Prediction of Energy Responses of Multi-bent Steel Frames By Equivalent Linear Method

by

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ABSTRACT

It is important to evaluate energy absorption capacity of frames required during a design earthquake. An inelastic computer analysis based on mathematical modelling of energy absorbing frames and elements makes it possible to evaluate required energy absorption capacity. But such an analysis sometimes consumes much computation time particularly in case of complicated structural system. This paper presents a proposal to predict energy absorption of multi-bent steel frames by simple equivalent linear method.

INTRODUCTION

In the earthquake resistant design for structures based on energy concepts, it is important to evaluate total input energy exerted by an earthquake and energy absorption of each structural element. According as computer capacity becomes greater, a numerical earthquake response analysis becomes applied better to complicated structural system. But it is difficult to evaluate energy absorption of earthquake resistant elements and frames accurately and efficiently. Many past researches about simple equivalent linear methods, which evaluate plastic energy absorption, were carried out[1][2][3]. This paper proposes a simple method to predict distribution of energy absorption over a multi-bent steel frame by the equivalent linearization technique, which replaces hysteretic energy absorption of structural element by equivalent viscous damping. As experienced in the past studies, equivalent linearizaton techniques are not always effective to predict peak displacement responses including large permanent sets. It is demonstrated here in, however, that energy absorption of each element is well predicted even by a simple equivalent linearization technique.

First of all, on the basis of the energy equilibrium in a linear-elastic multi-degree-of-

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freedom system, a process evaluating energy absorption is formulated. And hysteretic energy absorption and stiffness degrading after yielding of inelastic elements are replaced by equivalent viscous damping and equivalent stiffness, respectively, in the equivalent linear system. As for the arrangement of equivalent-linear parameters, this paper examines the validity of two ways of equivalent stiffness determination and also two ways of equivalent viscous damping determination for two types of hysteresis. And then, modal analysis on the equivalent linear system is repeated with reestimated equivalent stiffness and equivalent viscous damping coefficient until they converge to final values. Furthermore, the results are compared with substructuring on-line test results on four bent steel frames([4], Photo1) and the results of elastic and inelastic earthquake response analysis.

Energy responses in linear-elastic multi-degree-of-freedom system

For the equation of motion for a viscously damped linear system with multi-degrees of freedom is written in matrix notation as:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=-[M]\{1\}\cdot\ddot{y} \tag{1}$$

Where $[M]$, $[C]$, and $[K]$ are the mass, the damping, and the stiffness matrices of the system, respectively, and $\{\ddot{X}\}$, $\{\dot{X}\}$, and $\{X\}$ are the relative acceleration, the relative velocity, and the relative displacement vectors, respectively. $\ddot{y}$ is the acceleration of ground motion.

Here we introduce the following transformation to modal coordinates.

$$\{X\}=[\phi]\{q\} \tag{2}$$

where $[\phi]$ is the participation matrix obtained from the solution of the undamped free vibration system.

By subsituing Eq.(2) into Eq.(1), we obtain:

$$[\phi]^T[M][\phi]\{\ddot{q}\}+[\phi]^T[C][\phi]\{\dot{q}\}+[\phi]^T[K][\phi]\{q\}=-[\phi]^T[M][\phi]\ddot{y} \tag{3}$$

Considering the orthogonality between two different modes and Eq.(3) is rewritten as:

$$m^{(j)}\ddot{q}^{(j)}(t)+c^{(j)}\dot{q}^{(j)}(t)+k^{(j)}q^{(j)}(t)=-m^{(j)}\ddot{y}(t) \tag{4}$$

where $m^{(j)}$, $c^{(j)}$,and $k^{(j)}$ are j^{th} effective modal mass, j^{th} effective modal damping coefficient, and j^{th} effective modal stiffness, respectively. To carry out the multipling the term, $dq=\dot{q}dt$, and the integration $-\infty$ to ∞ in time domain, leads to:

$$m^{(j)}e_I^{(j)}=E_I^{(j)}=c^{(j)}\int_{-\infty}^{\infty}\dot{q}(t)^{(j)2}dt=-m^{(j)}\int_{-\infty}^{\infty}\ddot{y}(t)\dot{q}(t)^{(j)}dt \tag{5}$$

where $E_I^{(j)}$ denotes the j^{th} modal input energy and $e_I^{(j)}$ denotes the j^{th} modal input energy per effective unit mass. Eq.(5) means that input energy in the j^{th} mode is equal to the energy dissipated by damping in the j^{th} mode. Considering that the total energy of vibration system is invariant to the transformation of the coordinates, total input energy E_I is expressed as Eq.(6).

$$E_I = \sum E_I^{(j)} \tag{6}$$

Energy absorption of damping element K denoted by E_{CK} is expressed as:

$$E_{CK}{}^{(j)} = C_K \sum_{j=1}^{n} \delta_K{}^{(j)2} \frac{e_I{}^{(j)}}{2h^{(j)}\omega^{(j)}} \qquad (i = j) \tag{7}$$

where $\delta_K{}^{(j)}$ is j^{th} participation factor transformed into element coordinate for the damping element K and C_K is damping coefficient of the damping element K.

Vibration model and equation of motion

The vibration model for analysis is shown in Fig.1, which is modeled as a multi-bent steel frames composed of four planar bents and three shear floors. As for the planar bents, two of them are reinforced by diagonal braces, respectively. Three types of models are considered with different eccentricity due to brace arrangement. The first one is a model with large eccentricity by placing diagonal braces in the first and the second bents, the next is a small eccentricity model with diagonal braces in the first and the third bents, and the last is a symmetric model with diagonal braces at two ends of the floor. Two kinds of floors are considered; a rigid and a elastic flexible shear floor, where the stiffness of the flexible shear floor is set equal to the initial horizontal elastic stiffness of a diagonal brace. The rigid floor is considered only in the unsymmetrical model. There are 5 cases of the analysis.

Translational displacements only in direction of earthquake input and rotational movements as a rigid body in horizontal plane are considered here. The relative displacements between adjacent bents are caused by shear deformation and rotation of floors for flexible floors, while rigid rotation of the floor can not generate shear deformation. According to the d'Alembert's principle we can obtain the equation of motion Eq.(8) with 5 degrees-of-freedom. For rigid floors, only 2 degrees-of-freedom are remained.

$$[\tilde{M}]\{\ddot{X}\} + [C_{eq}]\{\dot{X}\} + [\tilde{K}]\{X\} + [\tilde{K}_{eq}]\{X\} = \{P\} \tag{8}$$

where

$$[\tilde{M}] = \begin{bmatrix} [M] & 0 \\ 0 & I_m \end{bmatrix}, \qquad [\tilde{K}] = \begin{bmatrix} [K_{xx}] & 0 \\ 0 & [K_{\theta x}] \end{bmatrix},$$

$$[\tilde{K}_{eq}] = \begin{bmatrix} [K_{eq}] & 0 \\ 0 & 0 \end{bmatrix}, \qquad \{P\} = -\begin{Bmatrix} [M] \\ 0 \end{Bmatrix} \ddot{y}, \qquad \{X\} = \begin{Bmatrix} \{x\} \\ \theta \end{Bmatrix}$$

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}, \qquad [K_{xx}] = \begin{bmatrix} k_A & -k_A & 0 & 0 \\ -k_A & k_A + k_B & -k_B & 0 \\ 0 & -k_B & k_B + k_C & -k_C \\ 0 & 0 & -k_C & k_C \end{bmatrix}$$

$$[K_{eq}] = \begin{bmatrix} K_{eq1} & & & \\ & K_{eq2} & & \\ & & K_{eq3} & \\ & & & K_{eq4} \end{bmatrix}, \qquad [C_{eq}] = \begin{bmatrix} C_{eq1} & & & \\ & C_{eq2} & & \\ & & C_{eq3} & \\ & & & C_{eq4} \end{bmatrix}$$

$$I_m = m_1 (S_A + S_B/2)^2 + (m_2 + m_3) S_B^2/4 + m_4 (S_C + S_B/2)^2, \qquad [K_{x\theta}] = [K_{\theta x}]^T$$

$$[K_{\theta x}] = [k_A S_A,\ \ k_B S_B - k_A S_A,\ \ k_C S_C - k_B S_B,\ \ -k_C S_C], \qquad K_{\theta\theta} = k_A \cdot S_A^2 + k_B \cdot S_B^2 + k_C \cdot S$$

k_A, k_B, k_C are the spring constants of floors between planar bent 1 and 2, 2 and 3, 3 and 4, respectively. S_A, S_B, S_C are the distance between the bents. $[K_{eq}]$ and $[C_{eq}]$ are equivalent stiffness matrix and equivalent damping coefficient matrix corresponding to hysteretic behavior of each bents and diagonal braces.

Arrangement of equivalent stiffness and equivalent damping coefficient

<u>Arrangement 1</u>

Equivalent stiffness is assumed as initial elastic stiffness. Equivalent damping coefficient is determined from hysteresis loops for bent and a pair of braces, which are assumed to be elastic-perfectly plastic.

$$K_{eqK} \doteq K_K \tag{9}$$

$$C_{eqK} = \frac{K_K}{2\pi^2} \cdot \frac{\hat{\dot{\eta}}_K}{\left(\frac{1}{T_{rK}} + \frac{\hat{\dot{\eta}}_K}{4}\right)^2} \tag{10}$$

where K_{eqK} and C_{eqK} are the equivalent stiffness and the equivalent damping coefficient of energy absorbing element K. $\hat{\dot{\eta}}_K$ is the average velocity of the accumulative plastic ductility during the period that main energy absorption occurs. And T_{rK} is the modal predominant period, which is the largest modal contribution to the energy absorption.

Arrangement 2

The formula to determine equivalent damping coefficient is same as Arrangement 1. But, equivalent stiffness is taken as secant modulus to the average displacement amplitude for the period of T_{rK}.

$$K_{eqK} = \frac{K_K}{\left(1 + \frac{T_{rK}\hat{\dot{\eta}}}{4}\right)} \tag{11}$$

Arrangement 3

It is assumed that equivalent damping coefficient is determined from the hyteresis loops for frames as elastic-perfectly plastic model and for a pair of braces as a combination of elastic-perfectly plastic model and slip model. And equivalent stiffness is taken as initial elastic stiffness.

$$K_{eqK}^{e-p} = K_K^{e-p} \tag{12}$$

$$K_{eqK}^{slip} = K_K^{slip} \tag{13}$$

$$C_{eqK}^{e-p} = \frac{K_K^{e-p}}{2\pi^2} \cdot \frac{\hat{\dot{\eta}}_K^{e-p}}{\left(\frac{1}{T_{rK}} + \frac{\hat{\dot{\eta}}_K^{e-p}}{4}\right)^2} \tag{14}$$

$$C_{eqK}^{slip} = \alpha \cdot \frac{K_K^{slip}}{2\pi^2} \cdot \frac{\hat{\dot{\eta}}_K^{sip}}{\left(\frac{1}{T_{rK}} + \frac{\hat{\dot{\eta}}_K^{slip}}{4}\right)^2} \tag{15}$$

where α is the modification coefficient to consider the difference of hysteresis shape between elastic-perfectly plastic model and slip model.

Arrangement 4

The formula to determine equivalent damping coefficient is same as Arrangement 3.

And, The formula to determine equivalent stiffness is same as Arrangement 2.

Iterative procedure of equivalent linear analysis

First of all, initial equivalent stiffness and equivalent damping coefficient are assumed as follows:

$$ {}_{Ini}K_{eqK} = K_K, \qquad {}_{Ini}C_{eqK} = \varepsilon\ K_K \tag{16} $$

Where ε is taken 1.0×10^{-4}. By assuming proportional damping, modal analysis is done, and j^{th} modal input energy per effective modal mass, $e_I^{(j)}$, is calculated using Eq.(5). After energy absorption of damping element, E_{CK}, the average velocity of the accumulative plastic ductility, $\hat{\eta}_K$, and the modal predominant period, T_{rK} are determined, renewal of equivalent linear parameters is carried out. Finally, if the equivalent linear parameters are converged to the values assumed in last step, the procedure is completed. At that time, E_{CK} can be used as the prediction of hysteretic energy absorption of the element *K*. If not converged, iterative procedure is continued until they converge to the values assumed in last step. The modification coefficient for slip hysterisis, α, is taken as 0.5. Fig.(2) shows schematic block diagram of the procedure proposed.

Results of the equivalent linear analysis

Fig.(3) shows the results of equivalent linear analysis based on the above-mentioned four arrangements in comparison with the substructuring on-line test results. It is found that any of the four arrangements can be used to predict energy absorption distribution over the structural system, and Arrangement 3 among them is particularly close to the test results. Fig.(4) compares the results of equivalent linear analysis by Arrangement 3 with elastic and inelastic earthquake response analysis by numerical integration. In the inelastic earthquake response analysis, 2-type of restoring force models are adopted. One is the simple restoring force model for braced bent, which is expressed as the parallel combination of two elastic-perfectly plastic model and elastic model for frames, and elastic-perfectly plastic model and slip model for a pair of braces. The other is a rather sophisticated model for braced bent and bent, skeleton shift model[5] for bents and modified Wakabayashi model[6] for braces. It is also found that the equivalent linear analysis based on Arrangement 3 provides a good prediction of energy responses as accurately as the numerical response analysis does.

CONCLUDINGS REMARKS

An iterative equivalent linear procedure without any assumption about response magnitude, such as response ductility expected in the design. Using this equivalent linear method, the energy absorption of multi-bent steel frames can be predicted accurately enough for practical use.

REFERENCES

[1] Shibata, A., Sozen, M.A.: "Substitute Structure Method for Seismic Design in R/C," Proc. of ASCE, Vol.102, No. ST1, January 1976

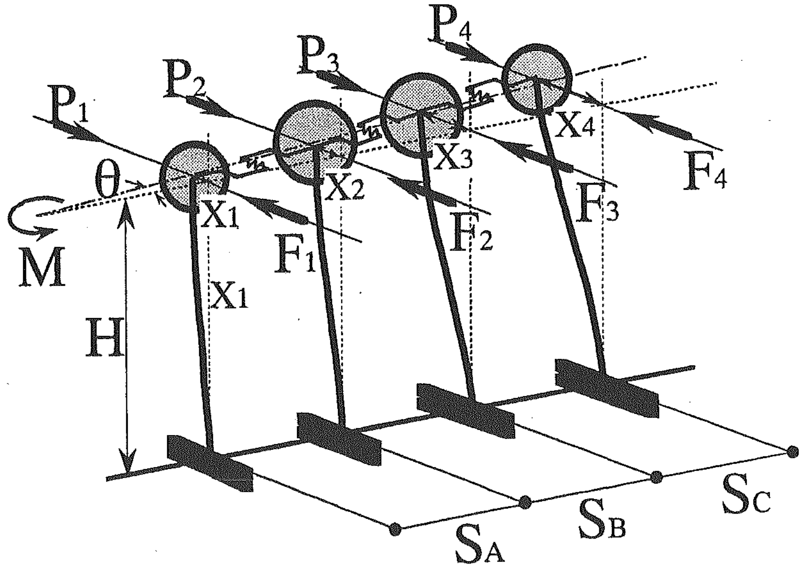
[2] Thomas K Caughey: "Equivalent Linearization Techniques," The Journal of the Acoustic Society of America, Vol.35, No.11, November 1963

[3] K. Ohi, H. Tanaka, K. Takanashi: "A Simple Method to Estimate the Statistical Parameters of Energy Input to Structures During Earthquakes," Journal of structural and construction engineering, AIJ, No. 347, January 1985

[4] K. Takanashi, X.G. Lin, S.J. Lee: "Substructuring On-line Test on Industrial Building," Proc. 11th WCEE, Mexico, July 1996

[5] K. Ohi, K. Takanashi: "Multi-Spring Joint Model for Inelastic Behavior of Steel Members With Local Buckling," Bull. of ERS, No.23, March 1991

[6] M. Wakabayashi, T. Nakamura, M. Shibata, H. Masuda: "Hysteretic Behavior of Steel Braces Subjected Horizontal Load Due to Earthquake," Proc. 6th WCEE, New Delhi. January 1977



Note: x_1-x_4 represent the vibration displacements, θ is the rotation angle of the floor as a rigid body, P_1-P_4 and M are external forces and moment, and F_1-F_4 are the restoring forces of the planar bents.

Fig. 1 Vibration model of 5 degrees-of-freedom

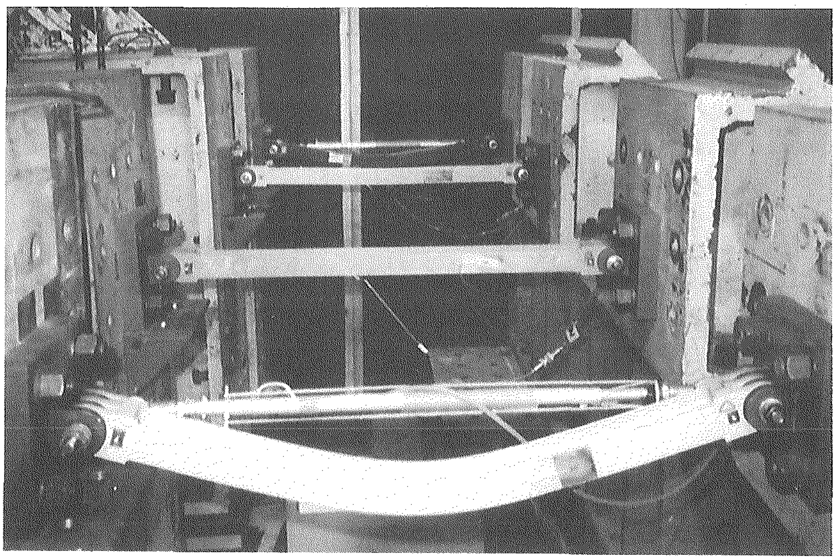


Photo 1 Substructurig on-line test [4]

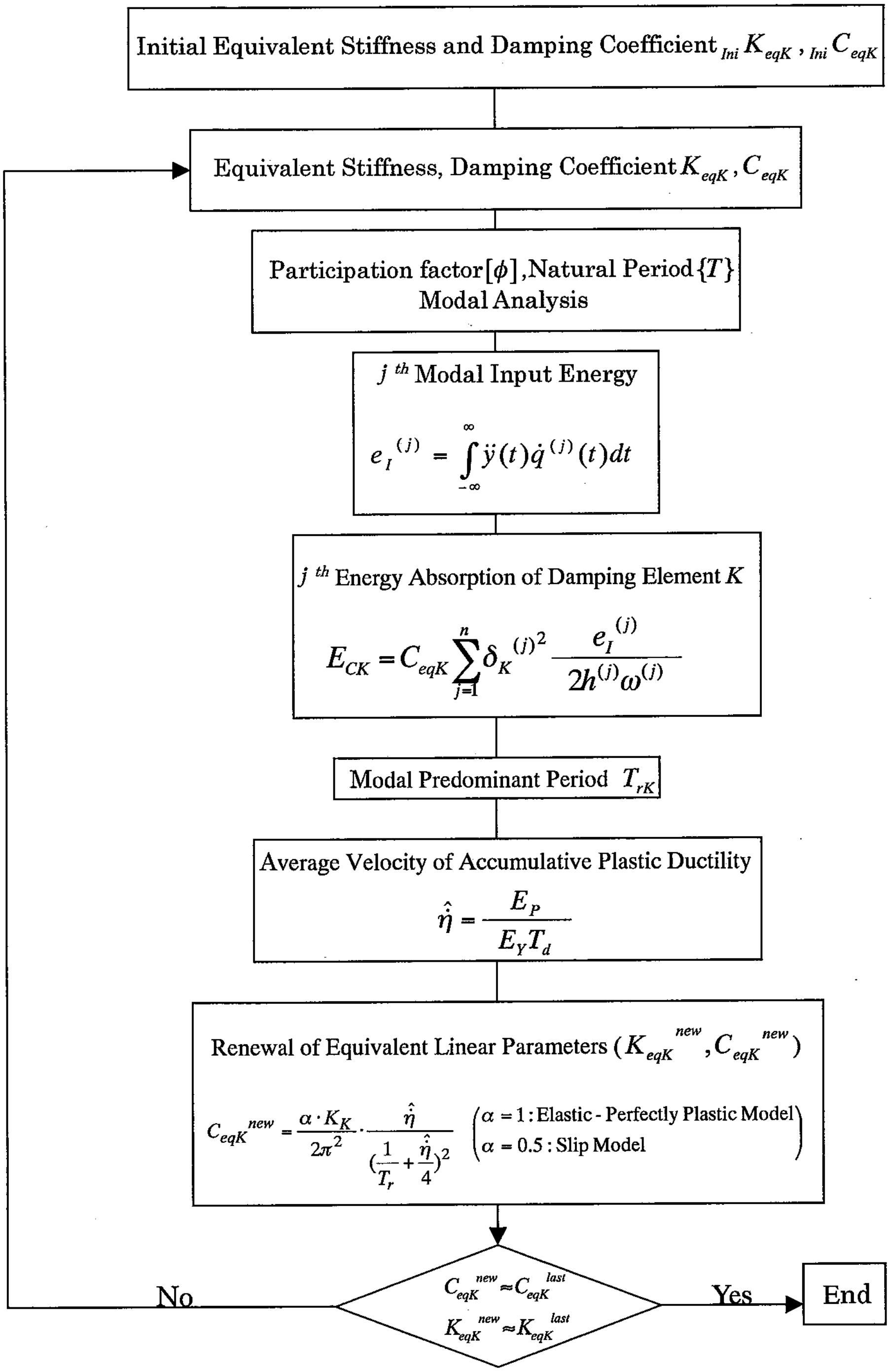


Fig. 2 Schematic block diagram of the procedure

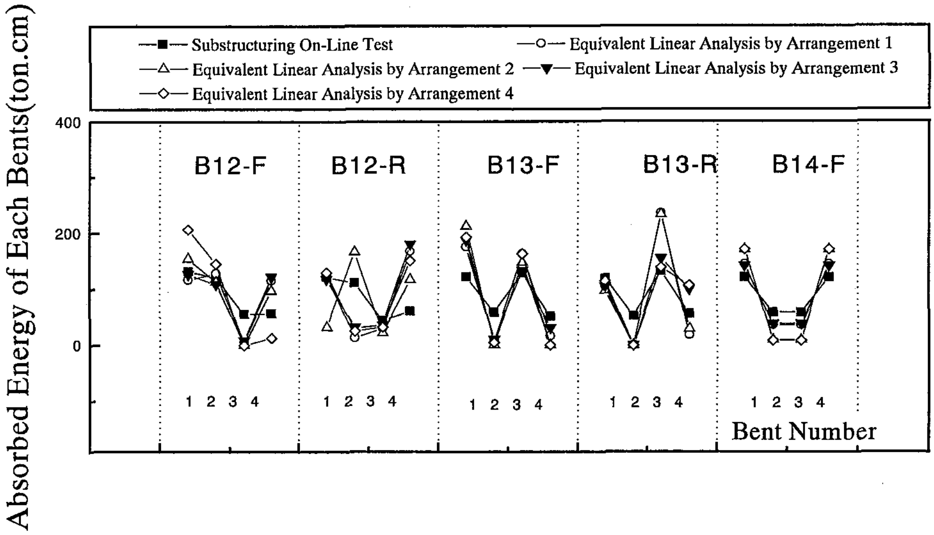
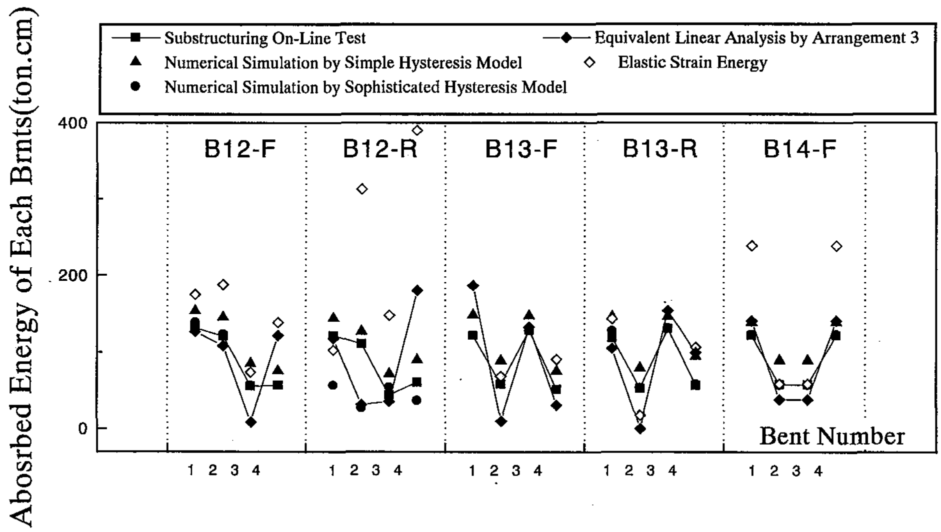


Fig. 3 Results of equivalent linear analysis by four arrangements



Note B□□-□: □□ shows the position of braced bents and □ shows the type of Floor(flexible or rigid).

Fig. 4 Comparisons of equivalent linear prediction with numerical simulation