology.

Herle, I., Gudehus, G. (1999). “Determination of parameters of a hypoplastic constitutive model from properties of grain assemblies.” *Mech. Cohesive-frictional Mater*. 4, 461–486.

Lu J, Yang Z, Elgamal A. (2006). “Openseespl 3D Lateral Pile-Ground Interaction ver. 1.00*” User's Manual*. Department of Structural Engineering, University of California, San Diego, La Jolla, CA.

Matsuoka, H., Nakai, T. (1977). “Stress-strain relationship of soil based on the SMP.” *9th International Conference on Soil Mechanics and Foundation Engineering (ICSMFE)*. pp. 153–162.

Niemunis, A., Herle, I. (1997). “Hypoplastic model for cohesionless soils with elastic strain range. Mech.” *Cohesive-frictional Mater*. 2, 279–299.

Velez, A., Gazetas, G., Krishnan, R. (1983). “Lateral Dynamic Response of Constrained-Head Piles.” J. *Geotech. Eng.* 109, pp. 1063–1081.

von Wolffersdorff, P. a. (1996). “A hypoplastic relation for granular materials with a predefined limit state surface”. *Mech. Cohesive-Frictional Mater*. 1, 251–271.

Wang N, Elgamal A, Lu J. (2014). “Three-Dimensional Finite Element Modeling of Pile and Pile Group System Response.” *Soil Behavior Fundamentals to Innovations in Geotechnical Engineering*, pp. 570-584.

Wang, M.C., Liao, W.P. (1987). “Active Length of Laterally Loaded Piles.” *J. Geotech Engg*, Vol. 113, No. 9, pp. 1044–1048.

Wakai, A., Gose, S., Ugai, K. (1999). “3-D Elasto-Plastic Finite Element Analyses of Pile Foundations.” *Soils and Foundations*, Vol. 39, No. 1, pp. 97–111.