PARAMETRIC ANALYSIS OF ACTIVE PILE LENGTH AND ITS APPLICATION TO THE ULTIMATE LATERAL PILE RESISTANCE

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ABSTRACT: When a flexible pile is subjected to a lateral load, it is deformed prominently near the ground surface and its deformation decreases with increasing depth. This region of significant deformation is called the active pile length, L_a. And during a nonlinear event, a soil wedge is pushed up in the passive region along this L_a. To simply investigate if the simple parameter, L_a can describe the ultimate side soil resistance, a plane strain condition using the 2D finite element method in nonlinear analysis is conducted. The subloading t_{ij} model is used to model the elasto-plastic behavior of the soil and the pile is modeled as a 2D continuum-based beam element. Effects of the pile stiffness and soil stiffness in the active pile length together with its application in describing the ultimate lateral pile resistance are presented for more practical approach in the seismic design and assessment of piles.

Key Words: active pile length, ultimate pile lateral resistance, soil-pile interaction

INTRODUCTION

Piles are usually used as deep foundations for important structures, or structures on weak soils. These piles are most susceptible to external lateral loads like seismic motions. With the mere presence of two elements: the soil and the pile, the lateral resistance of the piles is generally governed by the soil-pile interaction. The movements of grouped piles and their side soils are mutually dependent on each other such that when piles are subjected to a lateral load, they deform relative to the deformation of the surrounding soil and vice versa.

For flexible piles commonly used in engineering practice, the deformation of a vertical beam is observed to be significantly prominent in the region near the ground surface and decreases with increasing depth (Konagai, 2003). This region of significant deformation is defined as the active pile length, L_a, where the piles can be described as a cantilever beam assuming fixity at the negligible deformation. In this study, the point of negligible deformation is defined as the 3% of the maximum pile head deformation. This L_a is a parameter reflective of the soil-pile interaction as it is deemed to be related to the stiffness of the pile relative to the stiffness of the soil. This study aims to establish the relationship of the active pile length to the ratio of the pile stiffness and soil stiffness. Moreover, the effects of varying the pile thickness, b_t, Young's modulus, E_p, pile length, L_p, and the soil shear modulus, μ, on the active pile length, L_a, is investigated.

During nonlinear events like large seismic excitations, a soil wedge is pushed up along this L_a. This

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soil wedge along L_a is indicative of the ultimate lateral resistance. Thus, it can be inferred that the ultimate lateral pile resistance can be described by a simple parameter reflective of the soil-pile interaction. Many researches on soil-pile interactions have been done especially in the advent of high computing powers where rigorous solutions can be done for any complex soil-pile configuration. However, it is still warranted especially in the engineering practice that simple yet high caliber solutions can be available. Therefore, a simplified expression using the L_a as a key parameter to describe the ultimate lateral resistance of piles embedded in sand is presented for more practical approach in the seismic design and assessment of piles.

NUMERICAL ANALYSIS

In this paper, the soil-pile configuration is idealized in a plane strain condition similar to the works of Kok et al. (2008), Naveed et al. (2012) and Hazzar et al. (2013). Inaccuracies in the actual response of a single pile are expected given that pile foundations are principally a three-dimensional problem. Thus, this method may be limited to describe the behavior of piles considered as walls, row piles or sheet piles. Nevertheless, the intention to start off with the 2D platform is to simply investigate the formation of the passive soil wedge during nonlinear scenario and relate it to the active pile length. Should the idea be valuable, then it could be extended to 3D to realistically capture the behavior of piles.

Figure 1 shows the general layout of the soil-pile system in 2D plane strain condition using the finite element method. A pile with thickness, b_t, a unit thickness in the out of plane direction and length, L_p, is embedded in a homogeneous soil having a width, b_s = 20m, on each opposite side of the pile. The subloading t_{ij} model (Nakai et al., 2011, 2013 and Kyokawa, 2011) is used to describe the elasto-plastic behavior of soils. In this case, the homogeneous soil is taken as Toyoura sand (TS). The material parameters given in Table 1 are calibrated from the drained compression and extension tests shown in **Figure 2** (Kyokawa, 2011). A continuum based beam element (Yoon, 2012) is used for modelling the pile in elastic case. The pile head considered as fixed is slightly protruding from the ground level for the application of the lateral loading. The lateral load is applied monotonically until the pile head reaches 1.8m displacement. A joint element of virtual thickness is introduced between the soil and the pile to simulate the vertical slipping of soil against the pile during nonlinearity. Separation between the soil and pile is not taken into account. The angle of internal friction of the joint element is 25^o (Wakai, 1999). The bottom ends are considered as the hard strata, therefore, it is fixed in x and y directions. On the other hand, the sides are fixed only in x direction.

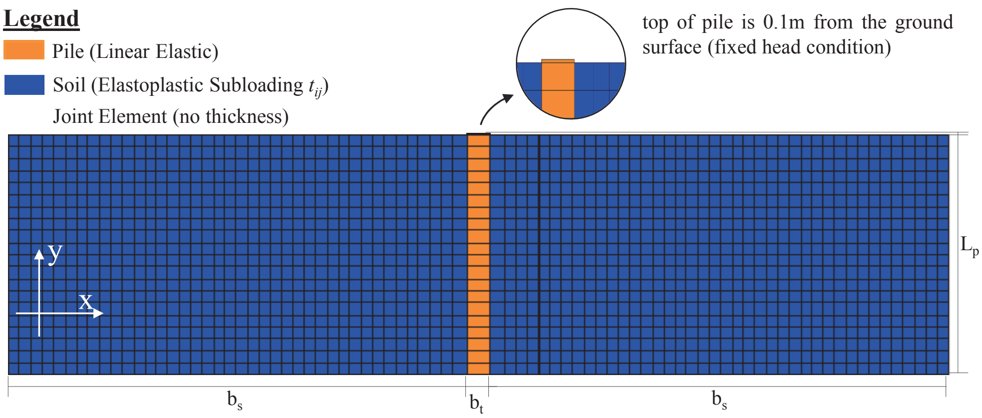

Figure 1. Soil-pile configuration

Table 1. Toyoura sand material parameters for the subloading t_{ij}

Material Parameters	
compression index, λ	0.070
swelling index, κ	0.0045
stress ratio at critical state, R_{cs}	3.2
shape of yield surface, β	2.0
void ratio at normal consolidation at Pa=98kPa, e_{NC}	1.10
atmospheric pressure (kPa), Pa	98
controlling decay rate of the influence of density, a	33
poisson ratio, ν	0.2

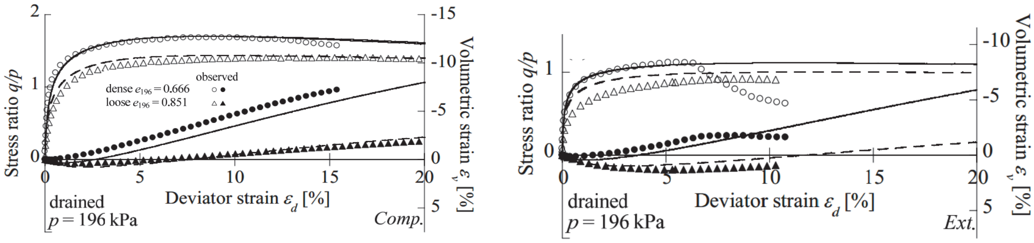

Figure 2. Drained compression and extension test for Toyoura Sand (Taken from Kyokawa 2011)

ACTIVE PILE LENGTH

The behavior of piles embedded in soil are generally governed by the deformation along its length. When the head of a flexible pile is induced by a lateral loading, its deformation at the region near the ground surface is remarkable. However it decreases with the increasing depth until it becomes negligible at some particular depth. In this study, the L_a is defined from the ground surface down to the point where the deformation is equal to 3% of the maximum pile head deformation. Within this region of significant deformation, pile can be described as a cantilever beam, assuming fixity for the deeper region of negligible deformation. In common engineering practice, Chang's formula as in Equation (1) is used to define the characteristic length in Equation (2).

$$\beta = \sqrt[4]{\frac{k_h b}{4EI_p}} \tag{1}$$

$$L_o = \frac{1}{\beta} \tag{2}$$

In which in Equation (1), k_h: coefficient of horizontal subgrade reaction, b: pile width or side edges considering a 3D pile and EI_p: pile stiffness. However, we note that in $k_h b$ is not an inherent property of the soil. Thus, a more rational expression is proposed by Konagai (2003) noting the stiffness of the pile relative to the surrounding soil. This is deemed linearly proportional to the active pile length, L_a. ($L_a = \alpha L_o$)

$$L_o = \sqrt[4]{\frac{EI_p}{\mu}} \tag{3}$$

In Equation (3), EI_p: pile stiffness and μ: soil shear modulus. The L_a is closely investigated on by varying the parameters directly affecting it such as the EI_p and μ. Particularly, the factors: pile thickness, b_t, Young's modulus, E_p, pile length, L_p, and the soil shear modulus, μ are varied.

Effect of pile stiffness

The L_a is largely affected by the EI_p. Since the EI_p is a function of the pile geometric properties, the effect of pile thickness, b_t, on the active length is investigated. In this case, a single pile with length, L_p =30m, having E_p = 30GPa, embedded in TS soil with initial void ratios of 0.6 and 0.9 were taken into account. While holding all parameters constant, the pile thickness, b_t, was set at 0.3m, 0.5m, 0.6m, 1.0m, 1.2m, 1.5m and 2.0m. This correspondingly varies the pile stiffness with respect to the same surrounding side soils.

Figure 3 shows the variation of the b_t and its corresponding L_a normalized with the L_p for piles embedded in the dense and loose condition. Generally, with increasing b_t, the EI_pI ncreases, and consequently, the L_a increases. However, looking closely, it can be seen that for piles with L_a/L_p < 0.75 and b_t/L_p <0.032, the difference in L_a of piles embedded in loose and dense sands increases with increasing b_t as seen by the divergence in **Figure 3**. But when the b_t/L_p reached 0.032, where the ratio, L_a/L_p, becomes greater than 0.75, the difference in the L_a*'s* for piles embedded in loose and dense sand starts to decrease and converges to a point. This scenario means that as the pile thickness increases, the pile stiffness increases, therefore, the active pile length increases eventually reaching a saturation point, where L_a is approximately equal to 0.9L_p. L_a becomes constant with increasing lateral pile head deformations as restrained by the fixity at the bottom of pile. In this study, flexible piles are of interests. Thus, piles having L_a/L_p < 0.75 are considered.

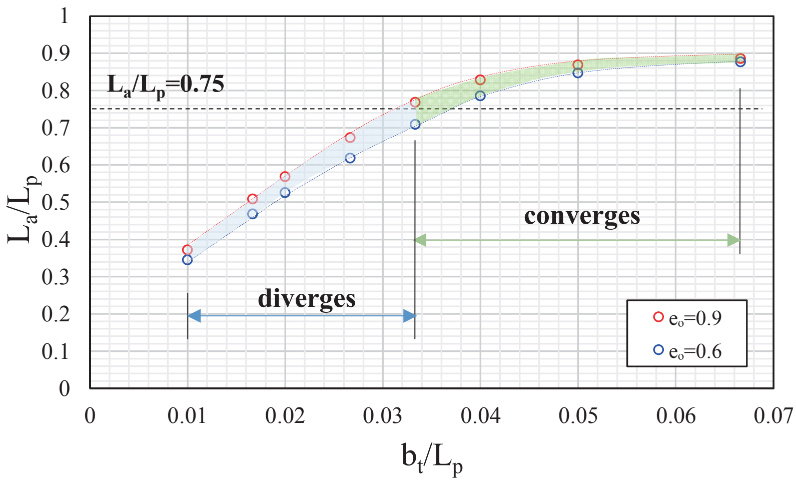

Figure 3. Variation of pile thickness, b_t, with the active pile length, L_a

The piles having E_p=200GPa with pile thickness, b_t, equal to 0.3m, 0.35m, 0.4m and 0.5m and embedded in loose Toyoura sand were considered. The obtained active pile lengths were superimposed to those of piles having E_p=30GPa with pile thickness, b_t, equal to 0.3m, 0.5m, 0.6m and 0.8m. The combined effect of the Young's modulus, E_p, and the pile thickness, b_t, using the pile stiffness parameter, EI_p, is plotted against the active pile length for all the cases considered in this particular parametric analysis as seen in **Figure 4**. All the data points lie on a unique line defined by y=2.891x + 3, where y and x are the active pile length and respectively. Thus, there's indeed a linear relationship between the active pile length, L_a and the $(EI_p)^{0.25}$.

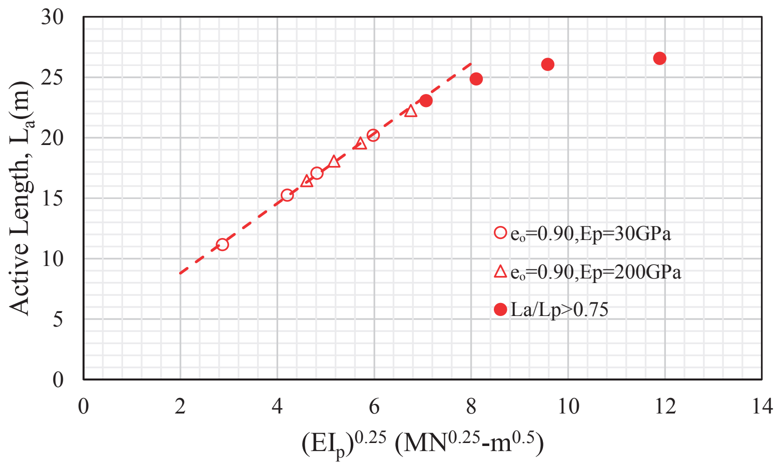

Figure 4. Effect of pile stiffness, EI_p, on active pile length, L_a

Effect of pile length, L_p

The effect of pile length on the active pile length is investigated. In this case, the single pile having EI_p equal to 312.5MN-m^2 and 2500MN-m^2 were taken into consideration. Each pile is embedded in the loose sand with e_0= 0.90, varying the pile lengths to 10m, 20m, 30m, 60m and 100m.

In **Figure 5**, for the piles with stiffness EI_p= 2500MN-m^2 given by the red line, the L_a continues to increase with increasing L_p until it reaches to a ratio of L_a/L_p=0.77 where the L_p is at 30m. The increase in active pile length means that it may actually need to be longer than the actual pile length but is restricted with the bottom boundary conditions, thus behaving as stiff piles. But when it reached L_p=60m, the effect of L_p on the L_a becomes negligible. This point is more evidenced for the pile stiffness equal to 312.5 MN-m^2 given by the blue line. Similar trend shows that as L_a/L_p becomes less than 0.76, the active pile length, L_a, becomes constant despite increasing the actual pile length, L_p. This means that the expected active pile length has been reached given the pile's actual pile length. Figure 6 shows the relationship between L_a/L_p and the actual pile length, L_p. For L_a/L_p <0.75 approximately, there is an asymptotic behavior with increasing L_p.

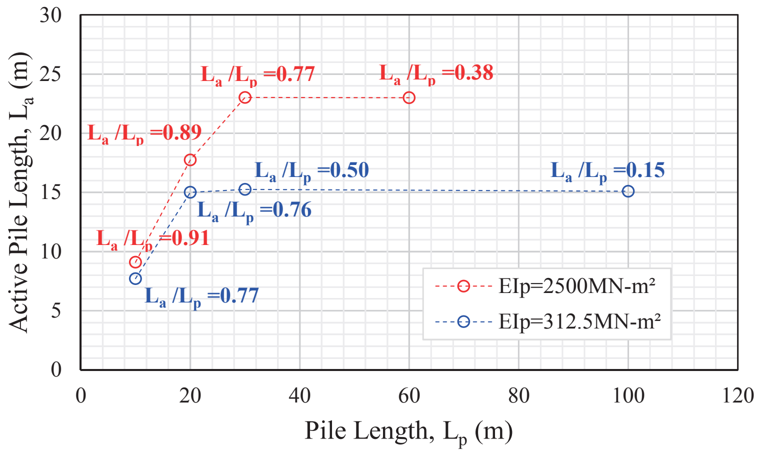

Figure 5. Variation of active pile length, L_a with pile length, L_p

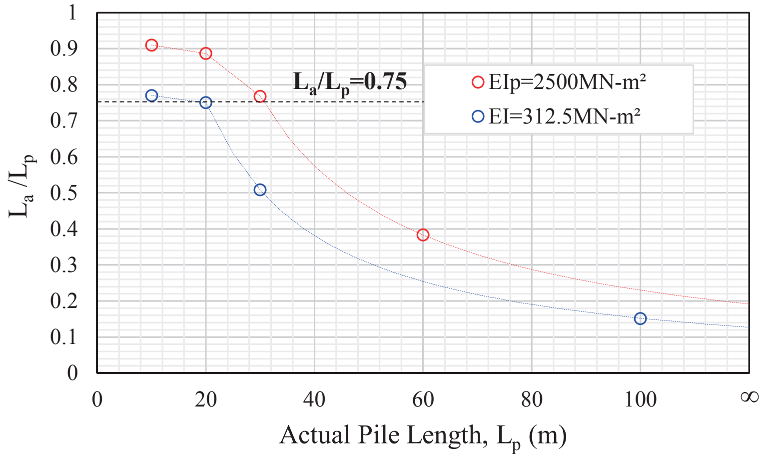

Figure 6. L_a/L_p with actual pile length, L_p

Effect of soil stiffness

The shear modulus needs to take account the shear degradation of the soil with increasing pile head deformation. In view of this, the following steps were undertaken. For each case considered, the shear strain is derived by dividing the current pile head displacement with the corresponding deformed L_a. Then, this shear strain is plotted in the shear degradation curves to get the corresponding shear modulus. **Figure 7** shows the variation of the L_a, with the L_o given by the fourth root of the ratio of EI_p with the μ at the progressive pile head deformation (particularly at u_y = 0.009m, 0.036m, 0.135m, 0.45m, 1.575m) for all the cases of piles with varying b_t, E_p embedded in dense and loose sand. It can be observed from this figure, that there is a linear relationship with the active pile length and characteristic length for various pile head displacements. Thus, at this stage, it can be established that L_a=αL_o holds true. There's a linear relationship between L_a and with $(EI_p/\mu)^{0.25}$.

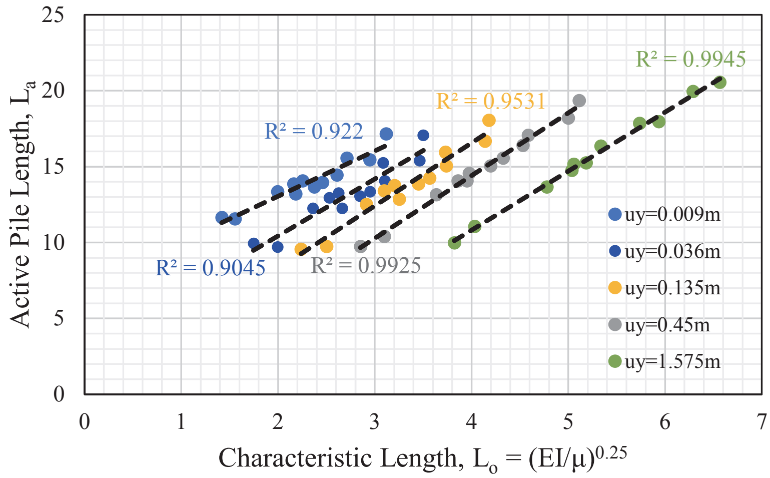

Figure 7. Variation of active pile length, L_a, with the L_o for all cases at different lateral pile head deformation

The progressive active pile length with the corresponding lateral pile head deformations were derived for piles embedded in dense and loose sand. These active pile length are normalized with the initial characteristic length, $L_o=(EI/\mu)^{0.25}$, where the shear modulus, μ, is the maximum shear modulus, μ_{max} which can be easily obtained in the field. The variation of the ratio of the active pile length to this initial characteristic length with the lateral pile head deformation is plotted in **Figure 8**. It can be seen that the curves reaches to a constant line at around α=6. Therefore, it can be said the initial active pile length is important as it can determine the active pile length at the ultimate stage by a factor of 6.

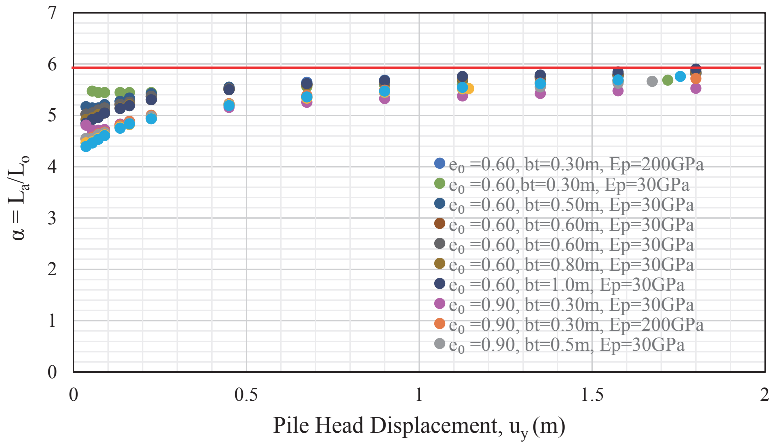

Figure 8. Variation of the ratio of the active pile length to the initial characteristic length with respect to the lateral pile head displacement.

APPLICATION OF La FOR THE ULTIMATE LATERAL PILE RESISTANCE

Based from the numerical simulations, upon application of lateral load to the pile head a soil wedge is progressively formed at the passive region. In **Figure 9**, the lateral force at the pile head is given by the black line. The pile resistance based on the active pile length at 1m pile head displacement (where the L_a starts to become constant and shear strain, γ is approximately greater than 5%) given by the blue line. The side soil resistance is derived by the difference of the lateral force at the pile head and the pile resistance. The soil resistance increase with increase in the pile head displacement and approaches a constant value. This is where the ultimate lateral pile resistance, P_{ultimate}, is derived for all cases.

The force representation of the wedge can be described by the unit weight, γ, and the Rankine passive earth pressure coefficient, K_p. The values for the γ and K_p are given in **Table 2**. Together with the L_a, the soil parameters such as the unit weight of the soil, γ_d, Rankine passive coefficient, K_p are used to estimate the ultimate lateral resistance. **Figure 10** shows the correlation of the ultimate side soil reaction force and the parameters $K_p\gamma_d L_a^2$ for all the cases considered in this analysis having L_a/L_p <0.75. In this figure, it can be observed that there is a high correlation between the $P_{ultimate}$ and $K_p\gamma_d L_a{}^2$ having an R^2=0.9807. Therefore, the P_{ultimate} can be given by this equation:

$$P_{ultimate} = 0.335 K_p \gamma_d L_a^2 \tag{4}$$

Table 2. Soil parameters of Toyoura sand (TS)

Initial void ratio, e_o	φ(deg)	γ_d (kN/m^3)
0.60	43.51	16.25
0.70	40.53	15.29
0.80	37.47	14.44
0.90	34.67	13.68

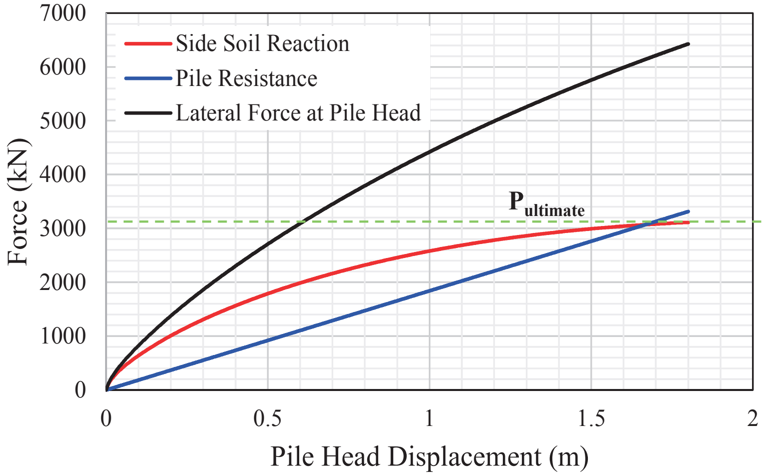

Figure 9. Force-deformation curve

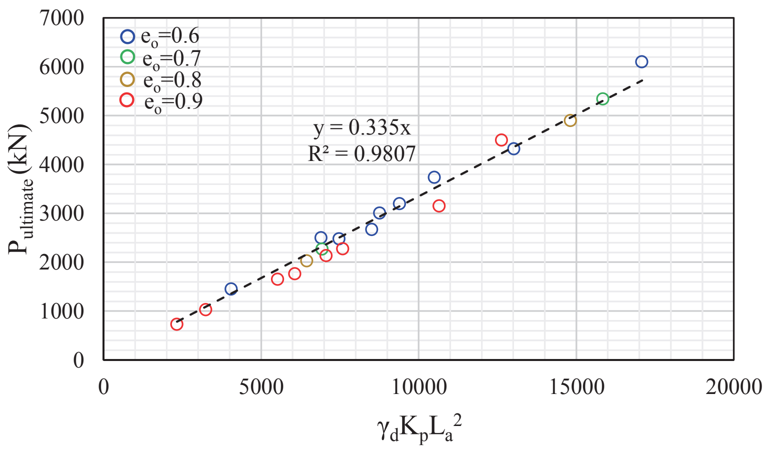

Figure 10. Correlation between $P_{ultimate}$ and $K_p\gamma_d L_a^2$

CONCLUSIONS

To ensure use of flexible piles, L_a/L_p<0.75 were taken into consideration to discount the limiting effect of the bottom boundary condition for shorter piles. For flexible piles, the active pile length is generally governed by the stiffness of the pile relative to the surrounding soil stiffness. Particularly, there is a linear relationship with the fourth root of the ratio of pile stiffness and soil stiffness at various lateral pile head deformation. Moreover, the active pile length at the initial stage is critical as it allows to estimate the active pile length at the ultimate stage. The use of active pile length to get the pile resistance and derive the side soil reaction from the load-deflection response curve at the pile head is indicative that the active pile length is a key parameter to define the ultimate lateral resistance. Together with other important soil parameters such as the soil unit weight and Rankine passive coefficient, the ultimate lateral resistance of the side soil can be estimated. This simplified expression can be useful for more practical approach in the seismic and assessment of piles. This idea can be extended to the 3D case and for a more complicated scenario i.e. non-homogeneous soil, for group piles, etc.

ACKNOWLEDGMENT

The author would like to thank the Japanese government (Monbukagakusho: MEXT) for making this study possible through their financial support.

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