Estimation of a Relation between Quality Factors and Standard Penetration Resistance using the Sompi Method

by

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ABSTRACT

The response of a structure to earthquake ground motion is strongly affected by the surface geology. It is recognized that coda parts of the seismic waves represent the surface geology conditions. In this study, the Sompi method, a relatively new technique for spectral analysis, is applied to the coda part. The great advantage of this method is that it evaluates not only the amplitude and phase spectrum but also the wave attenuation properties of the ground. Attenuation properties are represented by quality factors. A qualitative relation between the quality factors directly extracted from accelerograms and the averaged Standard Penetration Resistance (N-value) representing site geology conditions is found. This relation is independent of the distance between the epicenter and the recording station.

INTRODUCTION

Several models have been proposed to study the strong ground motion characteristics. The underlying assumption in all models is that the ground motion at the surface is given by the convolution of the source time function, wave propagation path, and site effect function. It has been observed that the surface ground motion is more sensitive to the latter. Due to the non-homogeneous conditions of the surface geology, site effect function is difficult to be assessed. This often leads to uncertainty in the evaluation of the input ground motion for the design of structures.

The characterization of the surface geology is important for the design of earthquake resistant structures. Field and laboratory tests are available for design parameters to be evaluated. One of the oldest and most commonly used in situ test in geotechnical engineering is the Standard Penetration Test (SPT). Relations between the Standard Penetration Resistance, N-value, and important soil characteristics have been developed. However, until now it has been difficult to relate it with soil attenuation properties such as the quality factor.

It has been recognized that the later portions of accelerograms appearing after the passage of body and surface waves i.e. the recorded coda waves reflect the surface geologic conditions. Using adequate spectral analysis techniques it is possible to recover attenuation characteristics of the soil from them. Since 1995 Hyogo-ken Nanbu earthquake, seismic observation station networks have been deployed in Japan. Geotechnical information at each station as well as strong ground motion records are now easily available. This information is useful for the determination of design parameters specifically quality factors.

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The aim of the present study is to find a relation between quality factors and N-values using strong ground motion records.

METHODOLOGY

The accelerograms recorded at certain number of stations by one particular event were analyzed. The Sompi method was used to obtain the quality factor at each location. The details of this technique are presented in the following section.

Only the later parts of the accelerograms or coda waves were analyzed. Until now there is not specific definition for coda wave. For the present study a simple definition is proposed. Coda wave is defined as the portion of the record after the peak ground acceleration (PGA) whose amplitude does not exceed a certain percentage of the PGA. The Sompi method was applied three times for each station considering three thresholds for the amplitude of coda waves, i.e. 10%, 20% and 30% of the PGA.

Standard Penetration Resistance, N-value, was obtained from the station network database. An average N-value was calculated for each recording station considering the characteristics of the upper 10m of the soil layer.

Several plots of epicentral distances, quality factor and frequency dependence of quality factor were prepared and analyzed.

Fig. 1 schematically shows the methodology of the study.

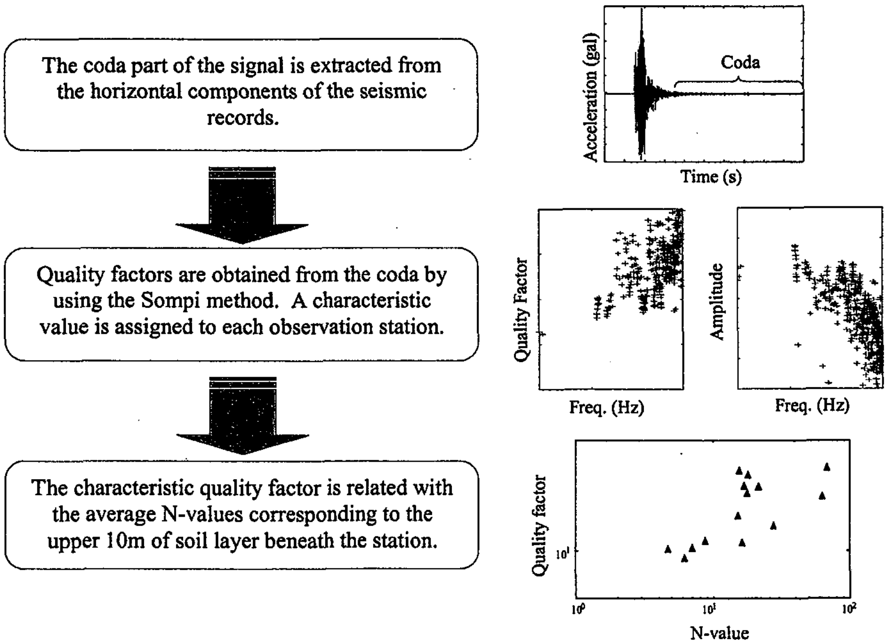

Figure 1. Methodology of the study

SOMPI METHOD

The Sompi method is a relatively new spectral analysis technique developed by Japanese seismologists[1]. The method is based on a non-conventional autoregressive (AR) model. Conventional AR-models estimate the present observation from the past observations by minimizing the estimation error. This implicitly assumes that past observations are noise free. On the other hand, the Sompi AR-model assumes that observations, both past and present, consist of signal and noise thus giving an unbiased estimation of the linear relationship among the successive observations.

The Sompi AR-model assumes that a time series is perfectly separated into mutually independent signal and noise parts. This assumption is expressed as:

$$x_n = u_n + e_n \qquad -N < n < N \tag{1}$$

where: x_n: time series
u_n: signal part
e_n: noise part

The signal part is required to follow a noise-free AR-model of order 'm', which is expressed by the following condition:

$$\sum_{j=-m}^{m} a_j u_{n-j} = 0 \tag{2}$$

A natural way to estimate the AR coefficients is to minimize the following extraction residual, S:

$$S = \frac{1}{2(N-m)+1} \sum_{n=-N+m}^{N-m} \left(\sum_{j=-m}^{m} a_j x_{n-j} \right)^2 \tag{3}$$

where the condition that the norm of the AR coefficients vector is equal to one is imposed in order to avoid the trivial solution. This corresponds to a conditional minimization problem, which can be solved by the Lagrange's method of undetermined multiplier. Introduction of a multiplier λ, leads to the following eigenvalue problem:

$$\sum_{j=-m}^{m} \left(P_{jk} - \lambda \delta_{jk} \right) \cdot a_j = 0 \qquad k = -m, \ldots, m \tag{4}$$

$$P_{jk} = P_{kj} = \frac{1}{2(N-m)+1} \sum_{i=-N+m}^{N-m} x_{i-j} x_{i-k} \qquad j,k = -m, \ldots, m \tag{5}$$

where: P_{jk}: positive, definite, and symmetric matrix

Each eigenvector gives a local extreme of the extraction residual S. Thus, the eigenvector corresponding to the minimum eigenvalue, hereinafter a_j, minimizes S.

Applying Z-transform to a_j and the signal, u_n,

$$A(z) = \sum_{j=-m}^{m} a_j z^{-j} \tag{6}$$

$$U(z) = \lim_{N \to \infty} \sum_{n=-N}^{N} u_n z^{-n} \tag{7}$$

Table 1. The epicenters and magnitudes (Kagoshima-ken Hokusei-bu Earthquake)

Occurrence date	Latitude (N)	Longitude (E)	Magnitude
3/26/1997	32.0	130.4	6.3
5/13/1997	32.0	130.3	6.2

Table 2. Selected seismic stations

Station code	Latitude (N)	Longitude (E)	Distance[1] (km)
KGS001	32.1913	130.1786	29.82
KGS002	32.0875	130.3555	10.61
KGS003	32.0525	130.5897	18.83
KGS004	32.0111	130.1950	19.39
KGS005	31.8972	130.4536	12.51
KGS006	31.9025	130.7044	30.73
KGS007	31.8108	130.3055	22.88
KGS008	31.7586	130.5722	31.42
KGS009	31.7333	130.7636	45.42
KGS010	31.7088	130.2758	34.48
KGS011	31.5861	130.3513	46.31
KGS016	31.4127	130.3247	65.77
KGS036	31.8325	129.8700	53.44

[1] Distance to the epicenter of 3/26 event

and assuming the final convergence of the power series for U(z), eq.(2) becomes:

$$A(z)\cdot U(z)=0 \tag{8}$$

This is the characteristic equation of the Sompi method. From the equation above, it is obvious that a signal may exist only at such z that satisfies A(z)=0, indicating that the signal is specified by a set of line spectra in the Sompi method.

In summary, the main difference between the conventional AR-models and the Sompi AR-model is that the former minimizes the prediction error whereas the latter minimizes the extraction residual. Thus, Sompi model assumes that both past and present observations consist of signal and noise. On the other hand, the conventional models assume that just the present observation has noise.

DATA

This study focused on the Kagoshima-ken Hokusei-bu earthquake, the biggest one that occurred in Japan after the 1995 Hyogo-ken Nanbu earthquake. It consisted of two large events on March 26 and May 13, 1997 causing damage to some structures. The epicenters and magnitudes of the events are shown in Table 1 and Fig. 2. Only the accelerograms corresponding to the March 26 event were analyzed.

The seismic stations considered in this study are part of the Kyoshin Net, a seismic station network installed all over Japan by Science and Technology Agency. The selected stations are located in the Kagoshima prefecture, within 70 km from the epicenter of the March 26 event since the most significant damages occurred in this area. The location of the stations is shown in Table 2 and Fig. 2. The accelerograms and surface soil conditions at each station were available from the Kyoshin Net database.

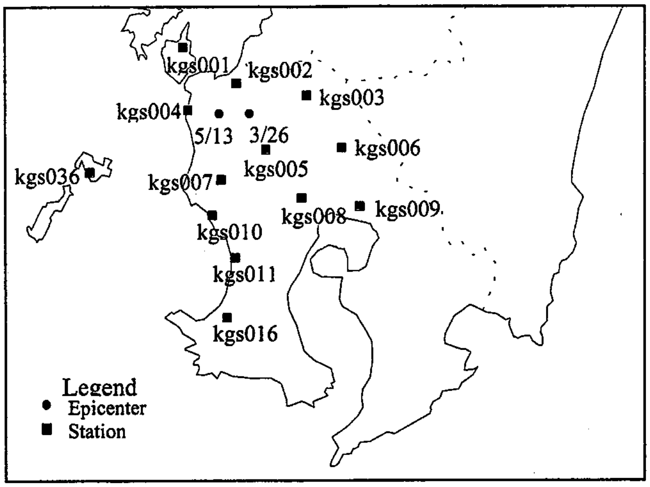

Figure 2. Location of the epicenters and stations considered in the study

RESULTS

For each station three relations between quality factors and frequencies, corresponding to the three amplitude thresholds of coda waves, were developed through the least-square method:

$$\log(Q)=A\log(f)+B \tag{9}$$

Quality factors for both NS and EW components are considered indistinctly since coda waves reflect the free oscillations of the surface layers. Vertical accelerograms were not considered because horizontal components are dominant over vertical ones.

In all cases the quality factors increase with the frequency, i.e. A is positive. Since higher frequency corresponds to waves with short wave length, this implies that the deeper zones beneath the site are more attenuative.

The variation of relation (9) at each station for the different coda wave amplitude thresholds is not significant.

The characteristic quality factor, Q_c, for each case was defined as the average value within the range from 1 to 5Hz as shown in Fig. 3.

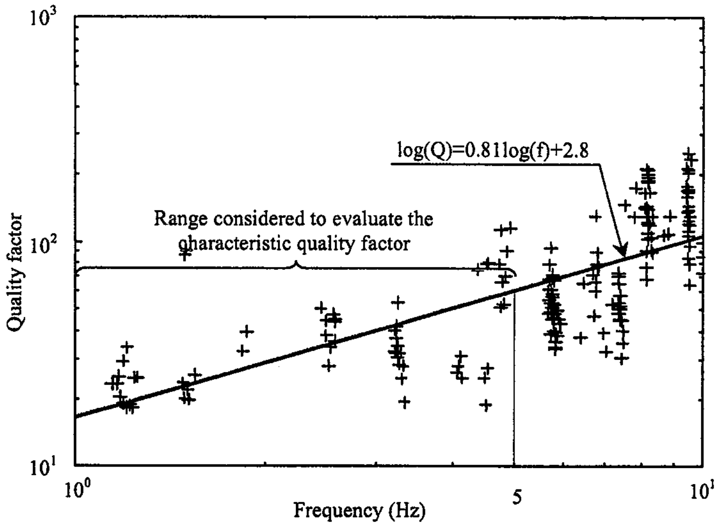

Figure 3. Results of the analysis for station KGS001
(Coda wave amplitude=10%PGA)

Table 3. Results of the analysis

Station code	Q_c	A	Average N-value
KGS001	22.87-38.97	0.812-1.091	17.7
KGS002	21.92-26.63	0.690-0.831	63.5
KGS003	31.60-50.97	0.638-1.318	15.0
KGS004	9.99-26.70	0.703-1.363	4.7
KGS005	33.49-34.98	0.594-0.715	68.0
KGS006	11.02-16.89	0.819-1.206	16.3
KGS007	11.31-33.77	0.565-0.635	8.8
KGS008	14.18-25.90	0.732-1.463	27.5
KGS009	10.29-16.77	0.755-0.986	7.1
KGS010	16.67-28.32	0.909-1.075	15.2
KGS011	24.96-66.10	0.788-1.193	16.7
KGS016	30.10-64.42	0.547-0.829	18.1
KGS036	8.88-13.66	0.611-1.694	6.2

A summary of the results of analysis by Sompi method and the average N-values for each station is shown in Table 3. Only maximum and minimum Q_c and A are shown.

Fig. 4, 5, and 6 show the relations among the previous parameters.

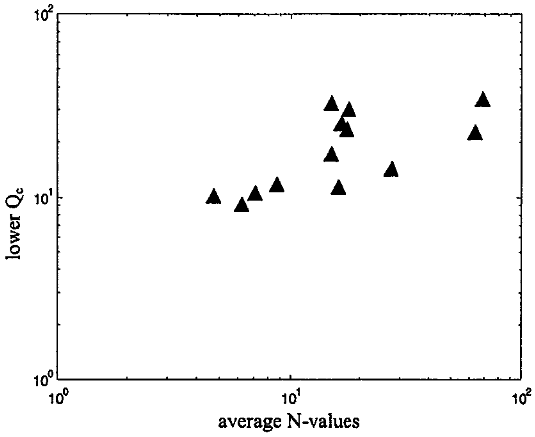

Figure 4. Average N-values versus lower Q_c

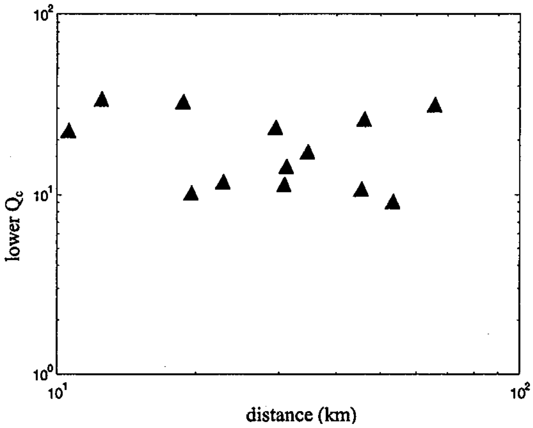

Figure 5. Distance versus lower Q_c

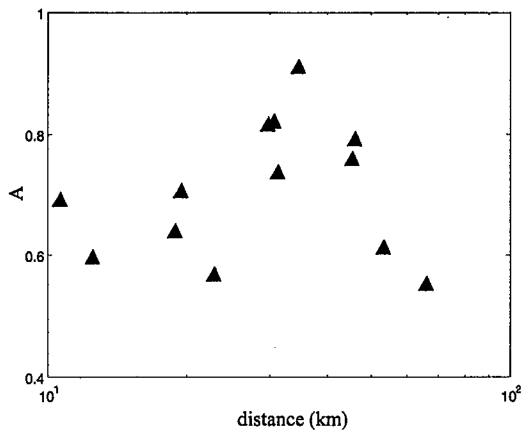

Figure 6. Distance versus A

From the previous plots it was found that there is a relation between the lower Q_c and the average N-values. Unfortunately, the number of analyzed stations was not enough to express this relationship quantitatively. More stations and events should be analyzed before proposing a reliable relation between Q_c and N-values.

On the other hand, no dependency was found between distance, A and lower Q_c. This implies that the distance between the seismic station and the epicenter, which is a measure of the coda wave excitation level, does not affect the quality factors or its dependency on the frequency.

SUMMARY

The aim of the present study was to find a relation between quality factor and N-value by using strong ground motion records. The records of the Kagoshima-ken Hokusei-bu earthquake at several stations of the Kyoshin Net were analyzed. The quality factor was obtained from the coda part of the seismic waves using Sompi method.

For this study, the coda wave was defined as the portion of the record after the peak ground acceleration (PGA) whose amplitude did not exceed a certain percentage of the PGA. Even though it was found that the variation of the amplitude threshold did not affect considerably the relation between quality factors and frequencies, further analysis is needed to validate the proposed definition of coda wave.

Dependence between quality factors and N-values was recognized. However, the number of stations considered was not enough to propose a quantitative relation between Q_c and N-value. Further analysis is needed to formulate such expression. On the other hand, it was detected that the epicentral distance does not affect the previously mentioned relation.

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