# STRESS-DILATANCY CHARACTERISTICS OF SAND IN DRAINED CYLIC TORSIONAL SHEAR TESTS

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**ABSTRACT**: Stress-dilatancy relationship of sand was investigated in drained cyclic torsional shear tests. It is presented in 2D form showing the relation between shear-stress ratios with the changes of volumetric strain. In order to obtain comprehensive results on stress-dilatancy relationship, a series of tests were conducted by changing several parameters such as the over-consolidation histories, specimen initial relative densities and strain amplitudes. It was found that over-consolidation history and specimen initial relative density have less effect on stress-dilatancy relationships, while shear strain amplitude has more predominant effect. The comprehensive studies from these experimental results may give important information, which later can be used in the modeling of the stress-dilatancy relationship. Volume change characteristics in stress-dilatancy relationship can be also used in predicting the excess of pore water pressure for liquefaction analysis.

**Key Words**: Stress-dilatancy characteristics, dilatation, contraction, torsional shear, shear stress ratio, shear strain and volumetric strain.

## INTRODUCTION

On the soil behavior subjected to drained shear, Rowe (1962) found a unique relationship between the stress ratio and the dilatancy ratio (strain increment ratio); so called stress-dilatancy relationship. It can be described as the change of volume in the sheared soil element as soil particles sheared override to one another.

In the early studies, researchers obtained the stress-dilatancy relationship by conducting tests under drained monotonic loading conditions (Rowe, 1962 and Roscoe et al., 1963, among others). However, as the behaviors of soil subjected to dynamic loading cases become more important on the recent issues in geotechnical engineering, more studies have been conducted under drained cyclic loading conditions (Pradhan et al., 1989, Shahnazari and Towhata, 2002, De Silva, 2008, among others). Under drained cyclic loading conditions, the stress-dilatancy relationship shows the characteristics of volume change in soils during each of the cyclic loading stages.

It was found that the stress-dilatancy relationship is also one of the key parameter in understanding the behavior of soil during liquefaction. The build-up of pore water pressure in undrained cyclic loading test in simulating the liquefaction process is linked to the accumulation of volumetric strain in drained cyclic loading test.

Tatsuoka et al. (1978) among others obtained the stress-dilatancy relationship by using triaxial apparatus. In this study, the stress-dilatancy relationship was obtained by conducting a series of tests using a hollow cyclinder torsional shear apparatus. Unlike triaxial shear test, torsional shear test is capable of simulating the rotation of major principal stress direction during loading process. Pradhan et al. (1989) and Shahnazari and Towhata (2002) also used torsional shear apparatus, by performing

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the tests under simple shear condition (TSS), where the radial and circumferential strain increments were kept to be zero ($\varepsilon_r = \varepsilon_\theta = 0$).

In order to obtain more comprehensive data on the of stress-dilatancy relationship, tests were performed in this study under drained cyclic torsional shear condition (TS) by changing several parameters such as over-consolidation histories, initial relative densities of specimen and shear strain amplitudes. The outcome from these experimental results can be used in modeling more precisely the stress-dilatancy relationship.

## MATERIAL, APPARATUS AND TEST PROCEDURE

Toyoura sand was used as the test material. Its particles have an angular or sub-angular shape with the following physical properties: specific gravity, $G_s$=2.656; mean diameter, $D_{50}$=0.162mm; fines content, $F_c$=0.1%; max. void ratio, $e_{max}$=0.992; min. void ratio, $e_{min}$=0.632. Tests were conducted on hollow cylindrical specimen, as shown in Fig. 1(a), having the height of 30cm, the inner and outer diameters of 12cm and 20cm, respectively.

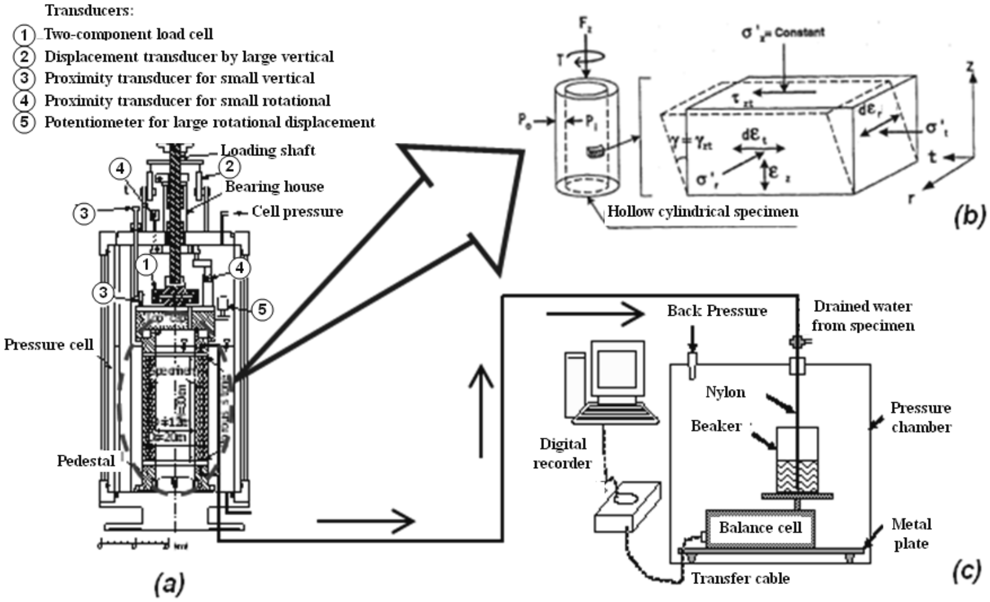


**Figure 1.** Hollow cylinder torsional shear apparatus

In the present study, five different tests were conducted to investigate the effects of over-consolidation histories, initial relative densities and strain amplitudes on the stress-dilatancy relationships. The detailed conditions of each test, named SD-1 to SD-5, can be seen in Table 1.

Specimens were prepared by pluviation of air-dried sand particles into a mold. In order to obtain the specimens with highly uniform density, their falling height was kept constant throughout the pluviation process. Double vacuuming was used during the saturation process considering the large size of the specimen, and thus B-value was achieved to be greater than 0.96. Volumetric change of the specimen during loading process was measured by high sensitive electronic balance, with the accuracy up to 1.0 mg. Unlike the volume change measured by LCDPT (Low Capacity Differential Pressure Transducer), measurement by the electronic balance was free from the effect of surface tension or meniscus forces within the water-retained beaker as illustrated in Fig. 1(c).

Specimens were consolidated isotropically from 35 to 100 kPa prior to the shear loading, while applying isotropic unloading and reloading cycles if necessary. Finally, drained cyclic torsional shear loading with the specified stress or strain amplitude was applied at constant shear strain rate ($d\gamma$) of 2.0 %/min., while attempting to maintain other stresses at $\sigma_z' = \sigma_r' = \sigma_\theta' = 100$ kPa.

**Table 1.** Test details on the stress-dilatancy study

| Test | $e_{ini}$ at $p'$=35 kPa | $D_{r(ini)}$ (%) at $p'$=35 kPa | B value | OCR | Loading conditions | Remarks |
|---|---|---|---|---|---|---|
| SD-1 | 0.792 | 55.6 (Loose) | 0.97 | 1 | CTS ($\tau_{z\theta} = \pm 50$ kPa) | Stress-ctrl. |
| SD-2 | 0.801 | 53.7 (Loose) | 0.97 | 4 | CTS ($\tau_{z\theta} = \pm 50$ kPa) | Stress-ctrl. |
| SD-3 | 0.691 | 82.1 (Dense) | 0.99 | 1 | CTS ($\tau_{z\theta} = \pm 60$ kPa) | Stress-ctrl. |
| SD-4 | 0.737 | 70.7 (Med. Dense) | 0.96 | 1 | CTS ($\gamma_{z\theta} = \pm 5.0$ %) | Strain-ctrl. |
| SD-5 | 0.728 | 73.4 (Med. Dense) | 0.98 | 1 | CTS ($d\gamma_{z\theta} = \pm 2.5$ %) | Strain-ctrl. |

note:

- OCR = Over-consolidation ratio
- CTS = Cyclic torsional shear loading while maintaining $\sigma_z' = \sigma_r' = \sigma_\theta' = 100$ kPa.

## DATA ANALYSIS PROCEDURES

There are several proposals among researchers to explain the stress-dilatancy relationship under different loading conditions such as triaxial, plane strain, etc. In this study, the investigation of stress-dilatancy relationship follows the one developed by Pradhan et al. (1989) in torsional shear test, presenting the relationship between shear stress ratio ($\tau/p'$) and dilatancy ratio ($-d\varepsilon^d/d\gamma^p$). The dilatancy ratio is defined as the ratio between volumetric strain increments due to dilatancy ($d\varepsilon^d$) and plastic shear strain increments ($d\gamma^p$).

Volumetric strain increment due to dilatancy ($d\varepsilon^d$): The total volumetric strain increment ($d\varepsilon_{vol}$) measured during cyclic loading consists of two components, which are due to dilatancy ($d\varepsilon^d$) and consolidation/swelling ($d\varepsilon^c$), as shown in Eq. 1:

$$d\varepsilon_{vol} = d\varepsilon^d + d\varepsilon^c \quad (1)$$

The component due to dilatancy is perfectly plastic in nature, while the other component due to consolidation/swelling consists of both elastic and plastic deformations.

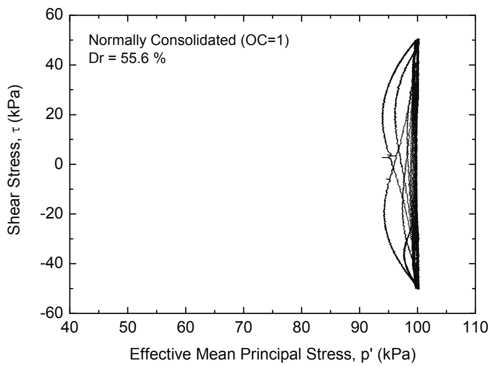


**Figure 2.** Change in $p'$ during drained cyclic torsional shear loading

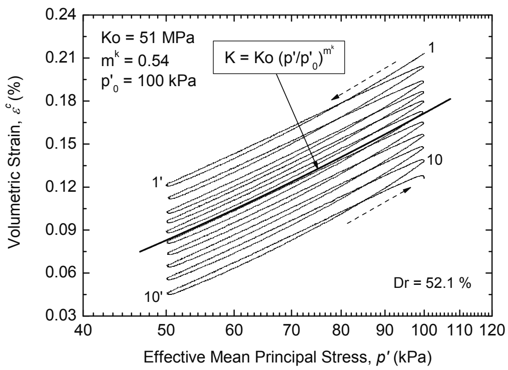


**Figure 3.** Swelling curve during isotropic unloading and reloading cycles

Due to limited accuracy of machine control during cyclic loading, the axial stress could not be maintained exactly at $\sigma_z'$= 100 kPa, resulting into a change in the effective mean principal stress, *p'*, in the range between 95 to 100 kPa, as typically shown in Fig. 2. Since the change in *p'* might induce the change of volume due to consolidation ($d\varepsilon^c$), the $d\varepsilon^c$ value was calculated by using Eq. (2) as shown below, while referring to the swelling curve measured during isotropic unloading and reloading cycles in the range of *p'* = 50 to 100 kPa, as shown in Fig. 3.

$$d\varepsilon^c = \frac{dp'}{K} = \frac{dp'}{K_o.\left(p'/p_0'\right)^{m^k}} \tag{2}$$

where *dp'* is the change of effective mean principal stress; *p'* and $p_0'$ are the current and the target effective mean principal stresses, respectively; $K_o$ is the reference bulk modulus (defined at $p' = p_0'$), and $m^k$ is material parameter obtained by curve fitting.

<u>Plastic shear strain increment ($d\gamma^p$):</u> The total shear strain increment ($d\gamma^t$) measured externally by potentiometer is also decomposed into two components, which are plastic component ($d\gamma^p$) and elastic component ($d\gamma^e$) as shown in Eq. (3):

$$d\gamma^t = d\gamma^p + d\gamma^e \tag{3}$$

The elastic component ($d\gamma^e$) was computed by using Eq. 4 and Eq. 5, formulated using the quasi-elastic constitutive model developed at the Institute of Industrial Science, University of Tokyo (Hong Nam, 2004).

$$d\gamma^e = d\tau / G \tag{4}$$

$$G = \{f(e)/f(e_o)\}.G_o.(\sigma_z'.\sigma_r')^{n/2}/(\sigma_o')^n \tag{5}$$

where: $G$ = shear modulus; $f(e) = (2.17 - e)^2/(1 + e)$, (Hardin and Richart, 1963); $e_o$ = initial void ratio at $\sigma_z' = \sigma_r' = \sigma_\theta'$ = 100 kPa, $G_o$ = initial shear modulus; $\sigma_o'$ = reference isotropic stress state (=100 kPa); $\sigma_z'$ and $\sigma_r'$ = vertical and radial effective stresses, respectively; and $n$ = stress induced anisotropy material parameter. In this study, $G_o$ and $n$ were set equal to 85 MPa and 0.508, respectively.

## TEST RESULT AND DISCUSSION

Effects of over-consolidation history, specimen initial relative density and shear strain amplitude on the stress-dilatancy relationship are discussed herein:

<u>Effects of over-consolidation histories:</u> Two specimens, named SD-1 and SD-2, were prepared with over-consolidation ratio of OC = 1 (normally consolidated, NC) and OC = 4 (over-consolidated, OC).

The relationships between the shear stress ratio ($\tau/p'$) and the plastic shear strain ($\gamma^p$) for NC and OC specimens are shown in Figs. 4(a) and 5(a), respectively. In these tests, shear loading was applied with constant shear stress amplitude of 50 kPa. It can be seen that during subsequent loading cycles, the hysteresis loops became smaller. This phenomenon cannot be attributed totally to the increase in density with cyclic loading (i.e., after applying 5 cycles, $Dr_{(N=5)}$ = 61.9 % for NC test and $Dr_{(N=10)}$ = 63.7 % for OC test), but it is probably due to the cyclic strain-hardening behavior.

The relationships between plastic volumetric strain due to dilatancy ($\varepsilon^d$) and plastic shear strain ($\gamma^p$) are shown in Figs. 4(b) and 5(b). During the subsequent cyclic loading, volumetric strains in both tests were accumulated in the positive side (contraction). After 5 cycles, NC and OC specimens attained total plastic volumetric strains of about 1.3% and 1.2%, respectively. Since both specimens were in the loose state, contractive behavior appeared predominantly in each cycle. It can be noticed that dilatation (i.e, $d\varepsilon^{p} < 0$) started after the second cycle in the case of OC specimen and in the third cycle in the case of NC one. During the first half cycle of cyclic loading (virgin loading), the NC specimen showed pure contractive behavior; on the other hand, the OC specimen initially behaved dilative.

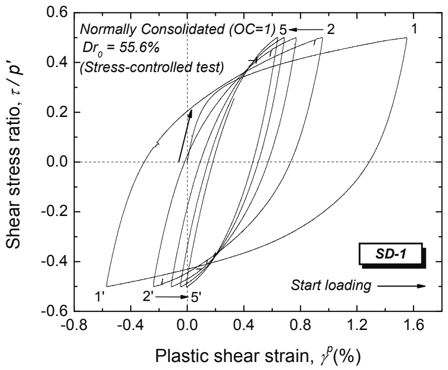


**Figure 4(a).** Shear stress-strain relationship for normally consolidated sand

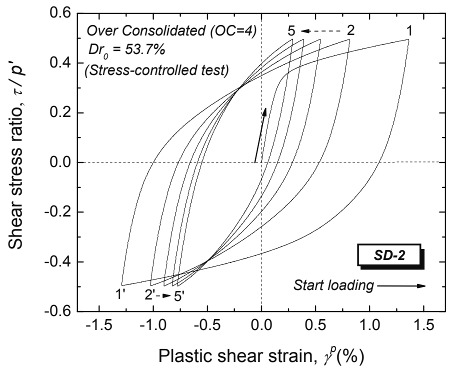


**Figure 5(a).** Shear stress-strain relationship for over-consolidated sand

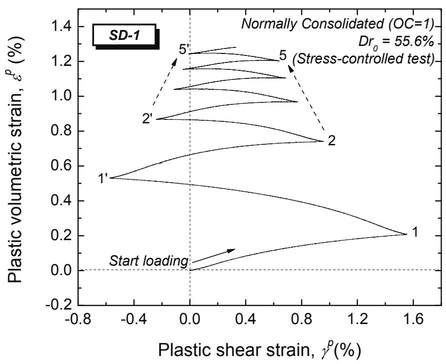


**Figure 4(b).** Volumetric strain-shear strain relationship for normally consolidated sand

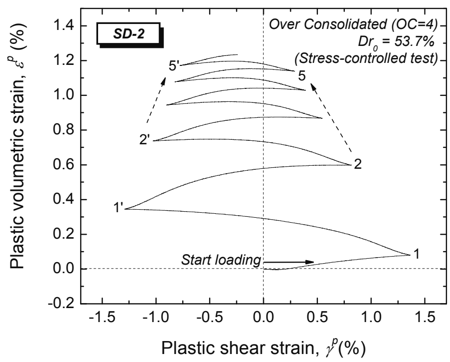


**Figure 5(b).** Volumetric strain-shear strain relationship for over-consolidated sand

Stress-dilatancy relationships for NC and OC specimens are presented in Figs. 4(c) and 5(c), respectively. In general, the stress-dilatancy curves for each cycle forming two parallel lines which consist of two segments of positive slopes (loading/reloading regime) and two nearly vertical segments (unloading regime). Numbers 1, 1', 2, 2' to 5 and 5' on Figs. 4 and 5 correspond to the points where the loading direction was reversed. It can be noticed that the dilatancy ratio ($-d\varepsilon^{d}/d\gamma^{p}$) changed discontinuously from negative to positive one ($-d\varepsilon^{d}$ means the specimen behaves dilative) immediately after each of the reversal loading.

The vertical line passing the origin at $\{-d\varepsilon^{d}/d\gamma^{p}\} = 0$ corresponds to the zero dilatancy state, which indicates the changes of behavior from contractive to dilative, or vice-versa. Ishihara et al. (1975) called this state as the phase transformation (PT).

The major impact of over-consolidation histories on the dilatancy characteristics of sand can be seen during the virgin loading in Figs. 6(a) and 6(b). During the virgin loading, the NC specimen shows a pure contractive behavior since the curve does not cross the zero dilatancy state; on the contrary, the OC specimen clearly shows a change in the dilatancy behavior from dilative to contractive. However, in the subsequent cyclic loading, the over-consolidation histories seem disappear, thus, both NC and OC specimens show similar dilatancy characteristics.

Figures 4(c) and 5(c) also show the shrinkage of the stress-dilatancy curves toward zero dilatancy state in the subsequent cyclic loading; soil becomes more dilative. Shahnazari and Towhata (2002) reported similar phenomenon under the torsional simple shear (TSS) condition. The reason of the shrinkage of this curve will be explained later based on the results from tests subjected to the constant and irregular shear strain amplitudes.

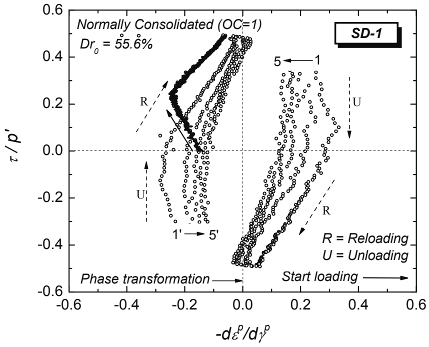


**Figure 4(c).** Stress-dilatancy relationship for normally consolidated sand

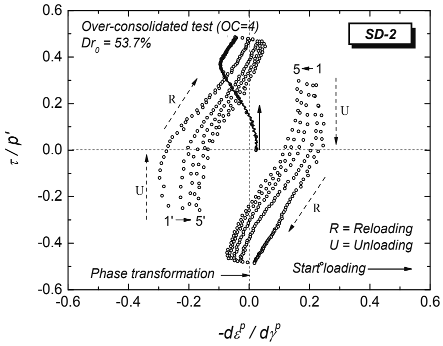


**Figure 5(c).** Stress-dilatancy relationship for over-consolidated sand

Effects of initial relative densities: The test results on a dense state specimen, SD-3, with the initial relative density of 82.1% subjected to constant shear stress amplitude of 60 kPa is presented.

The relationships between the shear stress ratio ($\tau/p'$) and the plastic shear strain ($\gamma^{p}$) are shown in Fig. 6(a). Similar with the ones conducted in loose state specimens (Fig. 4(a)), the hysteretic loops shrink in the subsequent cyclic loading (i.e., after applying 5 cycles, $Dr_{(N=5)} = 83.3$ %).

The relationship between plastic volumetric strain due to dilatancy ($\varepsilon^{d}$) and plastic shear strain ($\gamma^{p}$) is shown in Fig. 6(b). Different from the behavior observed with the loose state specimens, the dense state specimen exhibited both contraction and dilatation in each cycle. Even though dilatation appears from the first cycle up to the end, the total volumetric strain was still accumulated in the positive side. The dense state specimen attained less plastic volumetric strain of about 0.6% at the end of 5th cycle compared with the ones with the loose state specimens, which were about 1.2% to 1.3% (Figs. 4(b) and 5(b)).

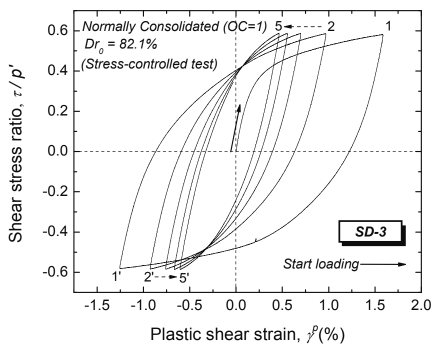


**Figure 6(a).** Shear stress-shear strain relationship for dense state sand

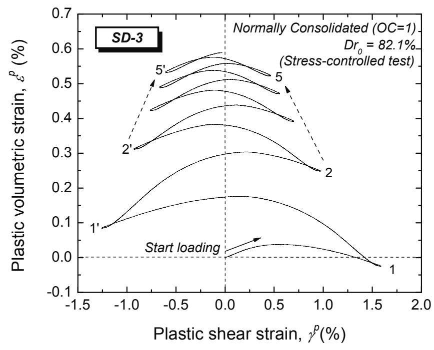


**Figure 6(b).** Volumetric strain-shear strain relationship for dense state sand

Stress-dilatancy relationship of dense state specimen is presented in Fig. 6(c). The shrinkage of the stress-dilatancy curve on the dense state specimen appears similarly with the one on loose state specimens. In dense state specimen, the dilatancy ratio ($-d\varepsilon^{d}/d\gamma^{p}$) attained a value of about 0.25 in the 1st cycle and the value of about 0.10 in the 5th cycle at the unloading regime. These values are similar with the ones achieved on loose state specimen. In relation to such behavior, Tatsuoka et al. (1978) confirmed that stress-dilatancy relationships are independent of specimen's initial relative densities or their stress levels.

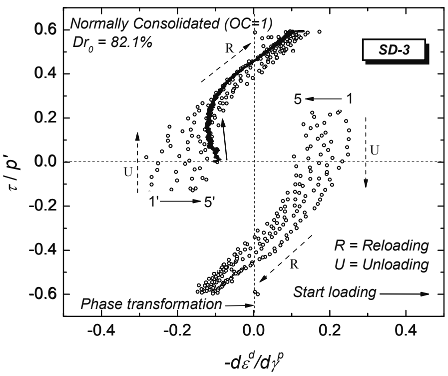


**Figure 6(c).** Stress-dilatancy relationship for dense state sand

Effects of shear strain amplitudes: Previous tests on both loose and dense state specimens were subjected to constant shear stress amplitude, and as a result the shear strain amplitude decreased during the subsequent cyclic loading. The results from these tests showed a phenomenon in which the dilatancy ratio changes towards the zero dilatancy state during subsequent cyclic loading. In order to understand such a behavior, two specimens, named SD-4 and SD-5, were subjected to the constant and irregular shear strain amplitudes, respectively. Specimen SD-4 was subjected to the constant shear strain amplitude of 5.0%, while specimen SD-5 was subjected to the progressive increase in the shear stain amplitude.

The relationships between the shear stress ratio ($\tau/p'$) and the plastic shear strain ($\gamma$□$^{p}$) for SD-4 and SD-5 specimens are shown in Figs. 7(a) and 8(a), respectively. It can be seen that both tests achieved higher shear stress ratio during subsequent cyclic loading. As discussed earlier, this behavior appears to be affected by densification and strain-hardening, hence, shear stress increased in the subsequent cyclic loading.

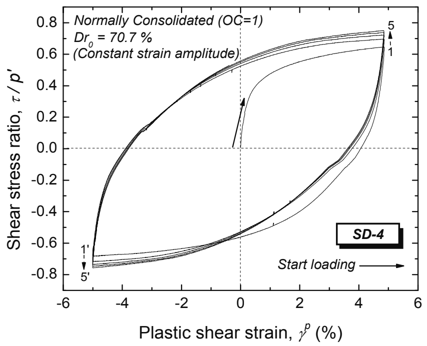


**Figure 7(a).** Shear stress-strain relationship for 5.0% shear strain amplitude test

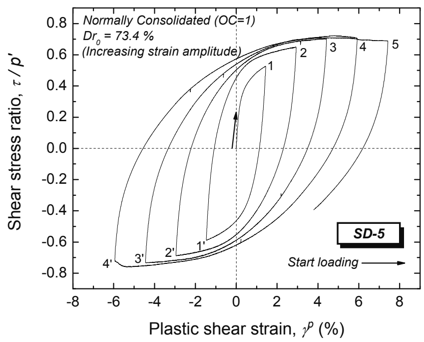


**Figure 8(a).** Shear stress-strain relationship for increasing shear strain amplitude test

The relationships between plastic volumetric strain due to dilatancy ($\varepsilon^d$) and plastic shear strain ($\gamma^p$) are shown in Figs. 7(b) and 8(b) for SD-4 and SD-5 specimens, respectively. Since both specimens were prepared at medium-dense state of about $Dr_0 \approx 70\%$, contraction and dilatation appeared in each cycle. However, SD-5 specimen showed remarkable increase of dilatation as shear strain amplitude increased. In terms of the accumulation of volumetric shear strain at the end of 5th cycle, SD-4 and SD-5 specimens attained about 2.5% and 1.4%, respectively. Such a difference between these two values seems to appear even during earlier cycles, where the specimen subjected to the constant shear strain amplitude exhibited larger contraction.

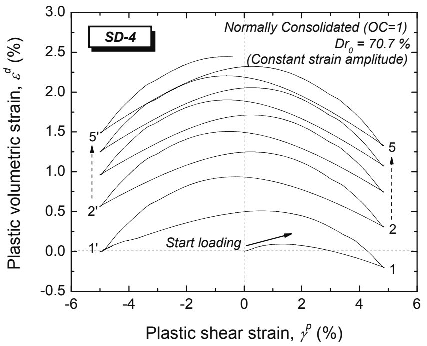


**Figure 7(b).** Volumetric strain-shear strain relationship for 5.0% shear strain amplitude test

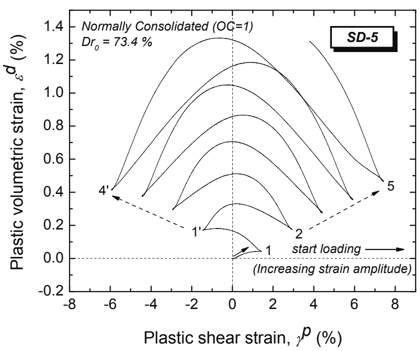


**Figure 8(b).** Volumetric strain-shear strain relationship for increasing shear strain amplitude test

Stress-dilatancy relationships for SD-4 and SD-5 specimens are shown in Figs. 7(c) and 8(c), respectively. The stress-dilatancy relationships in the constant shear strain amplitude test (SD-4) are not affected by the number of cycles, indicating that the stress ratio ($\tau/p'$) has a unique relationship with the dilatancy ratio ($d\varepsilon^d/d\gamma^p$). However, in the test where the shear strain amplitude increased progressively (SD-5), stress-dilatancy curves appear to be expanded, moving away from the zero dilatancy state.

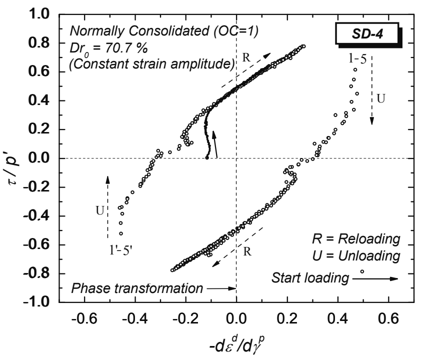


**Figure 7(c).** Stress-Dilatancy relationship for 5.0% shear strain amplitude test

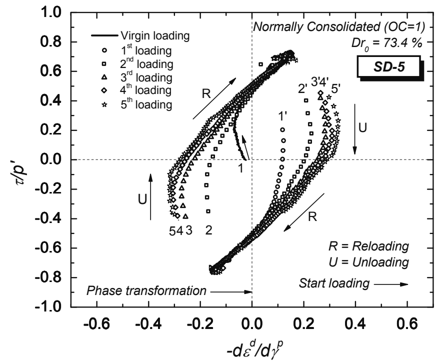


**Figure 8(c).** Stress-Dilatancy relationship for increasing shear strain amplitude test

These observations are consistent with the ones in the aforementioned stress-controlled tests where the progressive decrease in the shear strain amplitude induced shrinkage of the stress-dilatancy curves towards the zero dilatancy state, suggesting that the levels of strain amplitudes have a significant effect on the dilatancy characteristics. Such effect may be qualitatively explained by assuming the different extents of disturbance of the particle structure induced by cyclic loading. As the structure of particles is disturbed to larger extents when they are sheared with larger strain amplitude, soil specimen tends to exhibit more contractive behavior in the subsequent cycle, in particular at the states immediately after changing the loading direction. Therefore, the corresponding stress-dilatancy curves move away from zero dilatancy state (Fig. 8(c)). On the other hand, the extents of disturbance of the particle structure become smaller when the strain amplitude during cyclic loading is decreased, as is the case with the constant shear stress amplitude tests, resulting into less contractive response with the shrinkage in the stress-dilatancy curves (Figs. 4(c), 5(c) and 6(c)).

## CONCLUSIONS

The investigation on the effects of over-consolidation histories, initial relative densities and strain amplitudes on the stress-dilatancy relationships of sand under drained cyclic torsional shear conditions obtain several observations, which are:

1. The effect of over-consolidation histories only appear during the first half cycle of cyclic loading (virgin loading). During the virgin loading, NC specimen shows a pure contractive behavior, while the OC specimen initially behaves dilative. In addition, the effects of over-consolidation histories are disappear in the following subsequent cyclic loading, both NC and OC specimens exhibit similar behavior.
2. Initial relative densities of specimen do not affect the stress-dilatancy characteristics of sand. Both specimens in loose and dense states condition behave similarly.
3. The shear strain amplitude appears to be one of the major factors to affect stress-dilatancy relationships of sand. As soil particles are sheared with larger shear strain amplitude, specimen tends to exhibit more contractive behavior in the subsequent cycle. As a result, the stress-dilatancy curves expand, moving away from zero dilatancy state. On the other hand, the shrinkage of stress-dilatancy curves in constant shear stress amplitude tests is caused by their stress-strain relationships, in which the shear strain amplitudes decreased during subsequent cyclic loading.

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