An Adaptive Loading System for Hybrid Design Point Search on Steel Frames Subjected to Random Loads

by

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Introduction

Various types of failure modes have been observed in recent earthquake damage, and some of them seemed even not imagined during their design stages. A few might be resulted from human errors, but the main reason is the uncertainty of seismic action itself. To manage this uncertainty is a fundamental problem in structural design. On the other hand, standards of values in general are becoming more and more diverse, and structural designers also begin to consider other limit states than ordinary structural safety and serviceability, which are extended to repair-ability, immediate occupancy, various damage controls and so on. Consequently, it becomes harder to judge whether a structure reaches to one of its limit states or not by a simple criterion, such as how strong it is or how much it can deform. We need to look at and study carefully its performance, preferably by experiment, from various standards of values. Such a trend requires a development of a new experimental tool applicable to uncertain actions.

More than a quarter of century has passed after some of *ERS* members initiated a pseudo-dynamic on-line test technique [1]. This technique combined a loading test with a numerical response analysis. This paper targets to extend this technique to general uncertain actions by combining a loading test with a design point search analysis in the First-Order Reliability Method [2] abbreviated by *FORM* hereafter. The system developed drives a test structure to a design point or a most probable failure situation under specified random actions. As the first step of such an application, a load model is limited to an equivalent static one in this paper. Which load profile is most likely to drive a structure to its failure depends on how the structure fails, in other words, its failure mode as well as its non-linear behavior. The test system monitors inelastic behaviors of test structure and changes a loading pattern adaptively into a most likely load pattern. Two-story steel moment resisting frames are tested by this system considering different types of limit state functions.

Concept of Hybrid Design Point Search

Determination of a design point in *FORM* is basically identical to the following constrained optimization problem: *Minimize* $||\{u\}||$ *subject to* $G(\{u\})=0$ ··················(1)

where $\{u\}$: Standard random variables. They are normalized to have zero-mean and a unit-standard deviation, and they are uncorrelated with one another. In this paper all the structural parameters are deterministic, and all the basic variables consist of the parameters of actions.

$G(\cdot)$: Limit state function expressed in the standard space that gives positive values for safety or accepted domain and negative values for failure or not accepted domain.

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The design point denoted by $\{u^*\}$ is regarded as the most likely failure situation, which corresponds to the point with the highest probability density on the limit state surface.

Generally the limit state function is a non-linear function, and many algorithms are proposed to solve the optimization problem stated in Equation (1). For example, an iterative technique termed HL-RF method similar to the Newton-Raphson algorithm is sometimes used for this problem. The limit state function or the *G*-function need not be an explicit function, and any procedure, any algorithm, and any computer program will be used so far as it can give a value to judge the state of structure, accepted or not accepted. A basic proposal in this paper is to evaluate the value of *G*-function from the observation in a loading test carried out simultaneously. At each stage in the loading procedure, the design point is consecutively predicted from the information obtained so far, and the next loading step is targeted to the new design point predicted. While an ordinary *FORM* design point search is performed based on a behavior analysis on a mathematical model of a structure, the method proposed is based on a loading test on a physical model of a structure or a specimen, and then such a procedure is referred as *hybrid* design point search hereafter. Figure 1 illustrates such a concept of hybrid design point search.

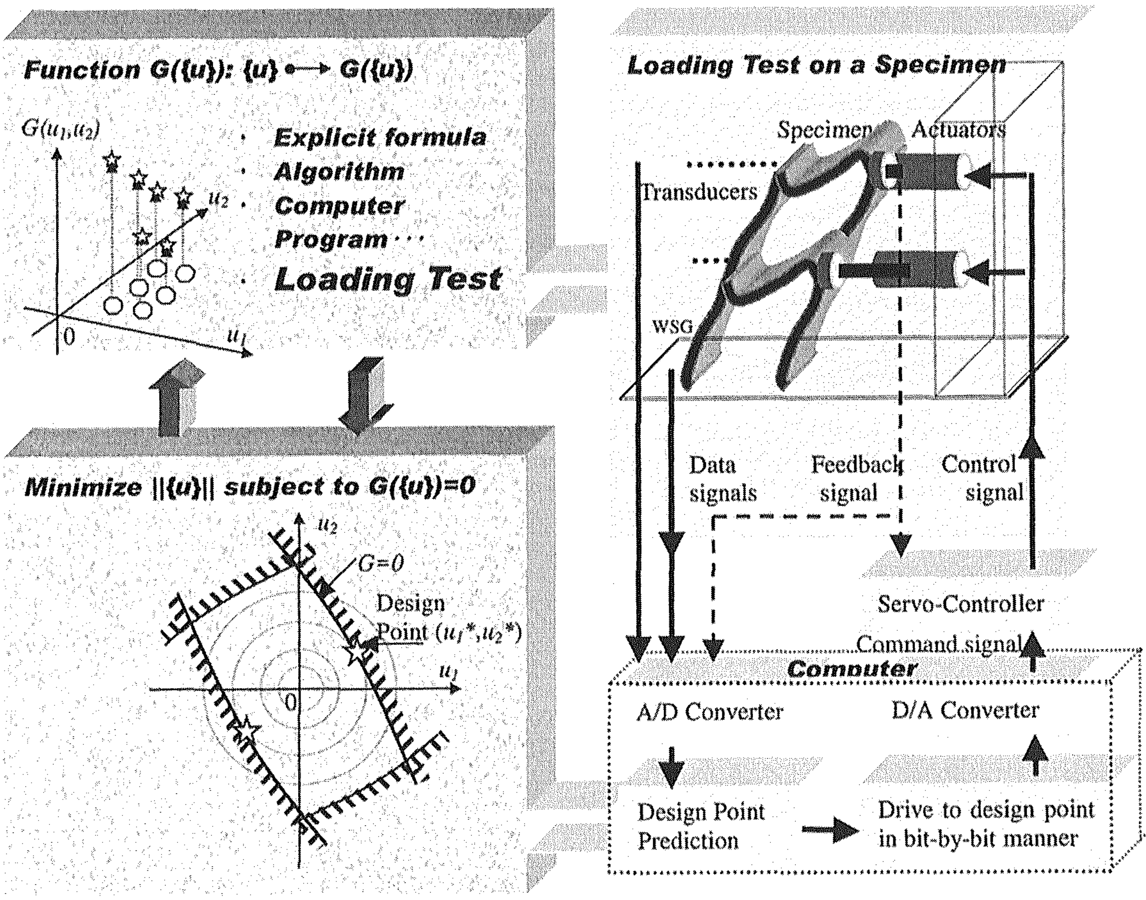


Figure 1: Concept of Hybrid Design Point Search

Example (I): Drift Limitation

Test structure: Test structure is a 2-story 1-bay moment resisting frame fabricated from steel plate columns (*PL-250x22*) and H-shaped beams (*H-150x150x6.5x9*) as shown in Figure 2. Steel material used is ordinary mild steel, JIS SS400 grade, and the mechanical properties of steel plate are shown in Table 1. The beam members are designed much stiffer and stronger than the column members, and then the test frame is regarded as a shear-type weak-column strong-beam frame.

Action model: Lateral forces are applied at both beam levels by electro-hydraulic actuators. The load vector denoted by $\{p_1, p_2\}^T$ is arranged as a multiple-mode random load: a linear combination of two deterministic load modes, $\{p_1, p_2\}^T = r_1\{1,1\}^T + r_2\{1,-1\}^T$, where r_1 and r_2 are zero-mean random multipliers uncorrelated with each other. Two cases for their standard deviations are tested as shown in Table 2, where the standard deviations of r_1 and r_2 are denoted by σ_1 and σ_2, respectively. The ratio of σ_2 to σ_1 is taken 1.0 for Case (1) and 5.0 for Case (2), respectively, and then the opposite-sign load mode of $\{1, -1\}^T$ is especially dominant in Case (2). Standard basic variables are non-dimensional and defined by $\{u_1, u_2\}^T = \{r_1/\sigma_1, r_2/\sigma_2\}^T$.

Limit state: The limit state considered is the limitation of inter-story drift denoted by δ_i in centimeter. Drift limit is taken 8 centimeters commonly for each story. When either story drift exceeds this limit, the system is defined to fail, that is, the system is regarded as a series system. The limit state at each story is judged by element limit state functions, such as $G_1=8.0-||\delta_1||$ or $G_2=8.0-||\delta_2||$. The system limit state function denoted by G is evaluated in the form: $G=G_1*G_2$ *for* $G_1>0$ *and* $G_2>0$, *otherwise* $G=0$
Each inter-story drift is a function of standard load variables, $\{u_1, u_2\}^T$, but the function is not given explicitly because the structural property of test structure is unknown before a loading test is performed. Inter-story drifts are directly measured during a loading test and used to evaluate the value of G-function. If a test structure would exhibit an idealistic bi-linear behavior, a view of G-function could be drawn as shown in Figure 3. In the following procedure, however, the value of G-function is directly evaluated from the loading test without any assumption on structural properties.

Search algorithm: The following non-derivative search algorithm was applied in the test procedure:
Step 1: The origin in u_1-u_2 space (a mean-value situation) is adopted as the starting load point, and a few sample values of G-function are evaluated at the load points locating with a small distance from the origin.
Step 2: The load point corresponding to the smallest G-value is chosen as the center load point, and a specified number of load points near to this point are also chosen from the past loading history. These load points plus current load point and the corresponding samples of G-values are used to approximate the G-function by a hyper plane. The form of G-function is approximated by $G=a_0+a_1u_1+a_2u_2$.
The coefficients, a_0, a_1 and a_2, in this equation are determined by least square method to minimize the sum of square errors from the sampled G-values for the load points chosen.
Step 3: The new design point denoted by (u_1^*, u_2^*) is predicted by use of the linear approximation of G-function in Step 2: *Minimize* $||\{u\}||$ *subject to* $a_0+a_1u_1+a_2u_2=0$
Step 4: A new load point is targeted in the direction to the new design point (u_1^*, u_2^*) with a specified small amount of load increment from the current load point.
Step 5: After actual loading is performed to the new load point, specimen response is measured and the G-value is monitored again at the new load point. When the monitored G-value becomes non-positive, that is, when the load point has reached to the design load point on a limit state surface, the test procedure is completed. Otherwise, the load point stays still in the safety or accepted domain and the test procedure is repeated again from Step 2. This non-derivative procedure search procedure is illustrated in Figure 4.

Test results: Figures 5 and 6 show the results of hybrid design point search for the two test cases. When an identical standard deviation is assigned for each load mode in Case (1), the story drift exceeds the limitation at the 1st/lower story. On the other hand, when a large value of standard deviation is assigned for the opposite-sign load mode in Case (2), the drift limitation is detected at the 2nd/upper story. Thus, a structure fails in a different manner depending on uncertainty of applied actions.

Table 1: Mechanical Properties of Steel Material for Columns

Yield stress	290 MPa
Tensile strength	440 Mpa
Elongation	30 %

Table 2: Summary of Hybrid Design Point Search for Drift Limit

	Case (1)	Case (2)
$E[r_1]$ $E[r_2]$	0.0	0.0
σ_2/σ_1	1.0	5.0
u_1*	2.133	0.108
u_2*	0.018	-0.824
u_1**	2.164	0.137
u_2**	0.007	-0.869

u_1* and u_2*: Results of hybrid search
u_1** and u_2**: Numerical solution based on an idealistic bi-linear behavior

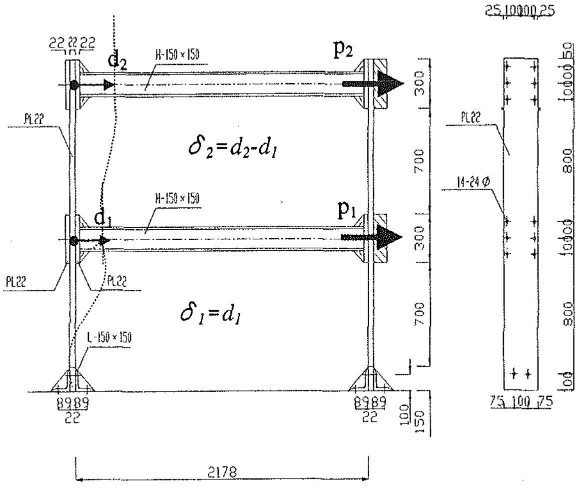


Figure 2: Test Frame for Drift Limitation

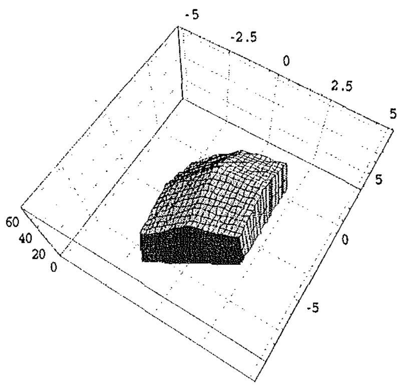


Figure 3: A View of Limit State Function Based on Idealistic Bi-linear Behavior

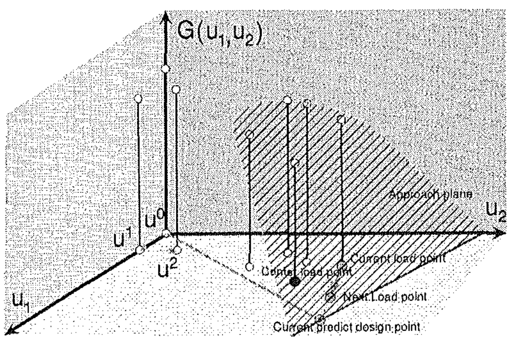


Figure 4: Non-derivative Procedure for Hybrid Design Point Search

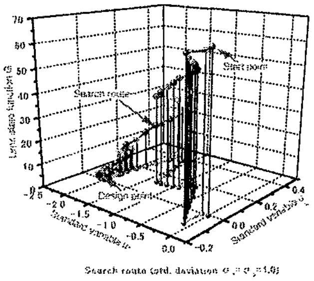


Figure 5: Hybrid Design Point Search (Drift Limitation, Case (1))

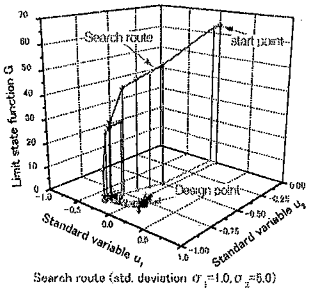


Figure 6: Hybrid Design Point Search (Drift Limitation, Case (2))

Example (II): Collapse Mechanism Formation

Test structure: Another test structure is a 2-story 1-bay moment resisting frame, which is fabricated from H-shaped members (commonly *H-125x125x5.5x9* cross-section bent about weak axis) as shown in Figure 7. Steel material used is JIS SS400 grade steel, and its mechanical properties are shown in Table 3. Over the beam-to-column node and the beam-ends, cover plates and splice plates were high-tension bolted from both sides of beam and column flanges, and these portions are regarded as rigid zones. In the design of test frame, by adjusting the size of rigid zone and the location of 2nd floor beam, three kinds of collapse mechanism are made possible to occur as shown in Figure 8: they are the 1st story local collapse, the 2nd story local collapse, and the overall collapse accompanied by yielding at 2nd floor beam-ends.

Action model: Lateral forces are applied at each beam levels, and they are modeled as a linear combination of two load modes similarly to Example (I). This multiple-mode load is herein regarded as a representative of seismic load effect to a linear-elastic system. Classical normal modes in linear-elastic vibration are used to determine the basic load modes as follows:

$$\{d\}=[\phi_1, \phi_2]\{q\}=[\Phi]\{q\} \quad \cdots\cdots\cdots\cdots\cdots(2)$$
$$\{p\}=[\psi_1, \psi_2]\{r\}=[\Psi]\{r\} \quad \cdots\cdots\cdots\cdots\cdots(3)$$

where $\{d\},\{p\}$: displacement and force vector in original coordinate, respectively
$\{q\},\{r\}$: displacement and force vector in modal coordinate, respectively
$\{\phi_j\}$: participation vector or classical normal mode vector multiplied by participation factor
$\{\psi_j\}$: column vector of matrix $[\Psi]$, the identity of total work requires $[\Psi]=[\Phi^T]^{-1}$
Considering the orthogonality of modes about mass matrix, $\{\psi_j\}$ is given by $[m]\{\phi_j\}/m_j^*$, where m_j^* is effective mass. Note: $\{1\}^T\{\psi_j\}=1$, then each modal multiplier r_j shows the contribution of each mode to base shear force.

In this example a random load vector is given by $\{p\}=r_1\{\psi_1\}+r_2\{\psi_2\}$, where r_1 and r_2 are random variables and $\{\psi_1\}$ and $\{\psi_2\}$ are deterministic basic load modes. These load modes are arranged by eigen-value analysis on a linear-elastic model of the test frame by assuming that an identical mass is placed at each floor. In a more rigorous analysis the modal force multiplier denoted by r_j may be modeled as a random process, but it is modeled herein simply as a random variable similar to the arbitrary-point-in-time value of a stationary random process. The stochastic features of modal multiplier

Table 3: Mechanical Properties of Steel Material-Example (II)

Yield stress	370 MPa
Tensile strength	490 Mpa
Elongation	29 %

Table 4: Summary of Hybrid Design Point Search for Drift Limit

	Case (3)	Case (4)
$\{\psi_1\}$	2^{nd} Fl: 0.204	
	R^{oof} Fl: 0.796	
$\{\psi_2\}$	2^{nd} Fl: 1.345	
	R^{oof} Fl: -0.345	
$E[r_1]$,$E[r_2]$	0.0	
σ_1/σ_2	3.0	1.0
Collapse mechanism identified by test system	Overall collapse with beam yielding	2nd story local collapse ↓ Overall collapse ↓ 1st story local collapse

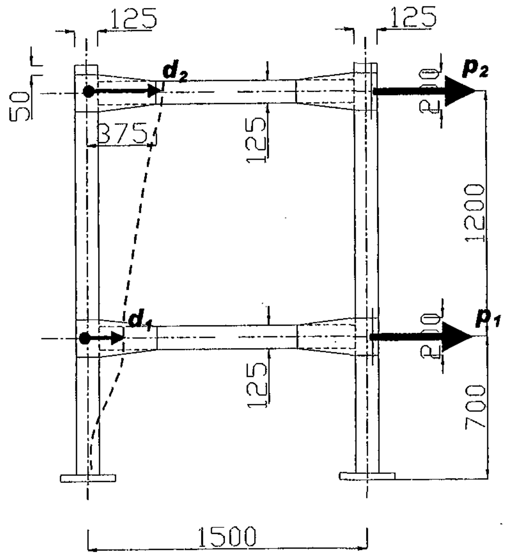


Figure 7: Test Frame for Limit State of Collapse Mechanism Formation

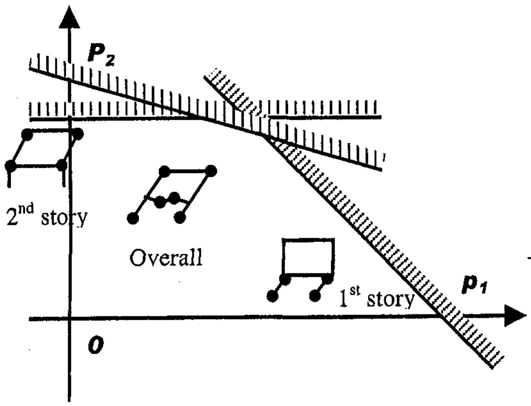


Figure 8: Limit state surface in original lateral load space

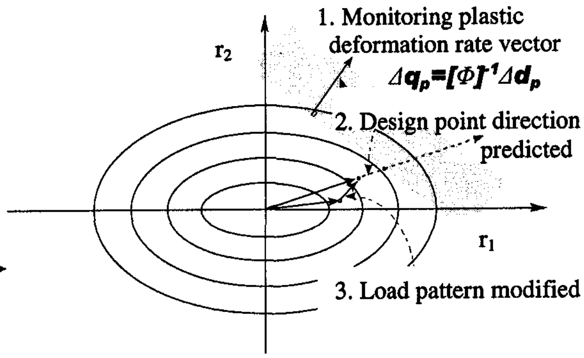


Figure 9: Control process in modal load space (Collapse Mechanism Formation)

might be adjusted consistent with a design earthquake response spectrum [3]. Two cases were tested here about the ratios of the 2nd mode standard deviation to the 1st mode as shown in Table 4.

Limit state and search algorithm: The limit state considered here is collapse mechanism formation. In the sense of rigid-plastic analysis, when a collapse mechanism is specified as a compatible set of plastic deformation rate/increment, we can write down a limit state function as:

$$G(\{u\})=\{\Delta\theta_P\}^T\{M_P\}-\{\Delta d_P\}^T\{p\}, \quad \text{(Note: } \{p\} \text{ is a function of } \{u\}\text{)} \qquad \cdots\cdots\cdots\cdots(4)$$

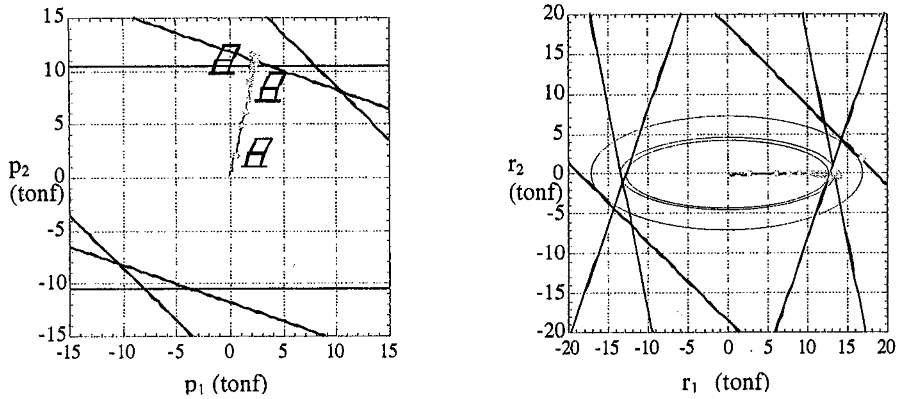
where $\{\Delta d_P\}$: nodal plastic displacement increment, $\{\Delta\theta_P\}$: plastic hinge rotation increment

When we test an unknown inelastic structure, however, the mechanism shape is unknown before testing. In the following procedure we monitor a set of nodal displacement denoted by $\{\Delta d\}$, from a loading test on an inelastic frame, and evaluate the plastic component, $\{\Delta d_P\}$ $(=\{\Delta d\}-\{\Delta d_E\})$, by subtracting an elastic component denoted by $\{\Delta d_E\}$ $(=[K_E]^{-1}\{\Delta p\})$. Here we assume that this plastic component, $\{\Delta d_P\}$, is forecasting the shape of collapse mechanism to be formed on this loading stage. Apparently, only by such a shape of mechanism, we cannot figure out the complete limit state function given by Equation (4). It is not enough to predict the position of limit state surface, but enough to grasp the direction in which the surface lies as shown in Figure 9. Then, we can modify the loading direction targeting the design point from such limited information. When starting this test procedure, the initial load pattern is taken the 1st load mode only, but the load pattern is soon modified by the procedure mentioned above, after a small plastic deformation is detected.

Test results: Figures 10 and 11 show the results of hybrid design point search for the two test cases. When a large standard deviation is assigned for the 1st load mode in Case (3), the load pattern is not so changed from the initial 1st load mode. The collapse mechanism shape identified by the test system is almost the overall mechanism during the whole test procedure. On the other hand, when a same standard deviation is assigned for the 1st and 2nd load mode in Case (4), the loading pattern was consecutively modified during the test. Early stage within elastic range, the 1st load mode was governing. After a while the 2nd story column began to yield, and the system identified it as the 2nd story local collapse mechanism. Then the system increased the contribution of 2nd load mode. After some yielding developed, the resistance of 2nd story column was increased due to strain hardening, and then the 1st story column in turn began to yield. The system identified this event first as the overall collapse mechanism, but soon changed it into the 1st story local collapse mechanism. Through such a sequence of judgments, the system reduced the contribution of 2nd load mode for a while, and increased it again to the opposite sign.

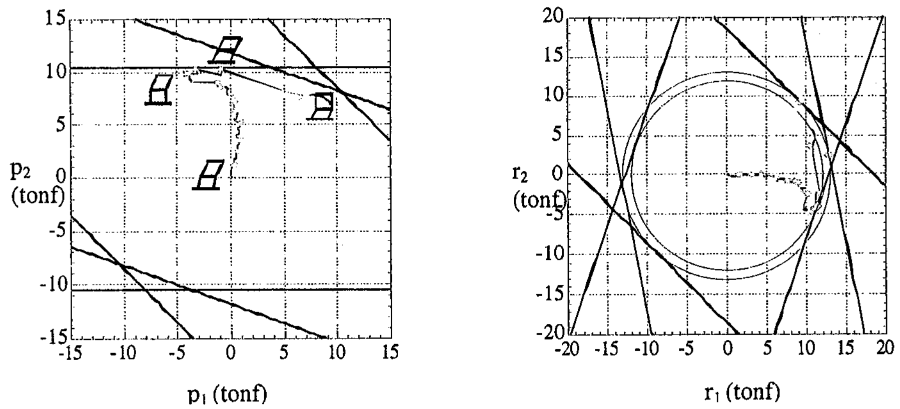
Concluding Remarks

A design point analysis by *FORM* has been combined with a loading test on a structural specimen. In this hybrid design point search procedure, a value and/or a constitution of limit state function is evaluated experimentally from actual inelastic structural behaviors. Two examples were demonstrated on two-story steel frame models: one limit state considered is drift limitation, and the other is collapse mechanism formation, both under multiple-mode random lateral loads. The test results show that the most stringent or the most likely load pattern depends on the failure mode of structure, in particular, when higher modes in vibration are considerably excited. This approach is regarded as a random push over test, and also useful to examine a load pattern used in a non-linear static procedure of performance based seismic design. The test system developed can adaptively change the load pattern by sensing structural behaviors, and it behaves like a sort of artificial intelligence.



(a) Loading process in original load space **(b) Loading process in modal load space**

Figure 10: Hybrid Design Point Search for Collapse Mechanism Formation (Case (3): 1st load mode dominant)



(a) Loading process in original load space **(b) Loading process in modal load space**

Figure 11: Hybrid Design Point Search for Collapse Mechanism Formation (Case (4): 2nd load mode excited)

References

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