

# Fundamental Analysis for Semi-Active Seismic Isolation System with Controllable Friction Damper Using Piezoelectric Actuators

by

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## ABSTRACT

In order to decrease a relative displacement between ground and a superstructure, a semi-active seismic isolation system using a controllable friction damper was developed [1,2]. However, if the controller and the actuator of the controllable friction damper break down when a great earthquake occurs, it can't demonstrate its full performance. In this case, the superstructure acts as a low damping seismic isolation system, and the relative displacement becomes great. As a result, serious damage may occur by a collision of the superstructure with surrounding structures. In this study, a new controllable friction damper using piezoelectric actuators is proposed. This damper has a fail-safe mechanism enabling the system to demonstrate damping effect in case of malfunction. This paper outlines the simulation results for the seismic isolation effect, the relative displacement reduction effect, and the performance of the fail-safe mechanism of the new controllable friction damper.

## 1. INTRODUCTION

In order to decrease response accelerations of a superstructure during an earthquake, a lot of base-isolated buildings using passive isolation systems have been constructed [3]. In these conventional isolation systems, large horizontal displacements between the ground and the superstructures are inevitable to achieve satisfactory reduction in response accelerations of the superstructures. To decrease the relative displacement, a semi-active seismic isolation system using a controllable friction damper was developed, which has the seismic isolation effect with smaller the relative displacement.

The friction force of the conventional system (hereafter referred as "controllable friction damper of holding type") is generated by the driving power of the actuators, and the damping effect can't be expect if the power source to the system is cut off because of a large earthquake. In such a case, the building acts as a low damping seismic isolation system, which means that the relative displacement is great and serious damage may occur by the collision with surrounding structures.

To solve this problem, a controllable friction damper with fail-safe mechanism is proposed. An experimental model of the semi-active seismic isolation system using the controllable friction damper with the fail-safe mechanism (hereafter referred as "controllable friction damper releasing type") is to be produced, and excitation tests is to be conducted. For a preliminary study, a fundamental analysis model for the experimental model was established, and the seismic isolation effect, the relative displacement reduction effect, and the performance of the fail-safe mechanism were examined.

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## 2. SEISMIC ISOLATION SYSTEM USING CONTROLLABLE FRICTION DAMPER

### 2.1 Analysis model

The seismic isolation system using the controllable friction damper was modeled as a single degree of freedom system, as shown in Fig.1. Equations of motion of the model are expressed in two phases, as shown below, considering changeover of static/dynamic friction due to presence or absence of sliding at the friction damper.

(1) Phase I : No sliding at friction damper

$$\begin{cases} x = \text{const.} \\ \dot{x} = 0 \\ \ddot{x} = 0 \end{cases} \quad (1)$$

(2) Phase II : Sliding occurs at friction damper

$$\ddot{x}(t) + 2\zeta\omega\dot{x}(t) + \omega^2x(t) + \text{sgn}(\dot{x})F(t) = -\ddot{z}(t) \quad (2)$$

And the changeover criteria between Phase I and Phase II are as follows.

(1) From Phase I to Phase II

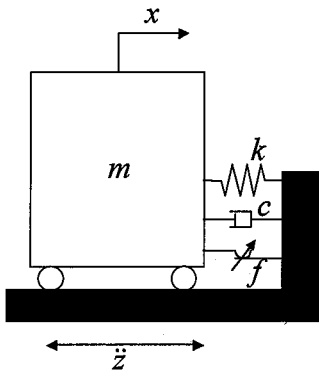
$$|kx + m\ddot{z}| > f \quad (3)$$

(2) From Phase II to Phase I

$$\dot{x} = 0 \quad \text{and} \quad |m\dot{x}| < 2f \quad (4)$$

where

$$\zeta = \frac{c}{2\sqrt{mk}}, \quad \omega = \sqrt{\frac{k}{m}}, \quad F(t) = \frac{f(t)}{m}$$



$x$  : Relative displacement of the superstructure to the ground  
 $m$  : Mass of the superstructure  
 $k$  : Spring constant of the passive isolation device  
 $c$  : Damping coefficient of the passive isolation device  
 $f$  : Friction force

Fig.1 Analysis model

## 2.2 Controllable friction damper

The controllable friction damper of holding type generates the friction force by pressing the friction material with actuators. The mechanism of the controllable friction damper of holding type is shown in Fig.2. The relationship between the actuator force and the friction force is expressed by Eq.(5).

$$f(t) = \mu p(t) \quad (5)$$

where

$f(t)$  : Friction force  
 $\mu$  : Friction coefficient  
 $p(t)$  : Actuator force

At a static situation, the actuator force is 0, therefore the friction force is 0. When an earthquake occurs, the damping force of the controllable friction damper is generated by controlling the actuator force. However, if a great earthquake occurs, the power source to the drivers of the controllable friction damper's controller and the actuators may be cut off. In such a case, the controllable friction damper of holding type can't generate the friction force, which means that the damping force of the damper is 0. So, the building acts as a low damping isolation system, which means that the relative displacement is great and serious damage may occur by the collision with surrounding structures.

To solve this problem, a controllable friction damper of releasing type with a fail-safe mechanism is proposed. The mechanism of the controllable friction damper of releasing type is shown in Fig.3. At the initial state, the friction materials are pressed. Once an earthquake occurs, the actuator force is applied to reduce the friction force by detaching the friction material. The relationship between the actuator force and the friction force is described as follows.

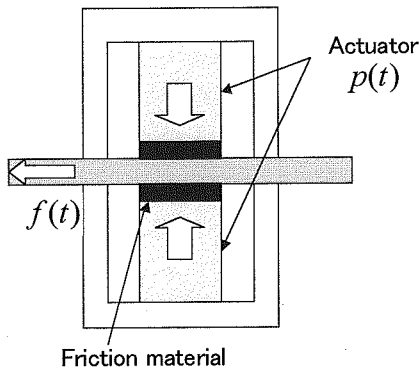


Fig.2 Controllable friction damper of holding type

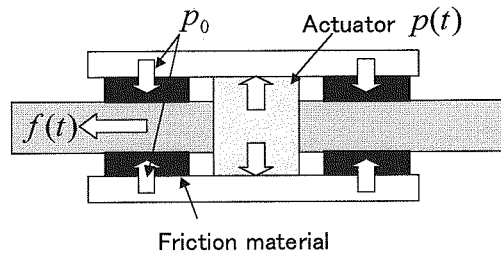


Fig.3 Controllable friction damper of releasing type

$$f(t) = \mu(p_0 - p(t)) \quad (6)$$

where

$$\begin{aligned} f(t) & : \text{Friction force} \\ \mu & : \text{Friction coefficient} \\ p_0 & : \text{Initial pressing force} \\ p(t) & : \text{Actuator force} \end{aligned}$$

If the actuator force  $p(t)$  is 0, the friction force  $f(t)$  is at its initial value of  $\mu p_0$ , and the damping effect can be expected. Therefore, this system can prevent serious damage of the base-isolated structure because of the collision with surrounding structures in case of power supply failure or malfunctions.

### 3. CONTROL RULE

In the semi-active seismic isolation system using the controllable friction damper of releasing type, the equations of motion are as follows:

$$\begin{aligned} \ddot{x}(t) + 2\zeta\omega\dot{x}(t) + \omega^2 x(t) + \text{sgn}(\dot{x}(t))F(t) &= -\ddot{z}(t) \\ F(t) &= \frac{\mu(p_0 - p(t))}{m} \end{aligned} \quad (7)$$

As shown above, the system has non-linear characteristics. Instantaneous Optimal Control algorithm (hereafter referred as "IOC"), which is effective to nonlinear system, is used to obtain optimal actuator force  $p(t)$ . As a performance index of IOC, the following function  $J(t)$  was defined.

$$J(t) = q_v \dot{x}^2(t) + q_d x^2(t) + q_f F^2(t) + p^2(t) \quad (8)$$

where

$$q_v \geq 0, \quad q_d \geq 0, \quad q_f \geq 0$$

Minimizing the performance index  $J(t)$  at every time instant  $t$  is examined. Over a small time interval  $\Delta t$ , the relative displacement  $x(t)$  and the relative velocity  $\dot{x}(t)$  can be expressed as follows:

$$x(t) = x(t - \Delta t) + \dot{x}(t - \Delta t)\Delta t + \frac{1}{2}(2\ddot{x}(t - \Delta t) + \ddot{x}(t))\Delta t^2 \quad (9)$$

$$\dot{x}(t) = \dot{x}(t - \Delta t) + \frac{1}{2}(\ddot{x}(t - \Delta t) + \ddot{x}(t))\Delta t \quad (10)$$

Eqs. (7), (9) and (10) lead to

$$\left(1 + \frac{\Delta t^2}{6}\omega^2\right)x(t) + \frac{\Delta t^2}{3}\zeta\omega\dot{x}(t) + \frac{\Delta t^2}{6}\text{sgn}(\dot{x}(t))F(t) + \frac{\Delta t^2}{6}\ddot{z}(t) + d_1(t - \Delta t) = 0 \quad (11)$$

$$\frac{\Delta t}{2} \omega^2 x(t) + (1 + \Delta t \zeta \omega) \dot{x}(t) + \frac{\Delta t \operatorname{sgn}(\dot{x}(t)) F(t)}{2} + \frac{\Delta t}{2} \ddot{z}(t) + d_2(t - \Delta t) = 0 \quad (12)$$

where

$$d_1(t - \Delta t) = -x(t - \Delta t) - \dot{x}(t - \Delta t) - \frac{\Delta t^2}{3} \ddot{x}(t - \Delta t) ,$$

$$d_2(t - \Delta t) = -x(t - \Delta t) - \frac{\Delta t}{2} \ddot{x}(t - \Delta t) \quad (13)$$

And, the second of Eqs.(7) immediately gives

$$F(t) - \frac{\mu}{m} p_0 + \frac{\mu}{m} p(t) = 0 \quad (14)$$

Lagrange's method of undetermined multipliers is applied to minimize the performance index  $J(t)$ . By introducing Lagrange multipliers  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , Lagrange function  $H(t)$  is generalized as follows:

$$\begin{aligned} H(t) = & q_v \dot{x}^2(t) + q_d x^2(t) + q_f F^2(t) + p^2(t) \\ & + \lambda_1 \left( \left( 1 + \frac{\Delta t^2}{6} \omega^2 \right) x(t) + \frac{\Delta t^2}{3} \zeta \omega \dot{x}(t) + \frac{\Delta t^2 \operatorname{sgn}(\dot{x}(t)) F(t)}{6} + \frac{\Delta t^2}{6} \ddot{z}(t) + d_1(t - \Delta t) \right) \\ & + \lambda_2 \left( \frac{\Delta t}{2} \omega^2 x(t) + (1 + \Delta t \zeta \omega) \dot{x}(t) + \frac{\Delta t \operatorname{sgn}(\dot{x}(t)) F(t)}{2} + \frac{\Delta t}{2} \ddot{z}(t) + d_2(t - \Delta t) \right) \\ & + \lambda_3 \left( F(t) - \frac{\mu}{m} p_0 + \frac{\mu}{m} p(t) \right) \end{aligned} \quad (15)$$

If Eq. (15) is minimized, the performance index  $J(t)$  is also minimized. The necessary conditions for minimizing Eq. (15) are as follows:

$$\frac{\partial H(t)}{\partial \dot{x}(t)} = 0, \quad \frac{\partial H(t)}{\partial x(t)} = 0, \quad \frac{\partial H(t)}{\partial F(t)} = 0, \quad \frac{\partial H(t)}{\partial p(t)} = 0 \quad (16)$$

which yield

$$\frac{\partial H(t)}{\partial \dot{x}(t)} = 2q_v \dot{x}(t) + \lambda_1 \frac{\Delta t^2}{3} \zeta \omega + \lambda_2 (1 + \Delta t \zeta \omega) = 0,$$

$$\frac{\partial H(t)}{\partial x(t)} = 2q_d x(t) + \lambda_1 \left( 1 + \frac{\Delta t^2}{6} \omega^2 \right) + \lambda_2 \left( \frac{\Delta t}{2} \omega^2 \right) = 0,$$

$$\frac{\partial H(t)}{\partial F(t)} = 2q_f F(t) + \lambda_1 \frac{\Delta t^2 \operatorname{sgn}(\dot{x}(t))}{6} + \lambda_2 \frac{\Delta t \operatorname{sgn}(\dot{x}(t))}{2} + \lambda_3 = 0,$$

$$\frac{\partial H(t)}{\partial p(t)} = 2p(t) + \lambda_3 \frac{\mu}{m} = 0 \quad (17)$$

Solving Eq.(17), the control input  $p^*(t)$  is obtained as follows :

$$p^*(t) = \frac{q_f \mu^2}{m^2 + q_f \mu^2} p_o - \frac{q_v m \mu \Delta t \operatorname{sgn}(\dot{x}(t))}{2(m^2 + q_f \mu^2) \left(1 + \frac{\Delta t^2}{6} \omega^2 + \Delta t \zeta \omega\right)} \dot{x}(t) - \frac{q_d m \mu \Delta t^2 \operatorname{sgn}(\dot{x}(t))}{6(m^2 + q_f \mu^2) \left(1 + \frac{\Delta t^2}{6} \omega^2 + \Delta t \zeta \omega\right)} x(t) \quad (18)$$

As the sufficient conditions to minimize the performance index  $J(t)$ , Hessian matrix should be positive.

$$\begin{aligned} \frac{\partial^2 H(t)}{\partial \dot{x}^2(t)} &= 2q_v, & \frac{\partial^2 H(t)}{\partial x^2(t)} &= 2q_d, & \frac{\partial^2 H(t)}{\partial F^2(t)} &= 2q_f, & \frac{\partial^2 H(t)}{\partial p^2(t)} &= 2, \\ \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial x(t)} &= \frac{\partial^2 H(t)}{\partial x(t) \partial \dot{x}(t)} = 0, & \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial F(t)} &= \frac{\partial^2 H(t)}{\partial F(t) \partial \dot{x}(t)} = 0, \\ \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial p(t)} &= \frac{\partial^2 H(t)}{\partial p(t) \partial \dot{x}(t)} = 0, & \frac{\partial^2 H(t)}{\partial x(t) \partial F(t)} &= \frac{\partial^2 H(t)}{\partial F(t) \partial x(t)} = 0, \\ \frac{\partial^2 H(t)}{\partial x(t) \partial p(t)} &= \frac{\partial^2 H(t)}{\partial p(t) \partial x(t)} = 0, & \frac{\partial^2 H(t)}{\partial F(t) \partial p(t)} &= \frac{\partial^2 H(t)}{\partial F(t) \partial x(t)} = 0 \end{aligned} \quad (19)$$

Hessian matrix is as follows :

$$\begin{bmatrix} \frac{\partial^2 H(t)}{\partial \dot{x}^2(t)} & \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial x(t)} & \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial F(t)} & \frac{\partial^2 H(t)}{\partial \dot{x}(t) \partial p(t)} \\ \frac{\partial^2 H(t)}{\partial x(t) \partial \dot{x}(t)} & \frac{\partial^2 H(t)}{\partial x^2(t)} & \frac{\partial^2 H(t)}{\partial x(t) \partial F(t)} & \frac{\partial^2 H(t)}{\partial x(t) \partial p(t)} \\ \frac{\partial^2 H(t)}{\partial F(t) \partial \dot{x}(t)} & \frac{\partial^2 H(t)}{\partial F(t) \partial x(t)} & \frac{\partial^2 H(t)}{\partial F^2(t)} & \frac{\partial^2 H(t)}{\partial F(t) \partial p(t)} \\ \frac{\partial^2 H(t)}{\partial p(t) \partial \dot{x}(t)} & \frac{\partial^2 H(t)}{\partial p(t) \partial x(t)} & \frac{\partial^2 H(t)}{\partial p(t) \partial F(t)} & \frac{\partial^2 H(t)}{\partial p^2(t)} \end{bmatrix} = 16q_v q_d q_f > 0 \quad (20)$$

As shown above, Hessian matrix is positive. Therefore the control input  $p^*(t)$  given by Eq. (18) minimizes the performance index  $J(t)$  given by Eq. (8)

## 4. SIMULATION RESULTS

### 4.1 Performance of seismic isolation system

For numerical simulations, the experimental model of the seismic isolation system was modeled as a single degree of freedom system. The mass of the model was 6,000 (kg), and the natural period and the damping ratio of the seismic isolation system were 3 (s) and 2 (%), respectively. The coefficients of the static and dynamic friction of the controllable friction damper were both 0.3. The initial pressing force of the controllable friction damper of releasing type was set to 30,000 [N], considering that the damper was designed to be effective when the input level of seismic ground motions was 75 [cm/s] to 100 [cm/s]. The piezoelectric actuator was used in the controllable friction damper, because the it is small, has high responsibility and can generate the large force.

The input waves are EL Centro NS (1940, Imperial Valley Earthquake), JMA NS (1995 Hyogoken-Nanbu Earthquake), Hachinohe NS (1968 Tokachi-Oki Earthquake) and Taft EW (1952 Arvin-Tahachapi Earthquake). The input levels were set to 25 [cm/s], 50 [cm/s], 75 [cm/s] and 100 [cm/s] for each wave.

The weighting coefficients of the relative velocity  $q_v$ , and the relative displacement  $q_d$  in the performance index  $J(t)$  given by Eq. (8) were set to 0 and  $2.0 \times 10^{20}$ , respectively. Fig.4 and 5 show the results of the simulations. For the comparison, the results of three other simulations are shown, which are 1) the semi-active seismic isolation system using the controllable friction damper of holding type whose controller was designed with the Linear Quadratic (LQ) optimal regulator problem, 2) the semi-active seismic isolation system using the controllable friction damper of holding type whose controller was designed with IOC, and 3) the passive seismic isolation system with damping ratio of 20 [%] Fig.4 shows maximum response accelerations of the superstructure and Fig.5 shows maximum relative displacements between the superstructure and the ground. The semi-active seismic isolation system using the controllable friction damper of releasing type could decrease the response accelerations to almost the same level as the passive seismic isolation system, as shown in Fig.4, and could decrease the relative displacements to about 1/2 of the passive seismic isolation system, as show in Fig.5. Moreover, the response accelerations are smaller than the other two semi-active seismic isolation systems, and the relative displacements are also smaller than the other two semi-active seismic isolation systems with the input level of up to 75 (cm/s). As for the input level of 100 (cm/s), the relative displacement of semi-active seismic isolation system using the controllable friction damper of releasing type is larger than the other two semi-active seismic isolation systems. This is because the controllable friction damper of holding type has no preset upper limit of the maximum friction force, while the controllable friction damper of releasing type has the upper limited corresponding to the initial friction force. To be effective at larger input level, a larger initial pressing force is needed, which means that a larger actuator is needed to release the friction material. The releasing force of 30,000[N] in this controllable friction damper of releasing type can be easily generated by existing piezoelectric actuators.

### 4.2 Performance at malfunction

In order to confirm the fail-safe mechanism of the controllable friction damper of releasing type, the simulations were executed under the condition that the piezoelectric actuator doesn't function. Fig.6 and 7 show the results of the simulations. For the comparison, the results of 1) the controllable friction

damper of holding type under the same condition, 2) the controllable friction damper of releasing type without malfunction, and 3) the passive seismic isolation system, are also shown in these figures. The controllable friction damper of holding type can't generate the damping force when the actuator doesn't function, and excessive relative displacement occurs as shown Fig. 7. On the other hand, the controllable friction damper of releasing type shows the damping effect by the friction force generated by the initial pressing force even if the piezoelectric actuator doesn't function, although the response acceleration is larger than the malfunctioned controllable friction damper of holding type. It was confirmed that because of the fail-safe mechanism, even if the controllable friction damper of the controllable friction damper of releasing type doesn't function, an excessive relative displacement doesn't occur.

## 5. CONCLUSIONS

It was confirmed that the semi-active seismic isolation system using the controllable friction damper of releasing type has the equal or more performance to the semi-active seismic isolation system using the controllable friction damper of holding type. Even if the controller and the actuator of the controllable friction damper doesn't function when a great earthquake occurs, the semi-active seismic isolation system using the controllable friction damper of releasing type can prevent excessive displacement, because of its fail-safe mechanism. This means that serious damage caused by the collision of the base-isolated structure with surrounding structures can be prevented.

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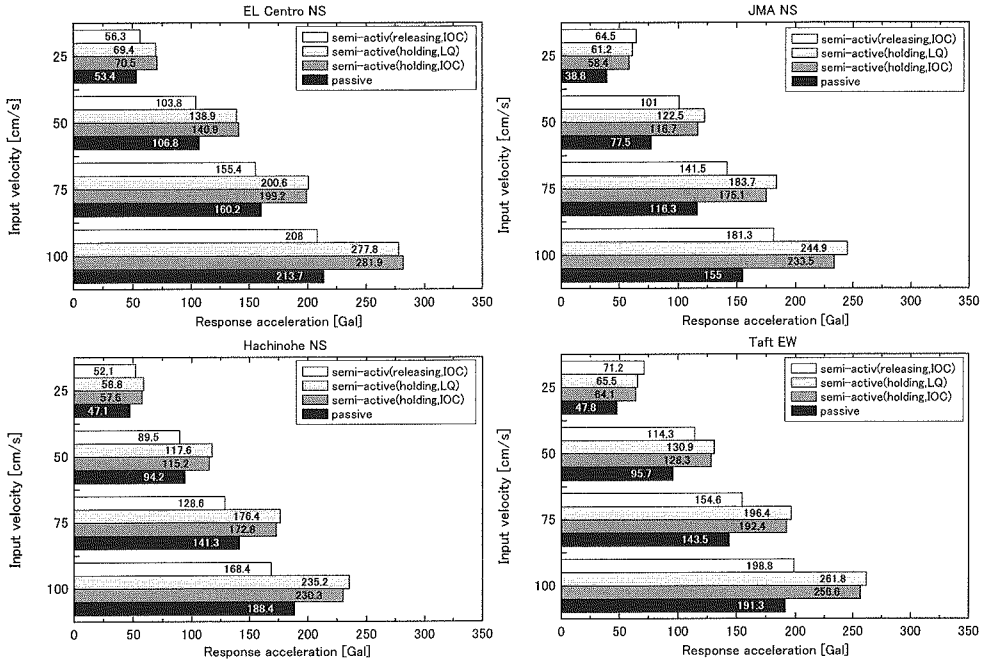


Fig.4 Simulation results of the response accelerations

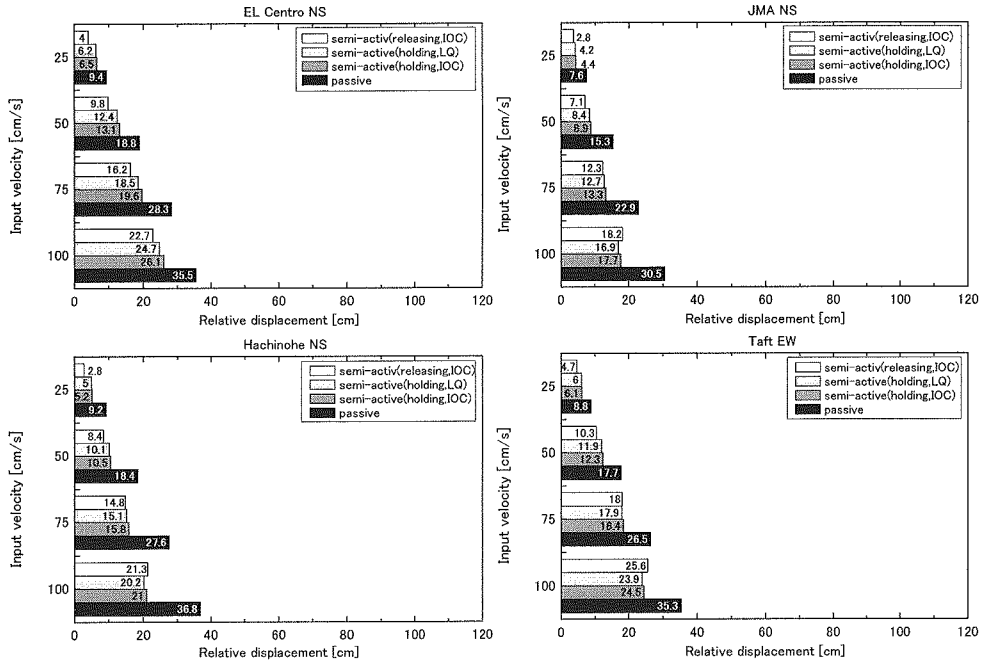


Fig.5 Simulation results of the relative displacements

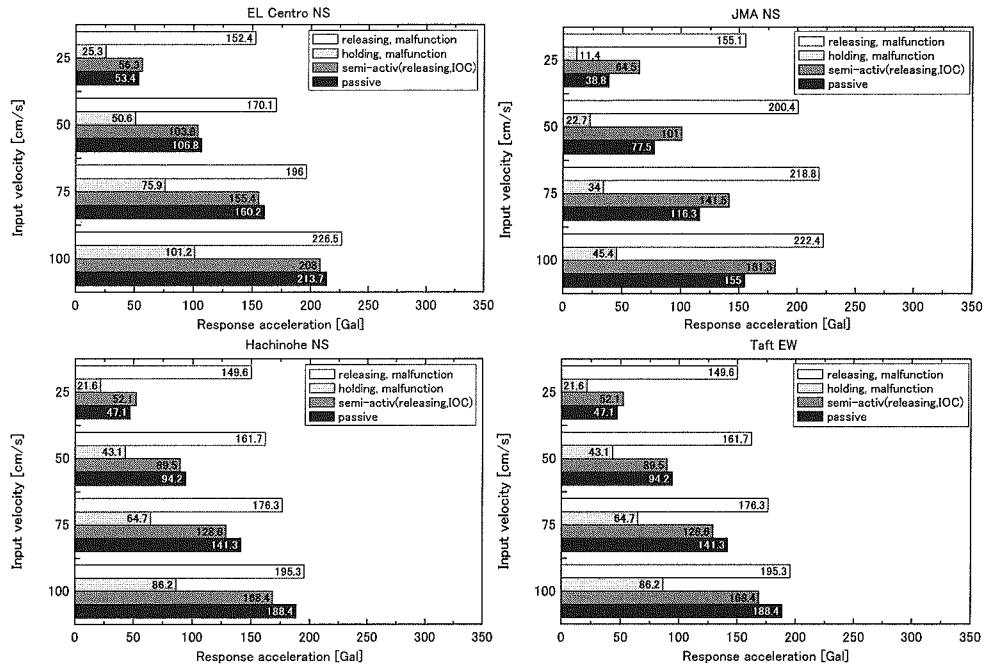


Fig.6 Simulation results for the response accelerations at malfunction

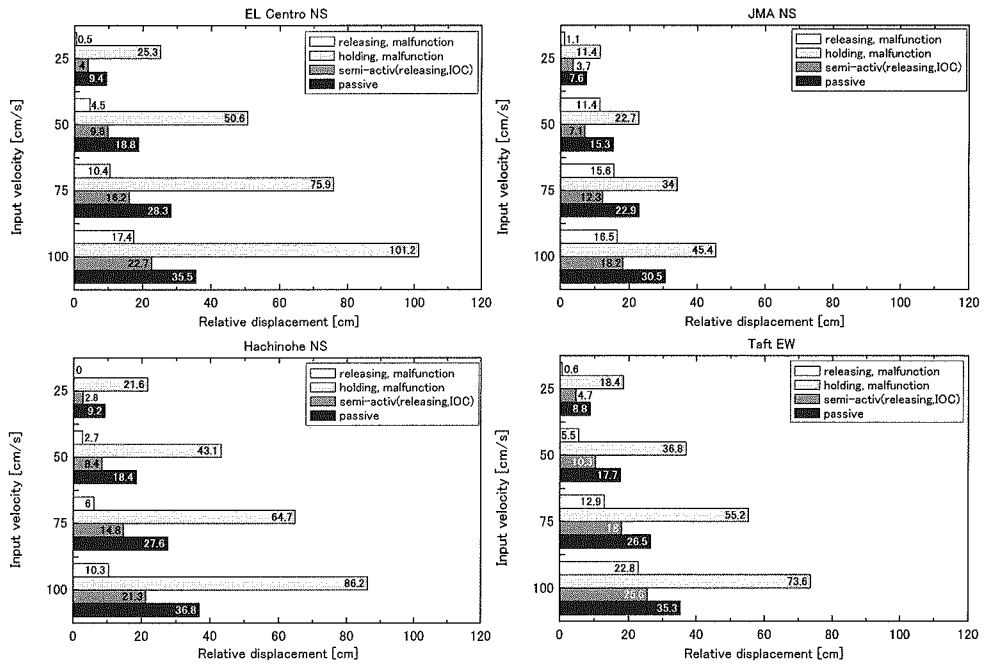


Fig.7 Simulation results for the relative displacements at malfunction