SPATIALLY DISPERSED TMDS BASED ON MTMD METHOD FOR VIBRATION CONTROL OF MULTI-MODES OF LARGE SPAN STRUCTURES

Susumu YOSHINAKA[1] and Ken'ichi KAWAGUCHI[2]

ABSTRACT: The purpose of this study was to develop a multi-mode vibration control system for large span structures. Large span structures have different vibration properties than building structures. Then, in this study, we propose a vibration control system using spatially dispersed arranged TMDs (Tuned mass dampers). In order to determine the design parameters of TMDs, the MTMD (Multiple TMD) method proposed by Abe and Fujino is applied. In order to verify the control effect of the proposed method, a numerical analysis using a flat plate model was executed. From the analytical results, we could see that under the condition of equal total TMD mass the proposed method was particularly effective.

Key Words: *Vibration Control, Large Span Structures, TMD, MTMD(Multiple TMD)*

INTRODUCTION

The TMD (Tuned mass damper) is a passive vibration control device consisting of a mass, a spring and a damper. It has been mainly used for tall buildings excited by wind or earthquake load, and its effectiveness has been well proved. However, recently it has been increasingly applied for controlling the ambient vibrations of large-span light-weight structures. For example, 29 pairs of TMDs have been installed in the London Millenium Bridge to avoid excessive oscillation caused by pedestrians [1].

The purpose of this study was to develop a TMD design method for large-span spatial structures that cover large spaces and areas, e.g. domes, and stadium roofs.

Generally, large-span spatial structures have various vibration modes whose frequencies are much more closely spaced than for building structures and whose modal shapes are very complex. The generally used single TMD has only one natural frequency and it is used for controlling one main mode. Therefore, in order to apply TMD to large-span spatial structures and to effectively control multi-modes, we need to develop a design method that is different from the conventional single TMD method.

In this study, we propose a vibration control system using spatially dispersed arranged TMDs [3]. For the design parameter set-up, the MTMD (Multiple TMD) method has already been developed for controlling one main mode. This method is robust property to changes of design parameters, especially the tuning ratio. We apply this method for controlling multi-modes and use it in this research.

A method using the superposed shape of the controlling modes is newly proposed for determining the MTMD arrangement.

[1] Research associate, Institute of Industrial Science, University of Tokyo
[2] Associate Professor, Institute of Industrial Science, University of Tokyo

This paper analytically compares the proposed method and the conventional single TMD method. All analyses shown in this paper are executed using the MSC-NASRAN finite element program.

DESIGN METHOD

As discussed above, large-span spatial structures have various vibration modes with complex modal shapes. Therefore, it is difficult to achieve overall structural control with only one TMD. Thus, we propose a vibration control system using spatially dispersed plural TMDs, as shown in Figure 1. Because small TMDs are used, it is possible to install TMD devices inside structural members or joints.

The detailed design method for TMD arrangement and parameters are described below.

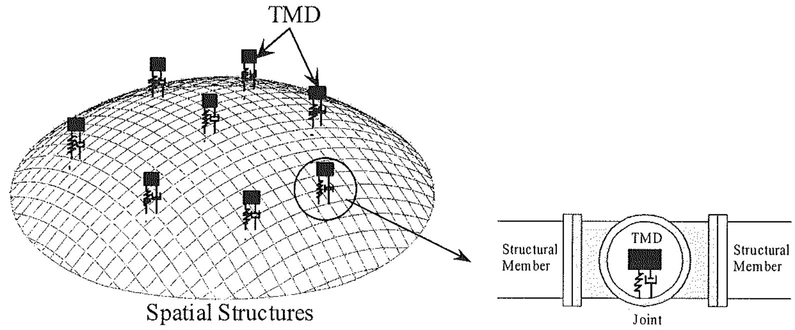


Figure 1. Proposed method (Spatially dispersed TMDs)

Design Parameters

In this study, the MTMD method is used to set up the design parameters, the tuning ratio and the damping ratio. It was developed by Abe and Fujino [3], and consists of a number of small TMDs with different natural frequencies distributed over a certain range around the natural frequency of the main structure. Its concept is shown in Figure 2. It has two important properties that are different from those of the conventional single TMD method, as described below.

a) Effectiveness at the resonance point of a main structure

b) Robustness against errors of design parameters

This study focuses on the robustness of the tuning ratio and applies it to control of multi-modes. Some other studies have aimed mainly at control of building structures [4]-[7]. A. Kareem and S. Kline executed a parameter study, e.g. total bandwidth of MTMD, tuning ratio of each MTMD and number of MTMDs, under random loading [4]. G. Chen and J. Wu proposed a sequential procedure for practical design and placement of the dampers in seismically excited building structures [5].

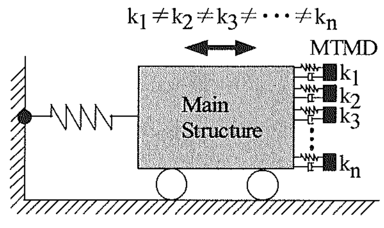


Figure 2. Concept of MTMD

In this method, all parameters are basically determined by the bandwidth of the MTMD (bandwidth between highest and lowest natural frequencies). Thus, the determination of the bandwidth is very important. Abe and Fujino determined the critical bandwidth by a criterion in which all MTMDs are strongly coupled with a main structure for all modes. In this study, in order to control multi-modes, the bandwidth of MTMD is re-adjusted to the proposed critical bandwidth to involve the natural frequencies of the controlling modes of a main structure. Detailed equations solving the critical bandwidth and damping ratio are described below.

The distribution of natural frequencies N $(=2n+1)$ MTMD on the frequency axis is shown in Figure 3.

The mass and damping ratio of the each MTMD, i.e. m and ξ_T, are equal, and the natural frequencies ω_j of the MTMD are evenly distributed from ω_{-n} to ω_n. ω_0 and ω_s are the central natural frequencies of the MTMD and the main structure, respectively.

The relation between ω_0 and ω_j is shown by Equation (1). The relation between ω_0 and ω_s are shown by Equation (2).

$$\omega_j = \omega_0(1+j\beta) \qquad (j=-n \sim n) \tag{1}$$

$$\omega_0 = \omega_s(1+\beta_0) \tag{2}$$

where β and β_0 are the non-dimensional frequency space. The non-dimensional bandwidth of the MTMD is defined as

$$B = (\omega_n - \omega_{-n})/\omega_s \tag{3}$$

The relation between β and B is expressed as

$$\beta = B/(N-1) = B/(2n) \tag{4}$$

The central frequency of the MTMD is taken as

$$\omega_0 = \omega_s \Big/ \sqrt{1+\mu_{total}} \tag{5}$$

where μ_{total} is the total mass ratio of the MTMD. The critical bandwidth B_c proposed by Abe and Fujino is expressed by

$$B_c = \sqrt{(\mu_{total}T)/2} \tag{6}$$

where $T = \gamma + \log(N)$ ($\gamma = 0.57721\cdots$: Euler's constant).

The damping ratio of the MTMD is proposed as

$$\xi_T = \sqrt{3}\beta/\pi \tag{7}$$

The critical bandwidth B_c is determined by the criterion that all MTMD are strongly coupled with the main structure for all modes. In case the adopted adjusted bandwidth B is greatly extended compared to the critical bandwidth B_c, the MTMD becomes decoupled with the main structure.

Then, the special characteristics of the MTMD method described above disappear and the MTMD method becomes similar to the conventional single TMD method. From the results obtained from the present study, about twice the critical bandwidth B_c is considered to be the standard application limit of the proposed method.

Therefore, the proposed method is considered to be more applicable to large-span spatial structures with various vibration modes whose frequencies are more closely spaced than those of building structures.

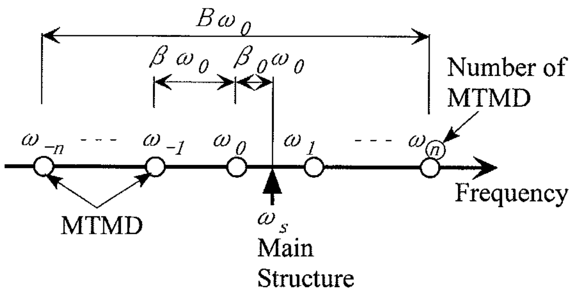


Figure 3. Distribution of natural frequencies of MTMD

MTMD Arrangement

As discussed above, in this study we utilize the MTMD's robust tuning property to control multi-modes with closely spaced frequencies. Therefore, in order to determine the MTMD's arrangement, we need to locate them at common large displacement points of controlling modes. Thus, as described in detail below, we propose a design method using the superposed shape of controlling modes.

Figure 4 compares (a) the conventional single TMD method and (b) the proposed MTMD method for arrangement of TMD and MTMD. In the former method, the TMD is usually put at the largest displacement point (antinode) of each controlling mode. In the latter method, the MTMD are dispersed among the largest displacement points of the superposed shape of the controlling modes. Using this method, under the condition of equal total mass of TMD, we can control structures more efficiently.

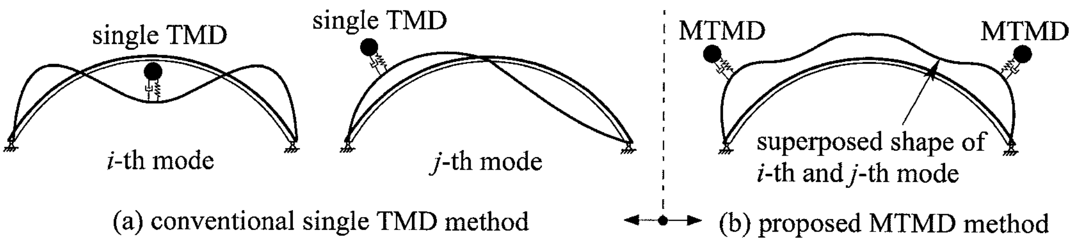


Figure 4. Comparison of two methods based on TMD and MTMD arrangements

Let me summarize the main points of the proposed method.

a) Design parameters

The bandwidth B of the MTMD is re-adjusted to the proposed critical bandwidth B_c to involve natural frequencies of controlling modes of the main structure. The damping ratio is calculated by Equation (7), determined by the bandwidth B.

b) MTMD arrangement

The MTMDs are dispersed among the largest displacement points of the superposed shape of the controlling modes.

NUMERICAL ANALYSIS

In order to confirm the control effect of the proposed method, we adopt a simple-shaped square plate model that covers a large area and use it to execute a frequency response analysis. To compare the efficiency of the conventional single TMD method and the proposed method, the total masses of the TMD and the MTMDs are set to be equal.

For a multi-degree-of-freedom system, an equivalent mass is usually used to set the mass ratio of the TMD. However, in this study we intend to control multi-modes simultaneously and it is difficult to determine an appropriate equivalent mass for each mode. Therefore, in this analysis we use an overall structural mass for the mass of the main structure and use it to explain the analytical results. It was confirmed from previous studies, for the MTMD method, the difference between results using an equivalent mass and an overall structural mass is smaller than for the conventional single TMD method.

Analytical Model

A square reinforced concrete slab 8.0 m square × 20.0 cm thick is chosen as an analytical model. The material properties, i.e. elastic modulus, shear modulus and Poisson's ratio, are assumed to be the values for concrete FC 240 with a mass density of 2.4 t/m^3. The overall structural mass is 3.07 t. The boundary condition is assumed to be pinned supported around the model.

Modal Analysis

In order to grasp the basic vibration properties of the plate alone, a linear modal analysis is executed. The obtained natural frequencies up to the sixth mode are shown in Table 1 and the modal shapes are shown in Figure 5. In Table 1, also shows the theoretical results of frequencies with a pinned supported rectangular plate calculated by Equation (8) and (9). The second and third modes, and the fifth and sixth modes are degenerated modes of equal natural frequencies.

$$p_{mn} = q_{mn}{}^2 h \sqrt{\frac{E}{3(1-\nu^2)\rho}} \tag{8}$$

$$q_{mn} = \pi \sqrt{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \tag{9}$$

where p_{mn} is the natural angular frequency and q_{mn} is the wave propagation constant, a and b are the lengths of the rectangular plate, m and n are the number of nodal lines parallel to side a and b, respectively, $2h$ is the plate thickness, ν is Poisson's ratio, ρ is the mass density of the plate, and E is the elastic modulus.

The controlling modes in this analysis are chosen in order to clarify the difference between the conventional single TMD method where one TMD is installed in one antinode of each mode and the proposed MTMD method where plural MTMDs are dispersed.

Then, the 4th and 5th modes in which the number of antinode is relatively many and in addition whose frequencies are closely spaced are chosen.

It can be considered that this method is applicable to all cases of controlling multi-modes under the condition in which the natural frequencies are not so separated and have the merits that we can choose optionally controlling modes to correspond to the vibration source.

Table 1. Natural frequency of plate model up to 6^{th} mode (Unit:Hz)

Modal number	Numerical analysis	Theoretical solution
1	2.77	2.81
2, 3	6.91	7.03
4	10.9	11.3
5, 6	13.8	14.1

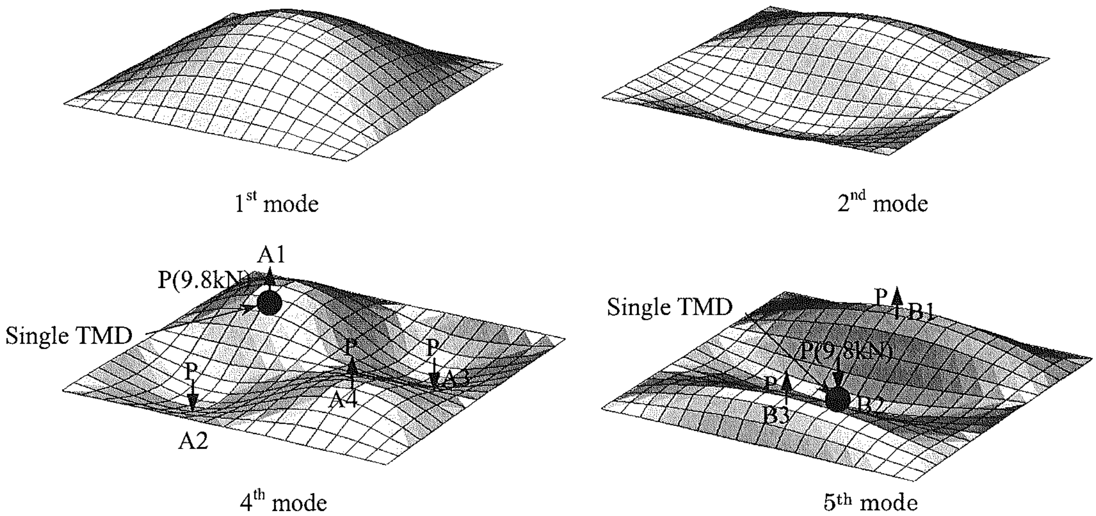


Figure 5. Modal shape

Design of Single TMD and MTMD

To compare the efficiency of the conventional single TMD method and the proposed MTMD method, the total masses of the TMD and MTMD are each set to 1.20 t. Therefore, the mass ratio to the overall structural mass is 3.91%.

(a) Single TMD method

In the single TMD method, one TMD is installed at antinodes of the 4^{th} and 5^{th} modes, as shown in Figure 5 (points A1 and B2). The design parameters, optimum tuning ratio γ_{opt} and optimum damping ratio ξ_{opt}, are determined by Equations (10) and (11) by Den Hartog. In these equations, μ is the mass ratio of one TMD to the overall structural mass of the main structure. Determined design parameters are shown in Table 2.

$$\gamma_{opt} = \frac{1}{1+\mu} \tag{10}$$

$$\xi_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{11}$$

Table 2. Design parameters for single TMD

Parameter	Value
Mass of one TMD	0.600 t (Mass ratio : 1.95%)
Tuning ratio	0.980
Damping ratio	0.0857

(b) MTMD method

MTMD are installed at the maximum displacement points of the superposed shape of the 4th and 5th modes obtained by modal analysis. Both modal shapes are normalized by the maximum displacement of each mode. The superposed shape and installation points of four MTMDs (from C1 to C4) are shown in Figure 6.

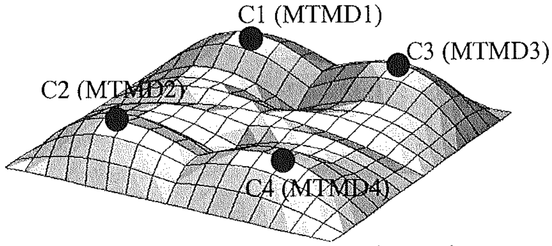


Figure 6. Superposed shape of 4th and 5th modes

The MTMD's design parameters are determined by Equations (1)-(7). When setting up its bandwidth and tuning ratios, the control target frequency is set to the middle frequency of the 4th and 5th modes (12.3Hz), and its bandwidth is magnified to the critical bandwidth B_c to include the natural frequencies of both modes. The re-adjusted bandwidth is twice the critical bandwidth and is from 10.2 to 13.9 Hz. The design parameters of four MTMDs are shown in Table 3 and their distributions with natural frequencies on the frequency axis are shown in Figure 7. Their instalation points are shown in Figure 6.

Table 3. Design parameters of four MTMDs

Parameter		Value
Mass of one MTMD		0.300 t (Mass ratio : 0.977%)
Tuning ratio	MTMD1	0.830
	MTMD2	0.930
	MTMD3	1.03
	MTMD4	1.13
Damping ratio		0.0558

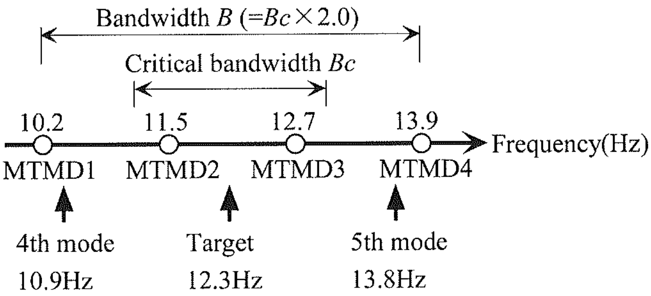


Figure 7. Frequency distributions of four MTMDs

Frequency Response Analysis

In order to compare the single TMD method and the proposed MTMD method, a frequency response analysis is executed. All the loading points are set to antinodes of the 4th and 5th modes shown in Figure 5 and magnitude P is assumed to be 9.8 kN. The damping coefficient is 3.0% for each mode.

The frequency response curve around the natural frequency of the 4th mode with the vertical displacement of point A1 at which a single TMD is installed in 4th mode loading is shown in Figure 8, and the frequency response curve around the natural frequency of the 5th mode of point B2 at which a single TMD is installed in 5th mode loading is shown in Figure 9. From these figures, it can be observed that the proposed MTMD method is efficient inside the MTMD bandwidth but the response is higher than that of the single TMD outside the MTMD bandwidth. The response peak occurs around 15.0 Hz in Figure 9 because the response is affected by the 10th mode (15.2Hz) of the plate-MTMD model. As mentioned below, we choose the natural frequencies of the 4th and 5th modes to compare the efficiency of two methods because these frequencies are target frequencies of a single TMD and it is easy to compare them. The MTMD design method is controlling throughout a given bandwidth and different to that of a single TMD that controls one target frequency.

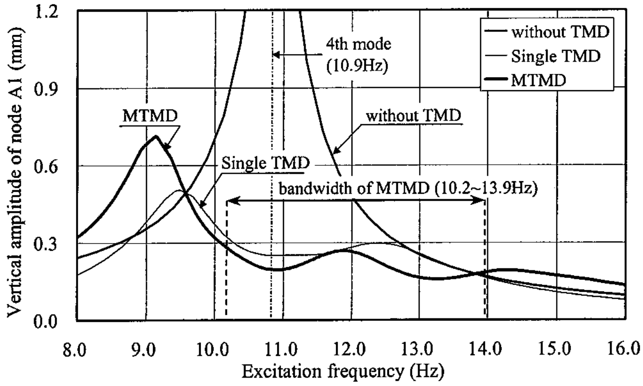


Figure 8. Frequency response curve (4th mode loading)

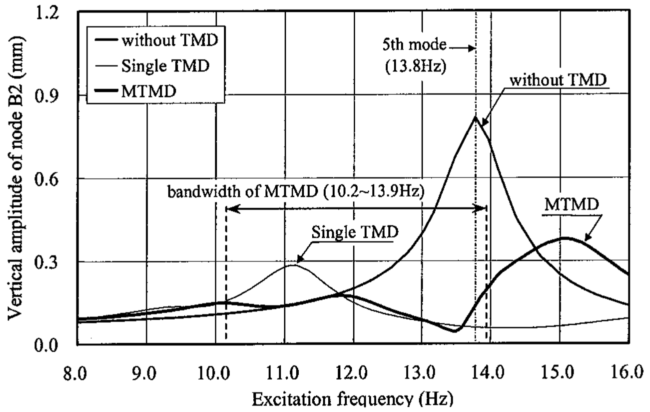


Figure 9. Frequency response curve (5th mode loading)

The comparison of amplitude from points A1 to A4 at the natural frequency of the 4th mode in 4th mode loading is shown in Figure 10. It can be observed that, for the single TMD method, the amplitude of point A1 at which a single TMD is installed is controlled efficiently but dispersion of amplitude with other points is very large. However, for the MTMD method, the amplitudes of all points are controlled efficiently. The reason why the efficiency of the MTMD is also higher at point A1 at which a single TMD is installed is that for the single TMD method the mass ratio of 1.95% used to control only the 4th mode is effective, but for the MTMD method a total mass ratio of 3.91% is effective for both modes. The planar distribution of frequency responses with vertical amplitude at the natural frequency of the 4th mode is shown in Figure 11. The scale showing the amplitude is set to be common. It can be observed from this figure that the MTMD is more efficient than a single TMD for overall plate control. The comparison of amplitude from points B1 to B3 at the natural frequency of the 5th mode in 5th mode loading is shown in Figure 12. In this case also, it can be observed that, for the single TMD method, the amplitude of point B2 at which a single TMD is installed is controlled efficiently but dispersion of amplitude of other points is much larger than with the MTMD method.

In order to compare the overall plate energy, Figure 13 compares the sums of the squares of the amplitudes of all points at the natural frequencies of the 4th and 5th modes. In both modes, the MTMD is more efficient than a single TMD.

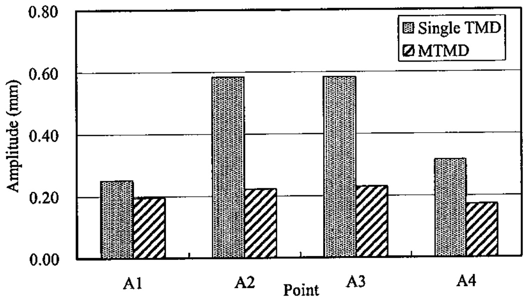


Figure 10. Comparison of amplitude (4th mode loading)

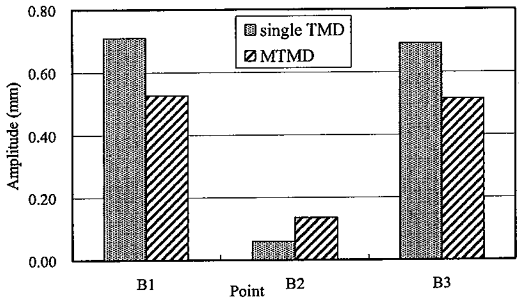
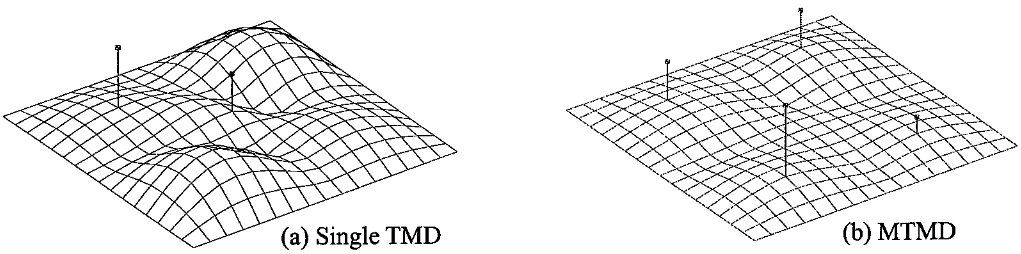


Figure 12. Comparison of amplitude (5th mode loading)



(a) Single TMD (b) MTMD

Figure 11. Comparison of planar distribution of amplitude (4th mode loading)

CONCLUSIONS

This paper has proposed a new vibration control system using an MTMD method. This method is applicable to structures in which multi-modes with closely spaced natural frequencies are oscillated. It is thus considered that this method is especially suitable for large-span spatial structures. It is also applicable to control of ambient vibration of large-span light-weight structures, e.g. large-span slabs constructed mainly in urban areas.

From the analytical results described in Chapter 3, it can be concluded that:

1) The proposed MTMD method is especially effective inside the MTMD bandwidth but its responses are a little high outside the MTMD bandwidth. Therefore, the effect of the proposed method is closely connected with the MTMD bandwidth.
2) The proposed method is much more effective than the conventional single TMD method for overall structural control.
3) Under the condition of equal total TMD mass, the proposed method is particularly effective compared to the single TMD method.

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REFERENCES

[1] P. Dallard, A. J. Fitzpatrick (2001). "The London Millennium Footbridge"; *The Structural Engineer*, vol. 79, pp. 17-33.
[2] Yoshinaka S., Kawaguchi K. (2003). "SOME SURVEYS OF MULTIPLE TMD SYSTEMS FOR LARGE SPAN STRUCTURES", *Bulletin of earthquake resistant structure research center*, No.36, pp. 143-148.
[3] Abe M., Fujino Y. (1994). "DYNAMIC CHARACTERIZATION OF MULTIPLE TUNED MASS DAMPERS AND SOME DESIGN FORMULAS", *Earthquake engineering and structural dynamics*, vol.23, pp. 813-835.
[4] A. Kareem, S. Kline (1995). "PERFORMANCE OF MULTUPLE MASS DAMPERS UNDER RANDOM LOADING", *Journal of structural engineering*, 121(2), pp. 348-361.
[5] G. Chen, J. Wu (2001). "OPTIMAL PLACEMENT OF MULTIPLE TUNE MASS DAMPERS FOR SEISMIC STRUCTURES", *Journal of structural engineering*, September 2001, pp. 1054-1062.
[6] Yamaguchi H., N. Harnpornchai (1993). "FUNDAMENTAL CHARACETRISTICS OF MULTIPLE TUNED MASS DAMPERS FOR SUPPRESSING HARMONICALLY OSCILLATIONS", *Earthquake engineering and structural dynamics*, Vol.22, pp. 51-62.
[7] T. Igusa, K.Xu (1994). "VIBRATION CONTROL USING MULTIPLE TUNED MASS DAMPERS", *Journal of sound and vibration*, 175(4), pp. 491-503.