A SIMPLIFIED EVALUATION METHOD OF DYNAMIC INTERACTION BETWEEN CLOSELY SPACED EMBEDDED FOUNDATIONS OF ARBITRARY SHAPES

by

A. MIKAMI[1] and K. KONAGAI[1]

ABSTRACT

A simplified method for the evaluation of dynamic interaction between closely spaced embedded foundations of arbitrary shapes is presented. In the method, these foundations are assumed to be rock situated on a non-deformable base. The overlying soil stratum surrounding these embedded foundations is replaced by an infinitely spread two-dimensional plane resting on Winkler-type springs. These springs take into account the fundamental shear vibration mode of the stratum. In addition, this model approximates stress-free ground surface conditions by assuming a plane-stress condition over the entire extent of the plane. Given these assumptions, the stiffness of the lateral soil at the sides of the foundations computed using the model was found to be in close agreement with results obtained by more rigorous means.

INTRODUCTION

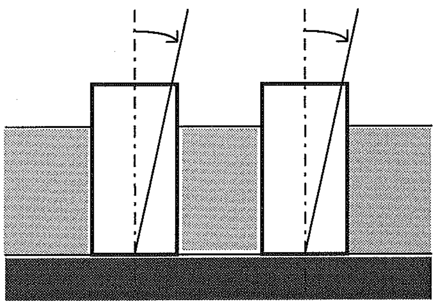


Fig.1 Closely Spaced Embedded Foundations

Many embedded structures are closely spaced to each other in urban areas. Although the effect of cross interaction between embedded structures is not negligible, the effect has not yet been incorporated into seismic design codes.

Kobori and Kusakabe(1) analyzed the dynamic cross-interaction between two embedded structures by using the so-called "Thin Layered Element Method.(2)" In this method, models of soil-structures were subdivided by several horizontal planes. The formulation was obtained by making use of the closed-form solution, based on exact displacement functions in the horizontal direction, and the finite element method in the vertical direction for the coupling of these layers. To analyze the dynamic cross-interaction between embedded foundations, they employed a method developed by J.E.Luco(3). Luco analyzed this problem by employing two cylindrical coordinate systems

[1] Atsushi Mikami, Kazuo Konagai, Institute of Industrial Science, University of Tokyo

so that the boundary condition between soil and structure is satisfied. Lin et al.[4] developed a hybrid method in which two rigid embedded foundations with rectangular cross sections were considered. The soil-foundation system is partitioned into a finite region, the near field, and the complementary infinite region, the far field, which is assumed to be horizontally layered.

The thin layered element method provides us with a semi-analytical solution which is considered to be very reliable if a stratum situated on a non-deformable base can be subdivided into horizontal layers of constant thickness. On the other hand, it is mathematically difficult to incorporate non-linearity of soil, spatial variation of soil-profiles, and embedded foundations of arbitrary shapes into the model.

Discretized methods, like the finite element method, are very widely used as multipurpose tools. However, these methods do not always aid in physical understanding of the dynamic soil-structure interaction. Also these numerically sophisticated methods do not always lead to reliable results because it is usually impossible to know all of the input soil parameters. Therefore, a rational simplified model which harmonizes the sophistication of the analytical model with the accuracy of available soil input parameters may be preferable.

In the present paper, a simple ground model is used to evaluate the dynamic cross interaction between two embedded rigid foundations of arbitrary shapes. The "Quasi-Three-Dimensional Ground Model" was originally developed by Tamura et al.[5] for earthquake response analysis of overlying stratum on an uneven rigid base. In the model, vertical motion of the ground is eliminated and the vibration mode of the surface layer is assumed to be the fundamental shear vibration mode. The latter assumption is endorsed by Kobori and Kusakabe's study which shows that the effect of cross interaction dominates at around the natural frequency of the surface layer. Given these assumptions, the stiffness of the lateral soil computed using the model is compared with results obtained by more rigorous means.

SIMPLIFICATION OF THE MODEL

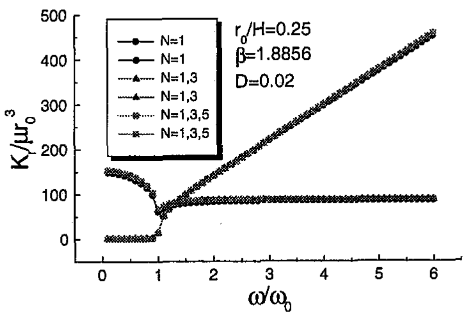
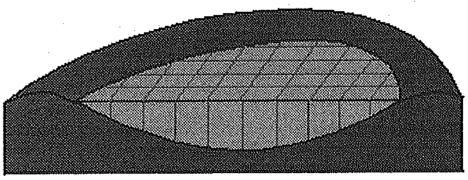


Fig.2 Contribution of the 1st vibration mode to the stiffness (*N*: Number of considered vibration mode, β: V_P / V_S, *D*: material damping, r_0: radius, *H*: thickness of the surface layer, μ : shear modulus)

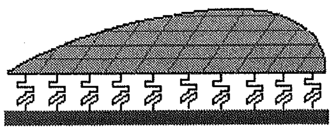
Neglecting vertical motion in a soil-structure interaction analysis has been adopted by many researchers. Tajimi[6] applied this assumption to the three dimensional wave propagation theory and obtained an analytical solution; The vibration mode of depth was represented by trigonometric series and the unknown constants were selected so that the boundary condition between the soil and the foundation was satisfied.

We examine how much the 1st vibration mode contributes to the dynamic stiffness of the soil. Fig.2 shows a normalized dynamic stiffness with normalized circular frequency for the horizontal axis. The figure indicates that the contribution of the 1st vibration mode to the stiffness dominates. The reason for this is that all vibration modes except the 1st vibration mode have mutual positive and negative components of depth. Therefore, the reaction moment by those modes around the rocking axis become negligible after the positive and negative components cancel each other out. Consequently, the model can be simplified by considering only the fundamental vibration mode.

QUASI-THREE-DIMENSIONAL MODEL



(a) Surface layer on a non-deformable base



(b) 2-D expanse on Winkler-type springs

Fig.3 Quasi-Three-Dimensional Ground Model

To model a soft surface layer overlying a non-deformable base, the soft surface soil deposit is divided into vertical soil columns(Fig.3(a)). Then, each soil column is replaced by a one-lumped-mass-spring system taking into account the fundamental mode of shear vibration for the column. A net of finite elements is used to link these oscillators together, thus forming a model of the alluvial surface layer (Fig.3(b)).

(1) Governing equation

Applying a cylindrical coordinate system to the model, the governing equation becomes:

$$(\lambda+2\mu)^* \frac{\partial(\Delta e^{i\omega t})}{\partial r} - \frac{2\mu^*}{r}\frac{\partial(\Omega_z e^{i\omega t})}{\partial\theta} = \rho^* \frac{\partial^2(u_r e^{i\omega t})}{\partial t^2} + c^*\frac{\partial(u_r e^{i\omega t})}{\partial t} + k^*(u_r e^{i\omega t})$$

$$(\lambda+2\mu)^* \frac{1}{r}\frac{\partial(\Delta e^{i\omega t})}{\partial\theta} + 2\mu^*\frac{\partial(\Omega_z e^{i\omega t})}{\partial r} = \rho^* \frac{\partial^2(u_\theta e^{i\omega t})}{\partial t^2} + c^*\frac{\partial(u_\theta e^{i\omega t})}{\partial t} + k^*(u_\theta e^{i\omega t})$$

... (1), (2)

Here,

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}$$

$$\Omega_z = \frac{1}{2r}\left[\frac{\partial}{\partial r}(ru_\theta) - \frac{\partial u_r}{\partial\theta}\right]$$

... (3), (4)

where, λ, μ identify Lamé's constant, ρ^* identifies density of elastic ground, c^* identifies viscous constant, k^* identifies spring constant, ω identifies the circular frequency of the excitation and (u_r, u_θ) identify the displacement in the (r,θ) direction, respectively.

These parameters are computed as follows taking only the fundamental shear vibration mode into account. That is, the vibration mode of the depth of the surface stratum is assumed to be

$$\varphi = \sum_{n=1}^{\infty} a_n \varphi_n \approx a_1 \varphi_1 \qquad \text{... (5)}$$

a_1 : Contribution of the 1st mode to the rigorous vibration mode

φ_1 : fundamental shear vibration mode

Thus, the parameters of the present model are computed from these equations.

$$(\lambda + 2\mu)^* = a_1^2 \int_0^H (\lambda + 2\mu)\phi_1^2 dz$$

$$\mu^* = a_1^2 \int_0^H \mu \phi_1^2 dz$$

$$\rho^* = a_1^2 \int_0^H \rho \phi_1^2 dz$$

$$\kappa^* = a_1^2 \int_0^H \kappa \left(\frac{d\phi_1}{dz} \right)^2 dz \qquad \text{... (6)-(9)}$$

(2) Incorporation of vertical motion

Neglecting vertical motion does not satisfy the stress-free condition at the ground surface. This assumption may leads to an overestimation of the stiffness. Therefore, if the surface layer is soft ground, the effect of the stress-free condition at the surface must be somehow incorporated in the analysis. The ratio between longitudinal and shear wave is expressed as

$$\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}} \qquad \text{... (10)}$$

where, V_P, V_S: Longitudinal and shear wave velocity, respectively and ν: Poisson's ratio

As Poisson's ratio approaches 0.5, the wave velocity ratio climbs toward infinity. When this occurs, the value for stiffness obtained ceases to be acceptable. Accordingly, we need to modify the value. Suggestions made by Gazetas et al.(7) leads to replacing the wave velocity ratio by letting

$$\frac{V_P}{V_S} = \sqrt{\frac{2}{1-\nu}} \qquad \text{... (11)}$$

which is equivalent to letting

$$\frac{\nu}{1-\nu} \rightarrow \nu \qquad \text{... (12)}$$

This replacement was also adopted by Veletsos et al.(8) in their study. Then, the value of the wave velocity ratio settles down to 2 when Poisson's ratio approaches 0.5. Konagai et al.(9) experimentally showed that the ratio

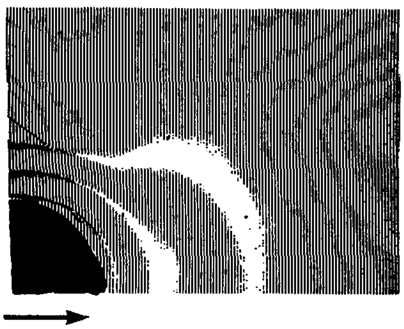


Photo.1 Visualization of wave propagation from an embedded foundation at the surface of the ground model by means of moiré technique (The arrow indicates the excitation direction of the foundation.)

becomes approximately two at the surface of the model ground when Poisson's ratio is 0.5 (Photo.1).

To neglect vertical motion corresponds to a plane-strain condition for the model. A plane-stress condition is another extreme condition which considers a stress-free condition at the surface. Then,

$$\sigma_{zz} = \tau_{rz} = \tau_{\theta z} = 0 \qquad \text{... (13)}$$

This assumption corresponds to a plane-stress condition in the Quasi-Three-Dimensional Model.

Plane-strain solutions can be transformed into plane-stress solutions only by changing Lame's constant λ in the plane-strain formulation to λ^*.

where,

$$\lambda \to \lambda^* = \frac{2\lambda\mu}{\lambda + 2\mu} \qquad \text{... (14)}$$

DYNAMIC INTERACTION BETWEEN CLOSELY SPACED FOUNDATIONS

The present section is devoted to applying the Quasi-Three-Dimensional Model to evaluate the cross interaction between two embedded structures. A local cylindrical coordinate system is set up and the partial differential equation is solved with the boundary condition between the structure and the ground.

(1) Solution of the Governing equation

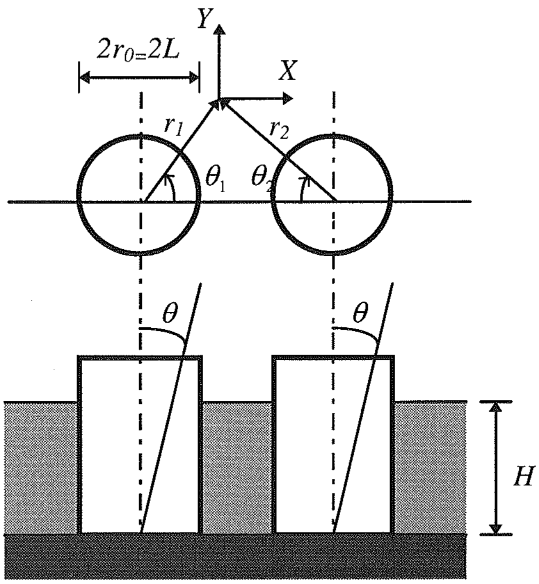


Fig.4 Soil-Structure System Condsidered
$2r_0$: Diameter for circular cross section
$2L$: Width for rectangular cross section

To apply the present model to the cross interaction problem, partial differential equation (1), (2) is solved. Potential functions are defined by

$$u_r = \frac{\partial\phi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$
$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{\partial\psi}{\partial\theta} \qquad \text{... (15), (16)}$$

Substituting equation(15), (16) into (1), (2), the following equations are obtained.

$$(\lambda + 2\mu)^* \nabla^2\phi = (-\rho^*\omega^2 + i\omega c^* + \kappa^*)\phi$$
$$\mu^*\nabla^2\psi = (-\rho^*\omega^2 + i\omega c^* + \kappa^*)\psi$$
$$\text{... (17), (18)}$$

Here,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \qquad \text{... (19)}$$

Potential functions ϕ and ψ are assumed to be separable.

$$\phi = R(r)\Theta(\theta) \qquad \dots (20)$$

$$\psi = R^*(r)\Theta^*(\theta) \qquad \dots (21)$$

Then, equation(17) yields two ordinary linear differential equations, namely,

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left\{\left(\frac{\omega^2\xi^2}{V_P^2}\right) + \frac{m^2}{r^2}\right\}\right]R(r) = 0 \qquad \dots (22)$$

$$\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = -m^2 \qquad \dots (23)$$

Here,

$$\xi^2 = 1 - i\frac{c^*}{\rho^*\omega} - \frac{k^*}{\rho^*\omega^2} \qquad \dots (24)$$

Its general solution is

$$R = A_{K,m}K_m(qr) + A_{I,m}I_m(qr)$$

$$\Theta = A_{s,m}\sin(m\theta) + A_{c,m}\cos(m\theta) \qquad \dots (25), (26)$$

where,

$$q^2 + \left(\frac{\omega\xi}{V_P}\right)^2 = 0 \qquad \dots (27)$$

For outward traveling waves, the potential function decays with horizontal distance and the term I_m must vanish. Then the potential function ϕ is obtained. The other function ψ is also obtained following a similar procedures.

$$\phi = \sum_{m=0}^{\infty} K_m(qr)\{B_{1,m}\sin(m\theta) + B^*_{1,m}\cos(m\theta)\}$$

$$\psi = \sum_{m=0}^{\infty} K_m(sr)\{B^*_{2,m}\sin(m\theta) + B_{2,m}\cos(m\theta)\} \qquad \dots (28), (29)$$

where,

$$s^2 = -\left(\frac{\omega\xi}{V_S}\right)^2 \qquad \dots (30)$$

Finally, we can obtain the displacement in the (r,θ) direction.

$$u_r = \sum_{m=0}^{\infty}\sum_{j=1}^{2}\{C_{m,j}B_{j,m} + C^*_{m,j}B^*_{j,m}\}$$

$$u_\theta = \sum_{m=0}^{\infty}\sum_{j=1}^{2}\{D_{m,j}B_{j,m} + D^*_{m,j}B^*_{j,m}\} \qquad \dots (31), (32)$$

where, $B_{j,m}$ and $B^*_{j,m}$ are unknown constants.

$$C_{m,1} = \{\frac{m}{r}K_m(qr) - qK_{m+1}(qr)\}$$

$$C_{m,2} = -\frac{m}{r}K_m(sr)$$

$$C^*_{m,1} = \{\frac{m}{r}K_m(qr) - qK_{m+1}(qr)\}\cos(m\theta)$$

$$C^*_{m,2} = \frac{m}{r} K_m(sr)\cos(m\theta)$$

$$D_{m,1} = \frac{m}{r} K_m(qr)\cos(m\theta)$$

$$D_{m,2} = \{-\frac{m}{r} K_m(sr) + sK_{m+1}(sr)\}\cos(m\theta)$$

$$D^*_{m,1} = -\frac{m}{r} K_m(qr)\sin(m\theta)$$

$$D^*_{m,2} = \{-\frac{m}{r} K_m(sr) + sK_{m+1}(sr)\}\sin(m\theta) \qquad ...(33)\text{-}(40)$$

(2) Dynamic stiffness of the soil

Normal and shear stresses were obtained from Hooke's law as follows:

$$\sigma_r = \lambda(\nabla^2\phi) + 2\mu\frac{\partial u_r}{\partial r}$$

$$\tau_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) + 2\frac{\partial u_r}{\partial r} \qquad ...\ (41), (42)$$

Thus, soil reaction force against the movement of the foundations is obtained by integrating the stress along the edge of the foundation.

$$P_x = -2\int_0^\pi \sum_{I=1}^{2} \{\sigma_r(r_I, \theta_I)\cos\theta_I - \tau_{r\theta}(r_I, \theta_I)\sin\theta_I\} r_I d\theta_I$$

$$P_y = -2\int_0^\pi \sum_{I=1}^{2} \{\sigma_r(r_I, \theta_I)\sin\theta_I + \tau_{r\theta}(r_I, \theta_I)\cos\theta_I\} r_I d\theta_I \qquad ...\ (43),(44)$$

Soil spring for the horizontal motion is defined as

$$K_H = \frac{P_x}{u_x} \qquad ...\ (45)$$

Then, soil spring for the rocking motion can be defined as follows, ignoring the shear force at the lateral surface of the foundation.

$$K_R = K_H \cdot H^2 \qquad ...\ (46)$$

The boundary condition for another foundation is considered by changing the coordinate system.

$$r_2 = \sqrt{D^2 + r_1^2 - 2Dr_1\cos\theta_1} \qquad ...\ (47)$$

$$\theta_2 = \tan^{-1}\frac{r_1\sin\theta_1}{D - r_1\cos\theta_1} \qquad ...\ (48)$$

These foundations vibrate in the X direction or in the Y direction, in phase or out of phase. Namely, there are four kinds of vibration pattern and the stiffness of the soil is obtained for each.

(3) Effect of the shapes

If the shape of the cross section of the foundation is circular, arbitrary constants can be accurately determined. But if the foundation has an arbitrary shape, for example, rectangular, these constants require that higher order Bessel functions approximate solutions with a sufficient degree of accuracy. Therefore, Bessel functions up to the 11th order were considered and the error was kept to within several percentage points.

RESULTS OF THE ANALYSIS

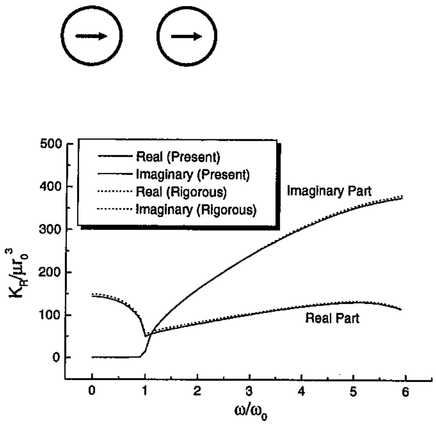
In this section, the dynamic stiffness computed by the present model is compared with the rigorous solution which treats the vibration mode of depth accurately.

(1) Results for the circular cross section

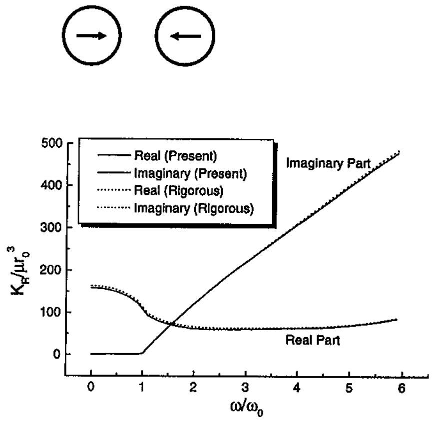
Dynamic stiffness of the ground for the rocking motion of the foundation computed by the present method and by the rigorous method is shown in Fig.5 in the frequency domain. In the rigorous method, up to 9th order trigonometric series were considered for the vibration mode of depth. The real portion of the result is slightly overestimated when compared with results obtained by the rigorous method. However, the imaginary portion shows perfect agreement with the rigorous one. The difference between the results obtained by the present method and by the rigorous method increases as the frequency increases. This means that the fundamental shear mode is the predominant factor for estimating stiffness, especially in the lower frequency range.

(2) Result for the rectangular cross section

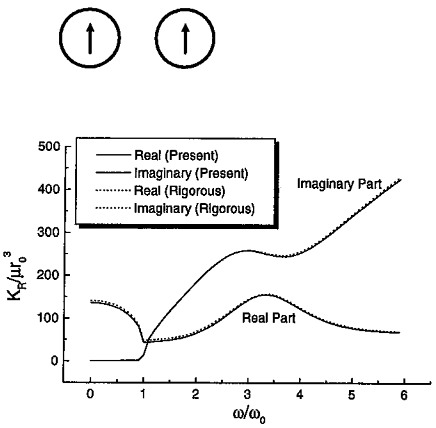
As applied to an example of arbitrary shape, a square-shaped cross section is considered. Stiffness is shown in Fig.6 for four vibration patterns compared with those results obtained by the rigorous method. The results obtained by the present method is 10-20 percent smaller than the results obtained by the rigorous method for the real portion, however, the imaginary portion is in almost perfect agreement with results obtained by using the rigorous method.



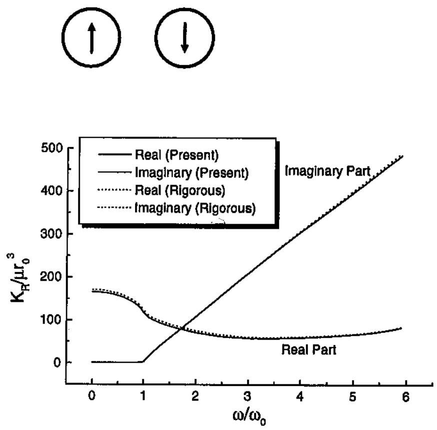
(a) In phase motion in the X direction



(b) Out of phase motion in the X direction



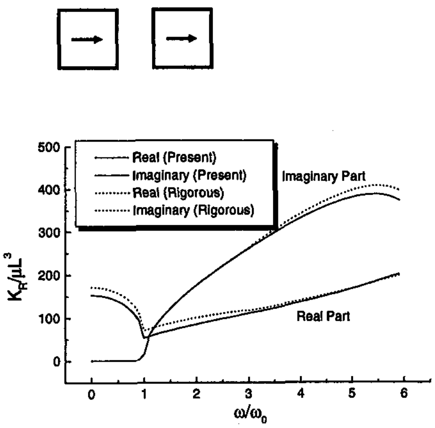
(c) In phase motion in the Y direction



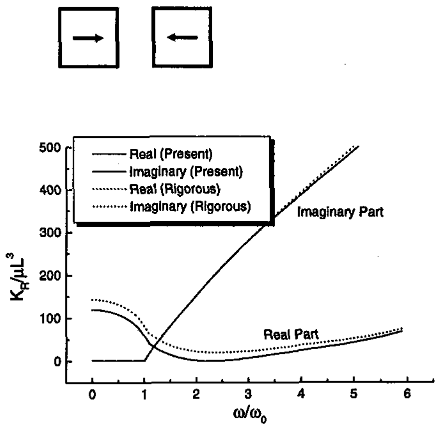
(d) Out of phase motion in the Y direction

Fig.5 Dynamic-stiffness versus frequency factor

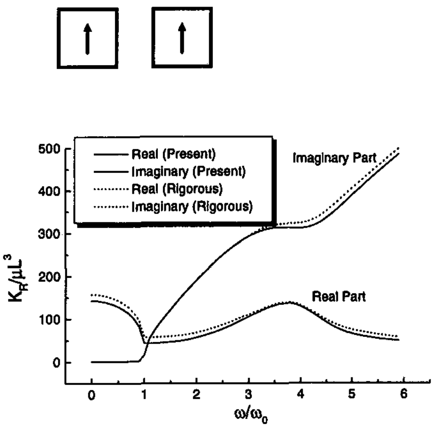
$(r_0 / H = 0.25,\ \beta = 1.8856,\ D = 0.02,\ \omega_0 = \pi V_S / 2H)$



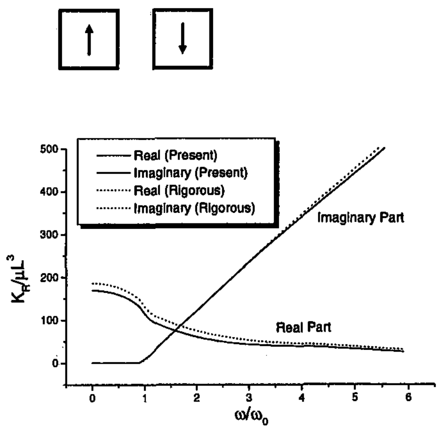
(a) In phase motion in the X direction



(b) Out of phase motion in the X direction



(c) In phase motion in the Y direction



(d) Out of phase motion in the Y direction

Fig.6 Dynamic-stiffness versus frequency factor

$(L/H = 0.25,\ \beta = 1.8856,\ D = 0.02,\ \omega_0 = \pi V_S / 2H)$.

CONCLUSIONS

Simplified evaluation method of dynamic cross-interaction between two foundations of arbitrary shapes was discussed. To evaluate the method, several assumptions were made, namely, neglecting vertical motion, and using only the fundamental shear vibration mode of depth. The effect of a stress-free condition at the surface was considered by assuming the plain-stress condition for the surface layer. Circular and square cross section shapes were taken into consideration. Given these assumptions, dynamic stiffness was evaluated. The results showed good agreement with the rigorous solution. Therefore, it should be relatively easy to incorporate complicated effects such as spatial variation of soil profiles and the non-linearity of the soil in future studies.

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