A New Simple and Accurate Technique for Failure Analysis of Structures

by

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ABSTRACT

A new method for failure and post failure analysis of structures is proposed. A structure is modeled as an assembly of distinct elements. These elements are connected by distributed springs in both normal and tangential directions. The main objective of this paper is to develop a new simple and efficient technique for failure of structures that can follow the structural behavior during failure in reasonable time with reliable accuracy. Although the proposed technique is simple, it is generalized method which can be applied for any type of structures or material. The idea of this technique depends mainly on determining the residual forces resulting from geometrical changes and material nonlinearity during loading. We developed the numerical technique and it is verified by comparing with many cases. In all cases, the results obtained show good agreement with the fracture behavior, separation of elements and the rigid body motion of failed parts of the structure.

INTRODUCTION

Previous researches showed that about 90% of death ratios in past earthquakes were because of structural failure. Although many numerical techniques are used for structural analysis, all of them are not capable to deal with failure behavior of structure. These techniques can be classified mainly into two groups. The first group is based on the assumption that the structural material is continuos while the other group assumes that the material is composed of "discrete" elements. The finite element method (FEM) is the most famous method based on the first assumption. However, this assumption makes the analyses restricted, in most of cases, to small deformation range. Large deformation analysis can be performed also by the FEM for continuos structures, however, with assumption that structural members usually are not separated during analysis. The applicability of FEM in large deformation analysis is restricted only to steel structures. Large deformation analysis of reinforced concrete (RC) or masonry structures requires taking into consideration of many issues like geometrical changes, material nonlinearity, cracking, separation of members, rigid body motion and collision of structural members. Dealing with these problems using the FEM makes the analysis very complicated and time consuming.

Many other techniques were developed based on assumption that structural elements are mainly separated and stresses are transferred through element edges by connecting springs. There are two types of methods which adopt this assumption. The first type is based on stiffness matrix type, like the Rigid Body and Spring Model, RBSM[1,2,3], while the other is based on "distinct" element type, like the Extended or Modified Distinct Element Method, EDEM or MDEM[4,5]. The accuracy of both types in small deformation range is questionable. However, only EDEM or MDEM can deal easily with the behavior in large deformation range accurately. The main complication that faces the EDEM is that the analysis requires large CPU time.

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A new method for failure and post failure analysis of structures such as (RC) and/or steel structures is proposed. This method combines the accuracy of FEM in small deformation range and simplicity together with accuracy in following large deformation behavior. For example in case of RC structures, concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions. The reinforcement bars are modeled as continuous springs connecting elements together. Local failure of concrete is modeled by failure of springs connecting elements when reaching critical principal stress. The accuracy of the model was verified in the range before rigid body motion starts[6]. This paper introduces a new technique to deal with post failure behavior of structure such as a process of change of structural behavior from continuum state to perfectly discrete state after total failure.

ELEMENT FORMULATION

We assume that the two elements shown in Fig. (1) are connected by distributed normal and shear springs at contact points. Each pair of springs fully represents deformation and failure of a certain area. The formulation and results of the element before rigid body motion stage were introduced in Ref. (6) and it was proved that the method could determine deformations and detect the initiation and propagation of cracks. The stiffness matrix developed in Ref. (6) is based on assumption that deformations are small. To develop the methodology for post failure analysis, the following steps are proposed.

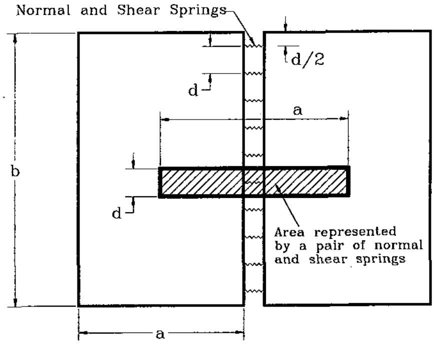

Fig. (1) Spring distributions and area of influence of each pair of springs

SMALL DEFORMATION AND LARGE DEFORMATION

To show the reason why the technique proposed before in Ref. (6) can not deal with large deformation case, the following example is presented. A beam simply supported by a hinge and a roller, as shown in Fig. (2), is loaded by a concentrated load in the middle of the beam. Assume that the material is elastic and Young's modulus is very low so that deformations become large and stresses are small. The behavior looks like behavior of rubber.

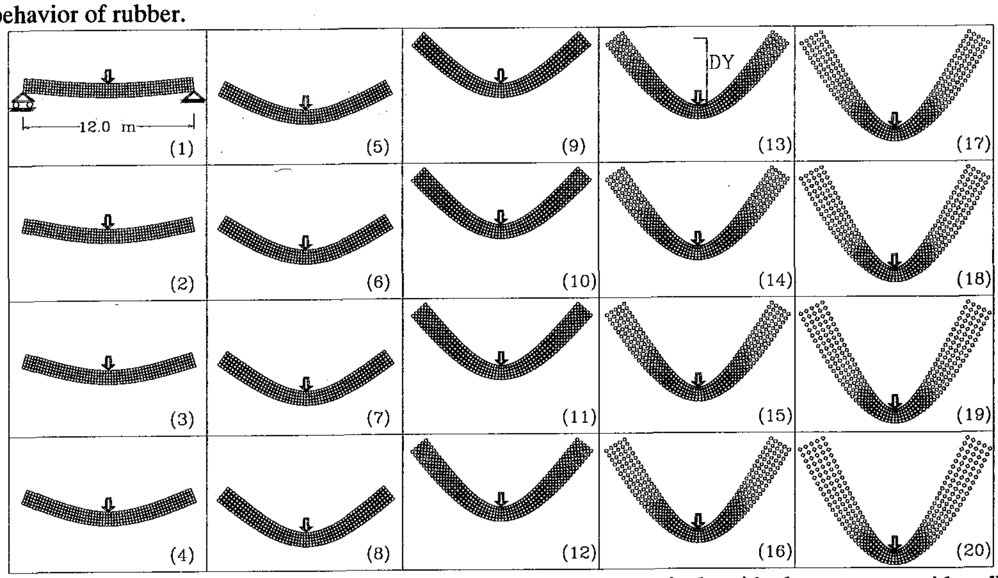

Fig. (2) Deformed shape of a three point bending beam (geometrical residuals are not considered)

The following can be noticed from the deformed shape of the beam of Fig. (2):

1. The deformation of the beam seems to be within a small deformation range.
2. The volume of the beam increases drastically when the analysis is continued till very large deformations.
3. The roller does not move with increased loading, which is not realistic.

NUMERICAL PROCEDURE

It is obvious from the last example that the method proposed in Ref. (6) can not be applied to very large deformations unless geometrical changes in shape of the structure are considered. However, these effects are taken into account in most of numerical techniques by adopting geometrical stiffness matrix. It is very difficult to deal with geometrical changes together with fracture problems especially if the studied region is a potentially of highly damaged material like bricks or concrete. In this technique, we do not have to determine the geometrical stiffness matrix resulting in making the method general and applicable for any case of loading or structure type. The technique can deal with the following problems:

1. The structural shape is changed during analysis.
2. The direction of internal stress vectors is changed because of geometrical changes.
3. Equilibrium should always be satisfied among external forces, gravity forces, internal forces, inertia forces and damping forces.
4. Separation of any structural member is allowed.
5. Rigid body motion of any structural part can be followed.

The main assumption in the technique used is that the direction of the applied external forces is constant. Follower loading condition, which means that applied load direction changes when the member buckles, can not be analyzed using the proposed technique. Moreover, the element contacts are not changed during analysis. This indicates that these modifications can not be applied with collision (or recontact) problems. The general equation of motion is:

$$[M][\Delta\ddot{U}]+[C][\Delta\dot{U}]+[K][\Delta U]=\Delta f(t)+R_m+R_G \tag{1}$$

Where [M] is mass matrix; [C] the damping matrix; [K] the nonlinear stiffness matrix; $\Delta f(t)$ the incremental applied load vector; $[\Delta U]$ the incremental displacement vector; and $[\Delta\dot{U}]$and $[\Delta\ddot{U}]$ the incremental velocity and acceleration vectors, respectively.

The term, R_m, is residual force vector due to cracking or incompatibility between strains and stresses at the spring location, while R_G is residual forces due to geometrical changes of the structure during loading. The method is applied using the following steps:

1. Assume that R_m and R_g are zeros and solve the equation to get incremental displacement. Newmark Beta method[7)] is used for accurate determination of incremental displacements.
2. Calculate incremental strains and stresses.
3. Calculate incremental and total velocities and accelerations.
4. Modify the geometry of the structure according to the calculated incremental displacements by modifying elements location.
5. Modify the direction of spring force vectors according to the new element configuration.
6. From the calculated stresses, check the situation of cracking and calculate the material residuals load vector R_m.
7. Calculate the element force vector from surrounding springs of each element F_m.
8. Calculate the geometrical residuals around each element from the equation below

$$R_G=f(t)-[M][\ddot{U}]-[C][\dot{U}]-F_m \tag{2}$$

Equation (2) means that the geometrical residuals account for the incompatibility between external applied and internal forces, damping and inertia forces due to the geometrical changes during analysis. It should be noted that residual forces are calculated based on total stress value. Gravity forces are considered as an external applied force. Small deformations are assumed during each increment.

9. Calculate the stiffness matrix for the structure in the new configuration considering stiffness changes at each spring location due to cracking or yield of reinforcement.
10. Apply again a new load increment and repeat the whole procedure.
11. Material and geometrical residuals calculated from the previous increment can be incorporated in solution of Eq. (1) to reduce the time of calculation.

It should be also emphasized that this technique can be applied in both static and dynamic loading conditions. In case of static loading condition, the mass and damping matrices are set equal to zero. The main limitation in static analysis is that separation of elements is not permitted during analysis as it makes the stiffness matrix singular. On the other hand, analyzing structures subjected to dynamic loading condition enables us to follow both geometrical changes of the structure and the rigid body motion during failure. As the deformations are assumed to be small in each load increment, small time increment should be used to reduce the error.

MATERIAL MODELING

One of main difficulties in this kind of analysis is how to deal with compression crushing of elements. Having crushing of elements at the support location means, from numerical view point, that the elements lose all its stiffness and the structure is not connected to the ground any more. In this analysis, Maekawa[8)] compression model is adopted till 1% strain in compression. After reaching this strain, minimum stiffness value (0.01 of the initial value) is assumed so that the connection of the structure to the ground is not lost. For reinforcement, the model illustrated in Ref. (9) is adopted. Namely after reaching 10% strain, it is assumed that the reinforcement bar is cut. The force carried by the reinforcement bar is redistributed by applying the redistributed force to the corresponding elements in the reverse direction. For cracking criteria, principal stress based failure criteria is adopted. For more details see Ref. (6).

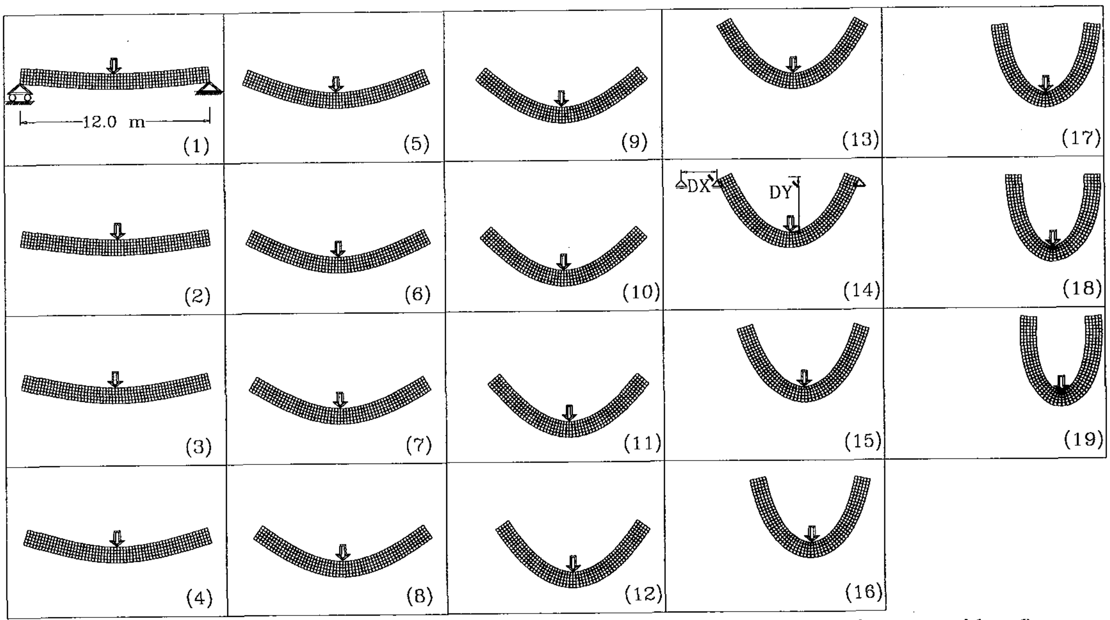

Fig. (3) Deformed shape of a three point bending beam (geometrical residuals are considered)

NUMERICAL RESULTS

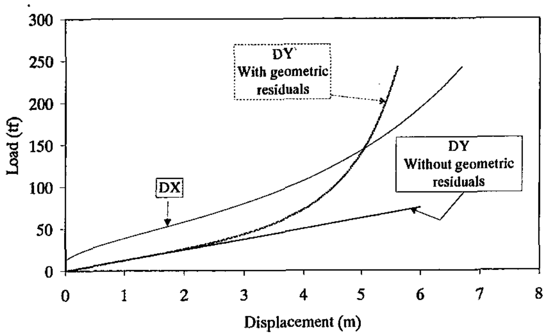

Fig. (4) Comparison between load-deformation relations of a three point bending beam (with and without geometrical residuals)

To verify the reliability of the proposed numerical technique, simulation is performed using the same simply supported beam in Fig. (2) under the same loading condition. Although unrealistic results were obtained when small deformation theory was applied, realistic results were simulated when the geometrical residuals technique is applied, as shown in Fig. (3). The load deformation relations before and after modifications are shown in Fig. (4). The "DY" and "DY'" values represent the mid span displacements while "DX'" represents the roller displacement in horizontal direction. It is obvious that more realistic results could be obtained. Based on the results, the following can be noticed:

1. In small deformation range, mid span displacement "DY" and "DY'" obtained from both cases are similar.
2. The roller displacement "DX'" highly increases when the applied load is increased. The roller displacement "DX" is almost zero in case of small deformations based analysis.
3. During loading, the beam shape is changed from straight line to arch. This illustrates why stiffness of the beam increases when the geometrical changes are considered.

To check the accuracy of the newly proposed method, large deformation analyses of different case studies are performed. The first case is harmonic motion analysis of a bar under its own weight. The main objective of this analysis is to show that the behavior of structural elements moving as rigid bodies after failure can be simulated. The bar configuration and results are shown in Fig. (5). Two different initial excitation angles were used (θ_o=0.05 and 0.3 rad). The result of small excitation angle (θ_o=0.05 rad) was compared with that obtained by theory based on assumption that $\mathrm{Sin}(\theta) \approx \theta$. The calculated X-displacement is almost the same as the one obtained from theoretical kinematics. This indicates that the geometrical residuals technique presented in this paper can simulate both the geometrical changes and rigid body motion. In case of (θ_o=0.3 rad), it is obvious that the amplitude of displacement and the oscillation period increases when the initial excitation angle increases.

To verify the numerical accuracy of the proposed technique with different damping ratios, analyses using initial angle (θ_o=0.05 rad) are performed using the same bar shown in Fig. (5). Figure (6) shows the relation between the angle θ with time. Damping ratio can be calculated also from the time-response relation shown in Fig. (6) by[7]:

$$\ln\left(\frac{u_i}{u_{i+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{3}$$

Where, ζ is damping ratio; u_i and u_{i+1} the displacement amplitude of two successive peaks.
From this equation, the applied and calculated damping ratios are compared in Fig. (7). The comparison is shown in Fig. (7). Nearly five values of damping ratios are calculated from each time response history. From Fig. (7), it is obvious that the applied and calculated values are very close except that when the damping ratio is less than 0.5%. For practical damping ratios, the accuracy is quite acceptable.

The second case is also harmonic motion of a "L" shaped bar under its own weight. The bar configuration and results are shown in Fig. (8). The damping ratio applied to the analysis is 4% to enable the structure to reach its stability condition. It is obvious that the bar starts oscillation around the stability position. Oscillation reduces gradually and finally stops at the equilibrium position. The simulated angle of final stability is the same as that calculated from theory.

These two analyses show that if some part of the structure failed, the rigid body motion of this part together with the final equilibrium position can be simulated automatically.

The third example shows the time history of failure process of a single bay RC frame. The frame is supported by hinged bearing (left) and hinged roller bearing (right). A concentrated load is applied at the center of the beam of the frame. The frame shape, dimensions, loading conditions and deformations under the applied load are shown in Figs. (9) and (10). The failure process can be summarized as follows:

1. Cracking starts from the center of the beam because of maximum bending moment. The failure load can be calculated accurately. (Ref. (6))
2. Reinforcement bars yield in the center of the beam.
3. Steel bars cut off after yield in the middle of the beam first followed by cut off in the left connection and finally the right one.
4. Refering to Fig. (10), displacements drastically increase after 0.7 seconds because of failure of reinforcement bars. At the same time, the structure begins unstable dynamic motion.
5. Tension cracks appear at the left connection first, because of difference of supporting conditions. After midspan cracking, the beam behaves as a double cantilevers connected to unstable columns. As the loading rate is very high, crack generation in connections is faster than the rigid body motion of the failed parts.
6. Tension cracks appear at the right connection together with motion of the roller.
7. The structural members lose curvature and move as three rigid bodies in the space.

Figure (11) shows failure pattern of a plain concrete simple beam subjected to three point bending. The beam is supported by two hinged rollers. It can be seen easily that realistic failure behavior can be obtained. After cracking of concrete in the mid span, the beam is separated into three parts, two beams and elements subjected to the load. The two beams rotate around the rollers till becoming vertical and then separated from the support and move as a rigid bodies in the space. The elements subjected to the load are separated and moves under gravity accelerations. After separation of beam segments, the rollers start inward motion inward. It should be emphasized that no previous guessing on the behavior should be done before the analysis. The crack separation location is arbitrary.

Figure (12) shows failure mechanism of a fixed-fixed frame subjected to lateral load. The analysis is performed till reaching collision with the ground. It is noticed that cracks mainly starts from the left connections. Compression failure occurs at the right support. After having crushing of the right column support, rigid body rotation of the two columns together with the attached beam start till crushing the ground. As the number of elements is small, it took only 10 minutes using a personal computer (CPU Pentium 267 MHz) to make such analysis.

This result shows that the crack initiation, crack propagation, failure of reinforcement, separation of structural members and rigid body motion of structural members after failure can be followed without any additional complications to the model.

CONCLUSIONS

In this study, a new technique was developed by which structure behavior can be followed during loading till complete failure. The failure process can be simulated for elastic region, nonlinear region, and even after separation of structural members. The main advantages of the proposed technique are:

1. This technique is general and can be applied for any material or structural shape.
2. The geometrical changes of the structure during loading together with the rigid body motion of failed structural elements can be followed with reliable accuracy and without any additional complications.
3. Unlike FEM, the crack location is arbitrary. This means that the structure can fail at any location.

At this moment, the main limitation of the model is that the collision effects are not taken into account. This means that elements can separate but new element contacts are not permitted. However, research is ongoing to consider the collision effects in the model.

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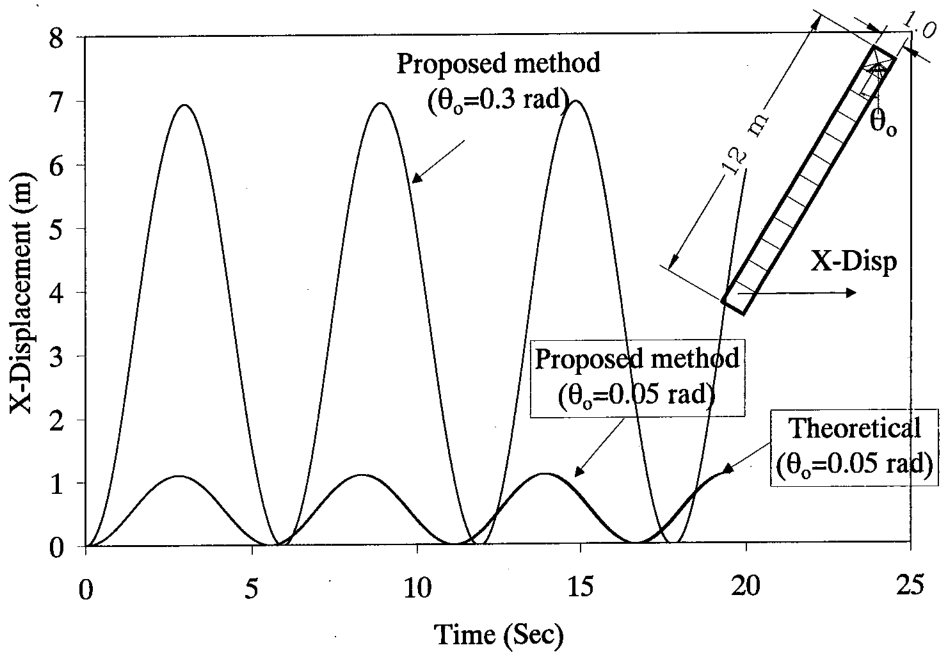

Fig. (5) Harmonic motion of a rigid bar under own weight and initial excitation (without damping).

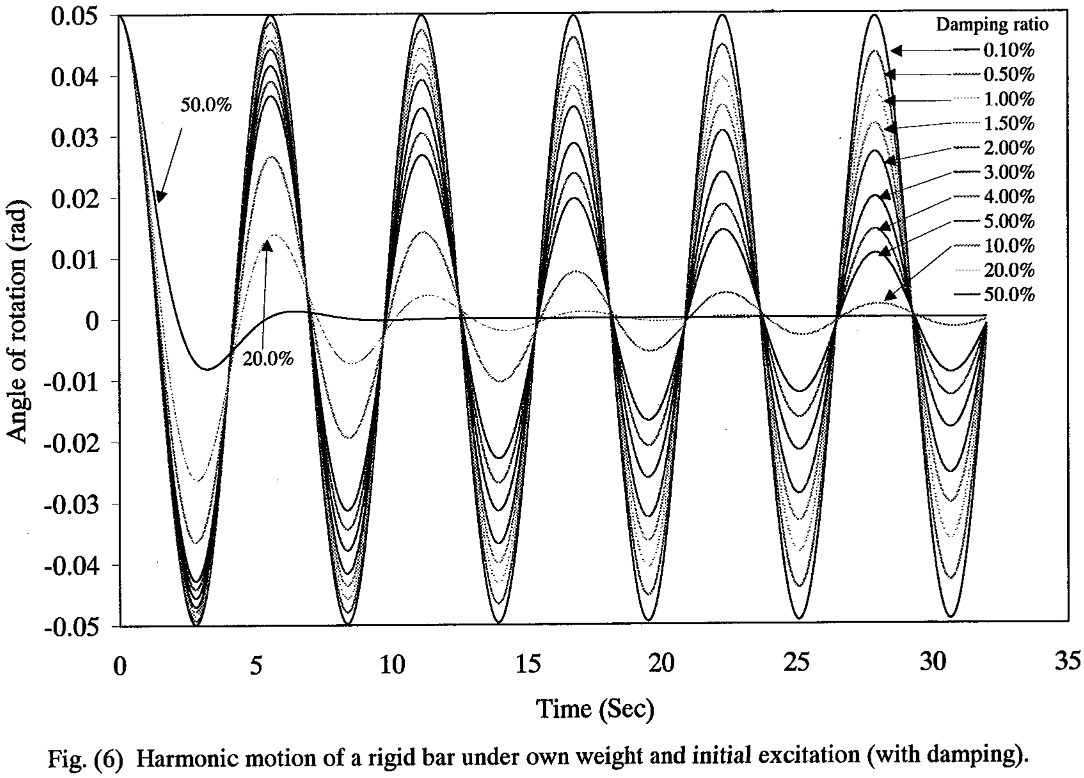

Fig. (6) Harmonic motion of a rigid bar under own weight and initial excitation (with damping).

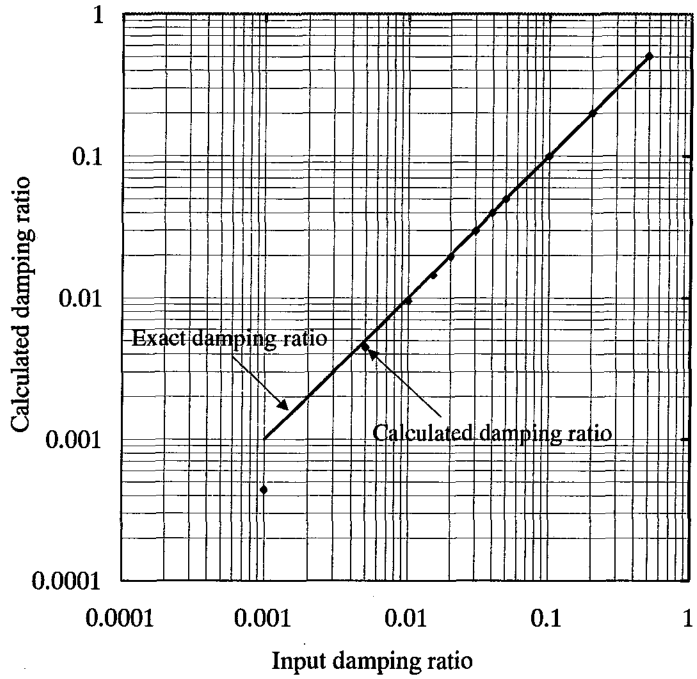

Fig. (7) Comparison between applied and calculated damping ratios

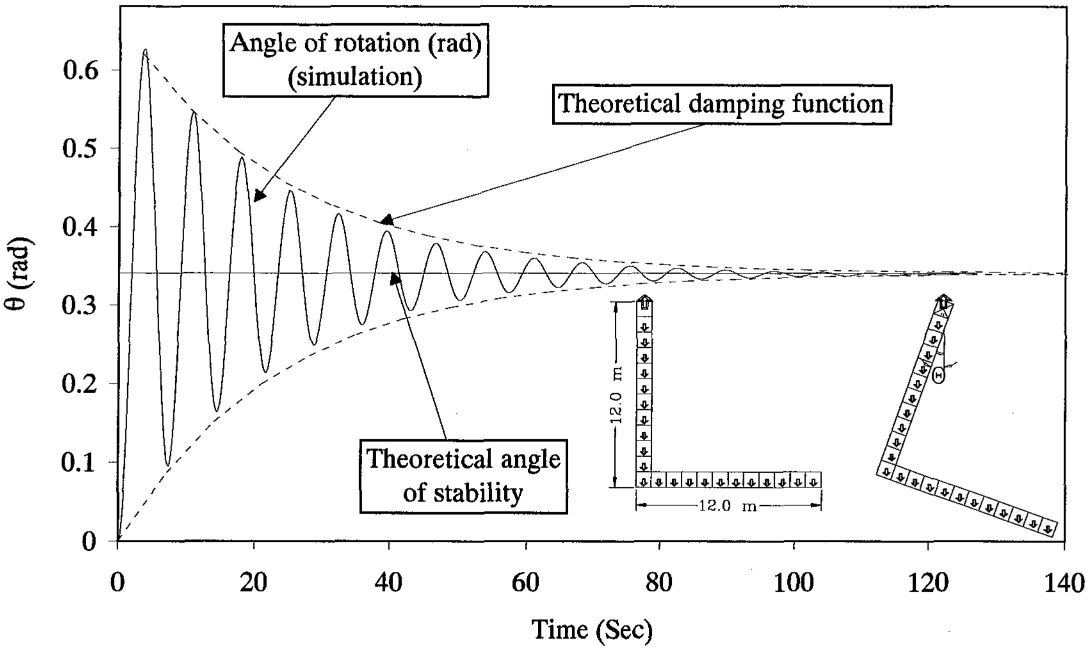

Fig. (8) Harmonic motion of a rigid "L" bar under its own weight. (Damping ratio is 4%)

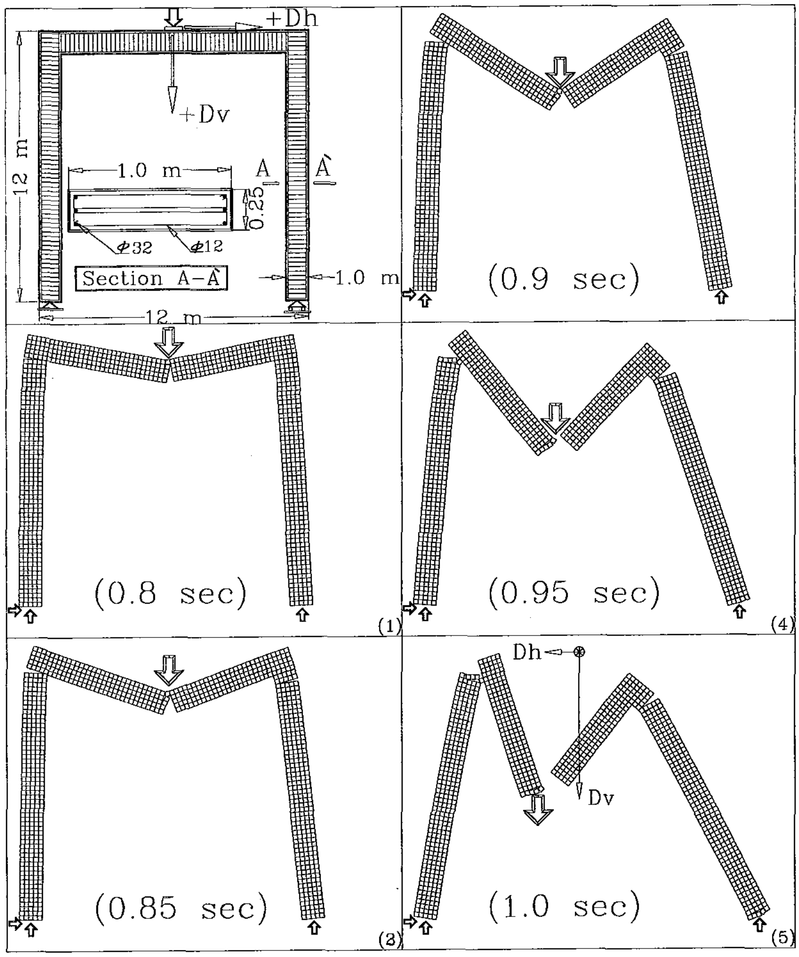

Fig. (9) Deformed shape and failure pattern of a hinged-roller RC frame

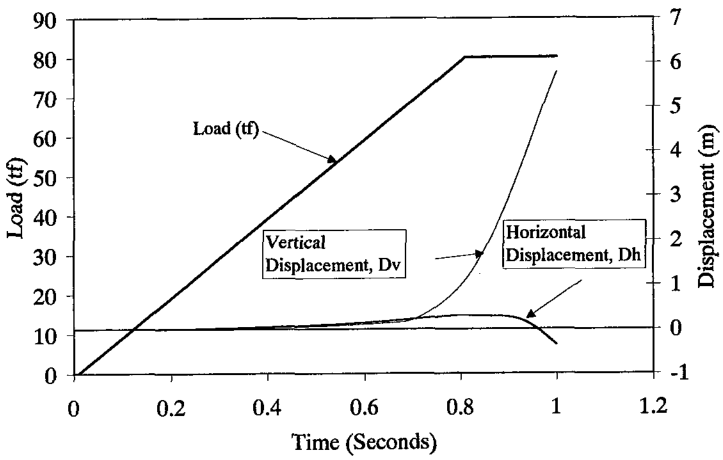

Fig. (10) Load, vertical and horizontal displacement at the loading point vs. time

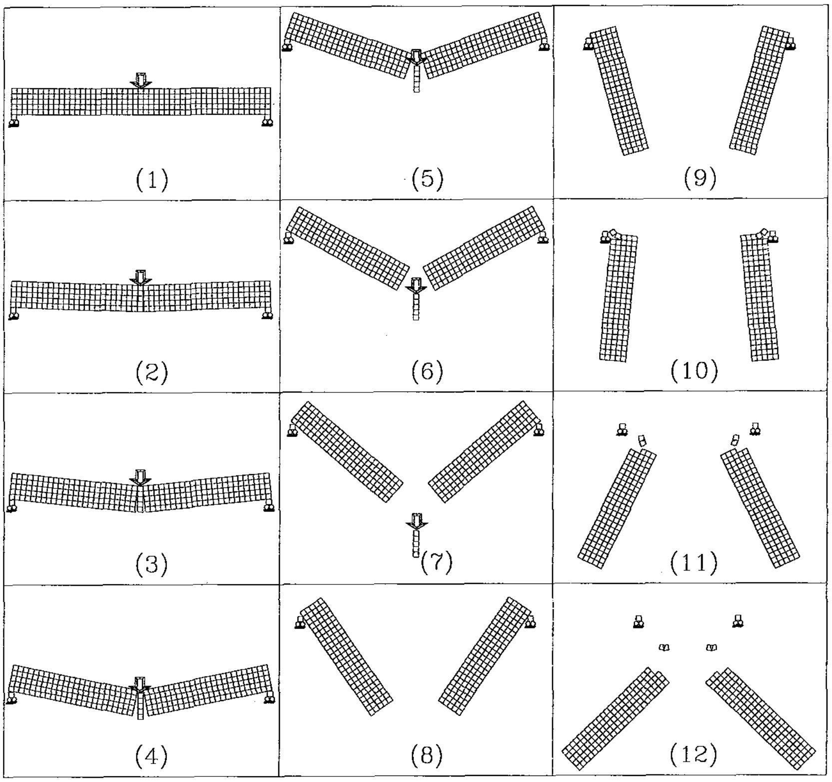

Fig. (11) Failure pattern of a plain concrete simple beam in three point bending

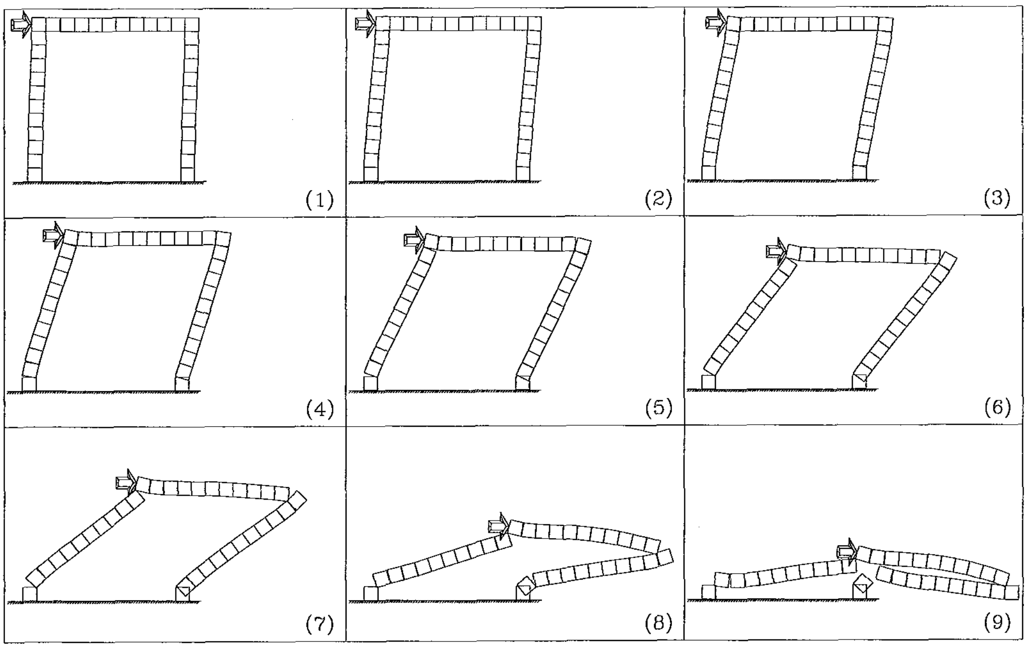

Fig. (12) Failure pattern of a RC concrete frame subjected to lateral load.