Real Time Control of a Shaking Table for Simulating Soil-Flexible Structure Interaction

by

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INTRODUCTION

Shaking tables are usually driven by either servo-hydraulic actuators or electro-magnetic actuators. A model on a shaking table interacts with the table, and often gives the table a great amount of force beyond the capacity of its actuators causing the motion of the table to deviate from the intended time history. For this reason, the input signal is often modified so that the motion of the table eventually follows the intended time history of displacement. However, the actual soil supporting a structure, which can be viewed as a natural shaking table, is never stiff enough for the seismic motion to be completely identical to the free-field earthquake motion. When the dominant frequency of a free-field motion is tuned to the resonance frequency of the structure, for example, the interaction results in the motion of the soil at the structure support point to stop naturally at this particular frequency.

With a model experiment on a shaking table, however, it is difficult to reflect the effect of soil-structure interaction in the experiment. One possible means of meeting the requirement would be to add to the input seismic motion to the shaking table a signal equivalent to the further displacement induced by the soil-structure interaction. This method, however, premises a device that generates signals identical to the transient responses of a soil medium of infinite extent. Konagai et al.[1), 2)] have recently presented an idea of the device which is capable of generating the basic transient response functions of soil. The device is a synthesis of analog circuits. To all intents and purposes, an analog circuit loses no time in generating its output. The device thus allows nonlinear superstructure model tests to be conducted on a shaking table incorporating the effect of wave dissipation without preparing any physical soil model.

This paper introduces, in its first half, a further-improved circuit for controlling shaking table to reflect soil-structure interaction. The latter half of the paper is addressed to a description of some findings and problems obtained through a test-try of the present circuit.

SIMULATION OF SOIL-STRUCTURE INTERACTION IN SHAKING TABLE TESTS

Figure 1 shows a schematic view of the set-up in a shaking table test for earthquake simulation, in which a superstructure model is placed directly on the table without a physical ground model. The soil-structure interaction effects are simulated by adding appropriate soil-structure interaction motions to the free-field ground motions at the shaking table. In the simulation, first, the transducers at the base of the foundation pick up the signals of the base forces, p_x, p_z and p_θ in sway, vertical and rocking motions, respectively. These three amplified signals are then applied to the circuits h_x, h_z and h_θ to produce the outputs corresponding to the soil-structure interaction motions, $\tilde{u}_x$, $\tilde{u}_z$ and $\tilde{u}_\theta$, respectively. The output signals are then added to the signals of free-field motions, u_x, u_z and u_θ, to

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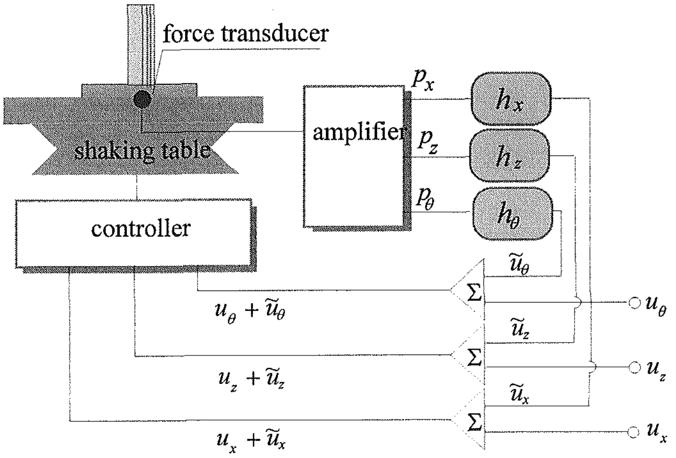


Figure 1 Simulation of soil-structure interaction on a shaking table

produce the signals of foundation motions, $u_x + \tilde{u}_x$, $u_z + \tilde{u}_z$ and $u_\theta + \tilde{u}_\theta$. The signals are translated into the shaking table motions by the shaking table controller.

A number of attempts to clarify various aspects of soil-structure interaction have been carried out mostly in the frequency domain, because impedance functions of soil for various vibration modes are completely dependent on excitement frequency. The attempt for the effect of soil-structure interaction to be reflected in a model test on a shaking table, however, requires expressions of transient response functions of soil. Meek and Wolf[5)-9)] have developed a unified approach for soil-structure interaction analysis by using truncated semi-infinite cone models representing an unbounded soil medium. By superimposing contributions of all the mirror images of the cone, their approach covers a wide variety of soil-structure systems including surface foundations, embedded bodies, flexible piles and so on. However, the very basic concept is found in a simple rigid circular foundation on the surface of a homogeneous soil half-space: which can also be viewed as an appropriate example explaining the soil-structure interaction at the base of an embedded foundation. According to their approach, the soil is idealized for each degree of freedom as a truncated semi-infinite elastic cone with its own apex height z_0 (**Figure 2**). The apex ratio z_0 / r_0, or the opening angle of the cone, is determined just by equating the static stiffness coefficient of the disk on the semi-infinite soil half-space to that of the corresponding cone: whereas the wave propagating through the cone with the velocity v dominates the stiffness within the considerably high frequency range. This approach allows unit impulse response functions for lateral, vertical and rocking modes of vibration to be approximated by an exponential or an exponentially decaying harmonic oscillation (Wolf[10)]). For a translational cone, the unit-impulse response function $h_x(t)$ is obtained as:

$$h_x(t) = \begin{cases} \dfrac{1}{K_{x,static}} \dfrac{v_T}{z_0} e^{-\frac{v_T}{z_0}t} & t > 0 \\ 0 & t < 0 \end{cases} \tag{1}$$

with $K_{x,static} = \rho v_T^2 \cdot \pi r_0^2 / z_0$. When rocking motion of the disk (moment of inertia: $I_0 = (\pi / 4) r_0^4$) is concerned, a rotational cone is to be discussed. The unit-impulse response function $h_\theta(t)$ for a rotational cone is obtained as:

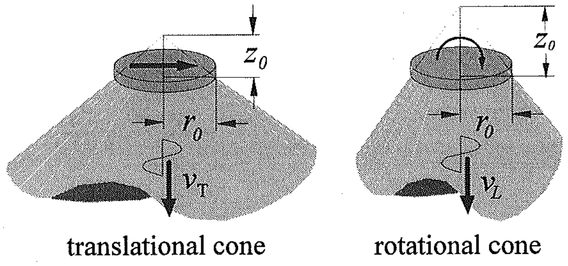


Figure 2 Cones for various degrees of freedom (Meek and Wolf (1992a-1993b))

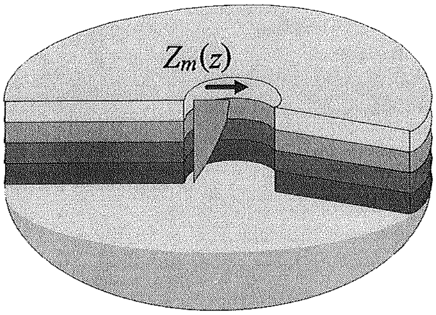


Figure. 3 Soil deposit surrounding an embedded body undergoing a particular vibration mode with respect to depth

$$h_\theta(t)=\begin{cases}\dfrac{1}{K_{\theta,static}}\dfrac{v_L^*}{z_0}e^{-\frac{3}{2}\frac{v_L^*}{z_0}t}\left(3\cos\dfrac{\sqrt{3}}{2}\dfrac{v_L^*}{z_0}t-\sqrt{3}\sin\dfrac{\sqrt{3}}{2}\dfrac{v_L^*}{z_0}t\right) & t>0 \\ 0 & t<0\end{cases} \tag{2}$$

where, $K_{\theta,static}=3\rho v_L^{\,2}I_0/z_0$

When the expression of soil flexibility at the side of an embedded body is concerned, analytical solutions (Tajimi[3], Nogami and Novak[4]) for the vibrations of a cylindrical hollow in an infinite horizontal layer on a stiff base (**Figure 3**) offer an important insights. Wave motion problems of this kind are best expressed in terms of cylindrical coordinates (r, θ, z). Reviewing the expression, it is found that the solution of lateral ground motion u_r of the soil medium at the side of the foundation is separable as:

$$u_r=\sum_m A_m(r,\theta,)\cdot Z_m(z) \tag{3}$$

The contribution of a particular vibration mode $Z_m(z)$ has been examined by Konagai (1996b, 1997), and it was found that the unit impulse response function for the particular vibration mode can be approximated by a linear combination of exponential and exponentially decaying cosine functions.

All above-mentioned expressions may be such an oversimplification of reality that they can not cover all cases. They, however, allow the impulse response function $h(t)$ for any of lateral, vertical or rotational vibration modes to be approximated by linear-combinations of basic response functions $h_m(t)$ as:

$$h(t) = \sum_{m=1}^{n} A_m h_m(t) \tag{4}$$

where, A_m is an unknown constant and

$$h_m(t) = \begin{cases} e^{-\alpha_m t}\cos(\omega_m t - \phi_m) & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{5}$$

The basic response functions expressed by equation (5) are simple exponential functions and/or exponentially decaying oscillations. Setting ϕ_m in equation (5) at $\pi/2$ yields an exponentially decaying sine function: which can be completely conformed to the simple-damped oscillation of any degree of freedom of a structure. Thus, these functions can also be used when a part of a structure or an attached device alone, like a tuned-mass damper on or in a building for example, is tested. Fourier transform of $h_m(t)$ in equation (5) is:

$$F(h_m(t)) = H_m(s) = \frac{s \cdot \cos\phi_m + (\alpha_m \cos\phi_m - \omega_m \sin\phi_m)}{s^2 + 2\alpha_m s + \alpha_m{}^2 + \omega_m{}^2} \tag{6}$$

where, $s = i\omega$, and F is the abbreviation of Fourier transformation. Needless to say, stiffness expression in the frequency domain is just an inverse of equation (6).

It is noted here that both the flexibility (equation (6)) and stiffness expressions in the frequency domain have the following common form as:

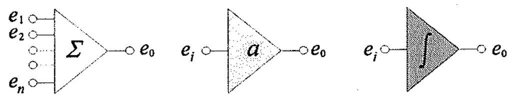
$$H(s) \text{ (or its inverse } 1/H(s)) = \frac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2} = \frac{e_o}{e_i} \tag{7}$$

where, e_i and e_o are input and output signals, respectively. Electric signals can be controlled by using analog circuits. The first-level units in analog circuits are operational amplifiers and passive elements (resisters and capacitors, respectively) (See Holman[11], for example). These units form a linear amplifier (**Figure 4a**), an adder (**Figure 4b**) and an integrator (**Figure 4c**), which are respectively to add several different signals together, to multiply an input signal by a scale factor *a*, and to integrate an input electric signal. An adder, a linear amplifier and an integrator are the key circuits used in designing analog electric circuits.

Introducing an unknown quantity *q*, the above equation (7) can be separated into the following two equations as:

$$e_o = a_0 \frac{q}{s^2} + a_1 \frac{q}{s} + a_2 q \quad \text{and} \quad e_i = b_0 \frac{q}{s^2} + b_1 \frac{q}{s} + b_2 q \qquad \text{(8a), (8b)}$$

With the expression in equations (8a) and (8b), the circuit that is capable of generating e_o to an arbitrary input signal e_i is designed as shown in **Figure 5**. The input signal e_i and two additional signals, later defined to be $-b_0 \cdot q / s^2$ and $-b_1 \cdot q / s$, are added together first by the adder **(a1)** and then multiplied by $1/b_2$ by the linear amplifier **(b1)**. The output signal in the above process is q according to equation (8b). Noting that integrating a signal is equivalent, in the frequency domain, to divide its Fourier spectrum by *s*, integrators **(c1)** and **(c2)** produce signals q/s and q/s^2, respectively. After these two signals go through linear amplifiers **(b2)** and **(b3)** with scale factors $-b_1$ and $-b_0$ respectively, they become $-b_1 \cdot q/s$ and $-b_0 \cdot q/s^2$, and returned to the adder **(a1)**: whereas linear amplifiers **(b4)**, **(b5)** and **(b6)** produce $a_0 q/s^2$, $a_1 q/s$ and $a_2 q$ respectively, and they are added together by the adder **(a2)**. It is now clear from equation (8a) that the output of the adder **(a2)** is identical to the signal e_o.



(a) adder (b) linear amplifier (c) integrator

Figure 4 Key circuits

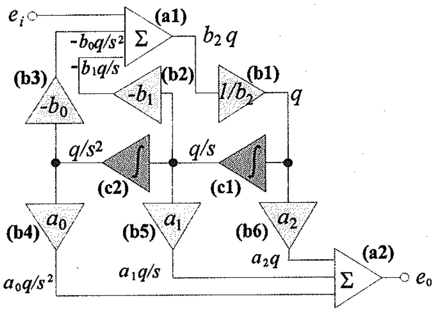


Figure 5 Analog circuit to generate basic response functions

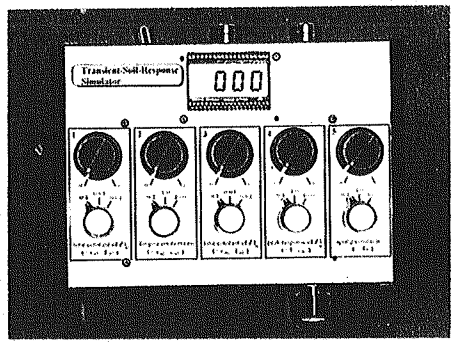
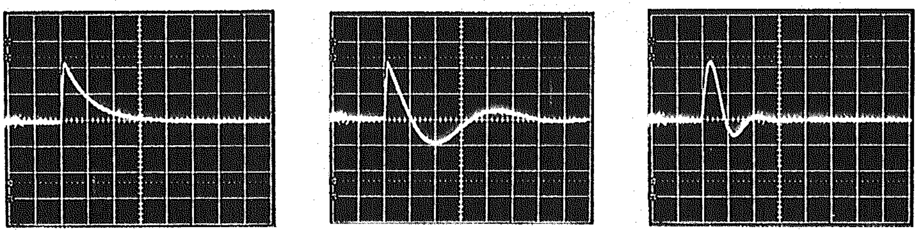


Figure 6 Basic response function generator



(a) $e^{-\alpha_m t}$ (b) $e^{-\alpha_m t}\cos\omega_m t$ (c) $e^{-\alpha_m t}\sin 2\omega_m t$

Figure 7 Basic response functions generated by the present analog circuit (0.1 s/div.)

Figure 6 shows a model for a test try of **Figure 5**-equivalent circuit. Since the circuit model is designed just to simulate flexibility functions (equation (5)) only, the linear amplifier **(b6)** with the scale factor a_2 is not built in. Five pairs of knobs are for tuning the five scale factors in **Figure 5**. In **Figure 7** examples are shown of transient response of the circuit to an impulse (rectangular pulse of 5V, duration time = 10 ms). Only tuning the parameters to prescribed values allows any of the basic response functions to be generated.

Since equation (4) implies that an impulse response function for any of lateral, vertical or rotational vibration modes can be approximated by a linear-combination of basic response functions generated by the present circuit, a necessary number of the circuits should be wired up so that an input signal p is applied to each one of these sub-circuits and outputs from them are added together by an adder (**Figure 8**).

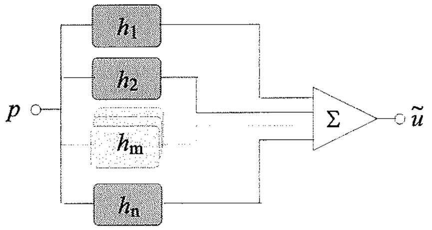


Figure 8 Assembly of h_m (m=1, 2,....n)

SYSTEM FOR CONTROLLING SHAKING TABLES

The assemblies of sub-circuits (**Figure 8**), that are designed to generate unit-impulse response functions for lateral, vertical and rocking motions, are used to produce the outputs corresponding to the soil-structure interaction motions, $\tilde{u}_x$, $\tilde{u}_z$ and $\tilde{u}_\theta$ in **Figure 1**.

An upright flexible cantilever (a steel strip: 2,000 mm × 300 mm × 8 mm) with an extremely small damping constant of only 0.2% (**Figure 9**) is put on a shaking table. The model is assumed to be

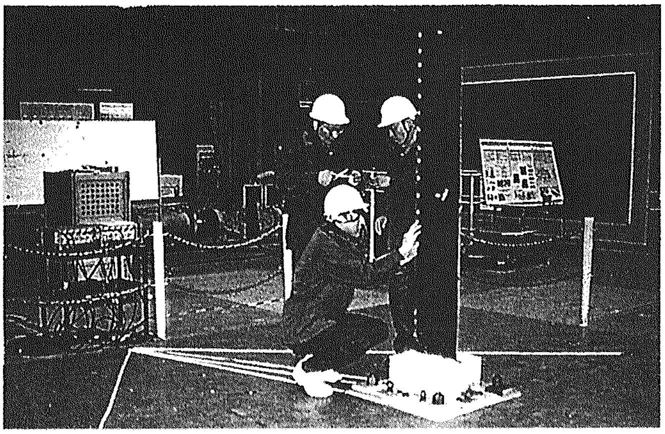


Figure 9 Upright beam on a shaking table

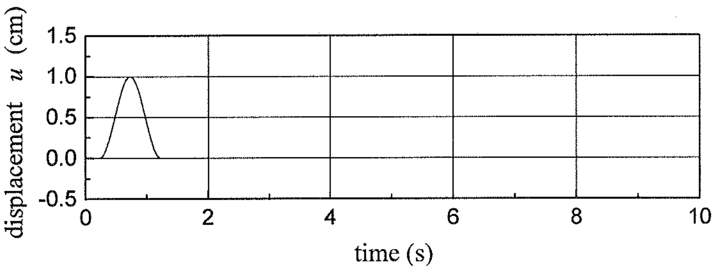


Figure 10 time history of applied displacement

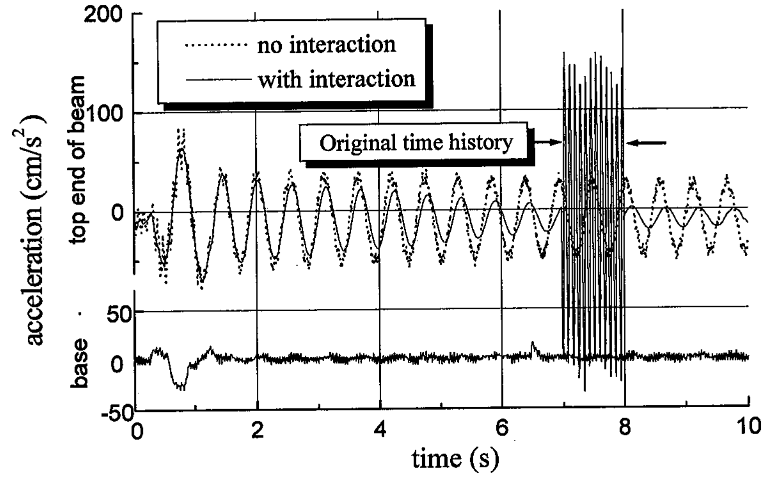


Fig. 11 Acceleration responses at top and bottom ends of beam

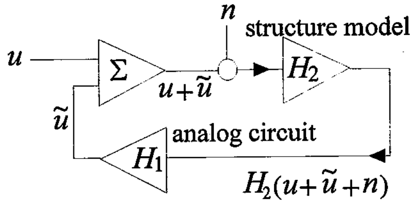


Figure 12 Signal flow for controlling shaking table

supported by a circular surface foundation ($v_T / r_0 = 10\,s^{-1}$, $K_{x,static} = 16.4$ kgf/cm, $K_{\theta,static}$ = 1.62×10^5 kgf·cm) whose unit-impulse response functions for sway and rocking motions are obtained by equations (1) and (2). An impulse shown in **Figure 10** is given to the shaking table as an input motion *u*. The dotted line in **Figure 11** shows the response of the model's top end to the input motion *u* without the interaction motion $\tilde{u}$ being added: whereas the thin line shows the response affected by the interaction motion $\tilde{u}$. Incorporating the effect of the interaction motion leads to the increase of damping up to 2.2% and to the slight decrease of natural frequency as well. However, what we should notice among all the features observed in this figure is the serious noise that has been intentionally left unfiltered in the time interval from 7 to 8 seconds. The predominant frequency of the noise is about 11 Hz, and is about identical to the fourth natural circular frequency of the model. This noise having the nearly constant and a bit higher frequency will be reduced to a great extent if it is integrated twice to become the time history of displacement. The noise, however, can be larger and more serious depending on soil-foundation systems to be studied.

This shows that unexpected noise amplification can cause serious problem in operating a shaking table system especially when a less-damped structure model is tested. **Figure 12** shows the basic signal flow of the present system in which u and $\tilde{u}$ are the free-field ground motion and the soil-structure interaction motion, respectively. A structure model, responding to the input ground motion $u+\tilde{u}$, is expected to produce interaction forces $H_0 \cdot (u+\tilde{u})$. In actuality however, the motion of a shaking table normally contains some steady noise *n*, which is added together with $u+\tilde{u}$. Therefore, after the actual signal $H_0 \cdot (u+\tilde{u}+n)$ goes through the present analog circuit with a transfer function H_1, it becomes $H_1 \cdot H_0 \cdot (u+\tilde{u}+n)$ that is exactly identical to the interaction motion $\tilde{u}$. Equating $H_1 \cdot H_0 \cdot (u+\tilde{u}+n)$ to $\tilde{u}$ results in:

$$\tilde{u} = \frac{H(u+n)}{1-H} \tag{9}$$

where, $H = H_1 \cdot H_0$ (10)

Equation (9) shows that the interaction motion $\tilde{u}$ can be seriously affected by the presence of the noise *n* when $|H| >> |1-H|$. Thus, the transfer function H $(= H_0 \cdot H_1)$ must be examined beforehand to avoid bringing the value of *H* close to 1.

CONCLUSIONS

A new method for a model experiment on a shaking table has been presented. The present method allows soil-structure or base-structure interaction to be simulated. The conclusions of this study are summarized as follows:

(1) Unit-impulse response functions at the side or at the base of an embedded body for lateral, vertical and rotational vibration modes are approximated by signals produced by one analog circuit with five parameters being adjusted to the prescribed values.

(2) The above-mentioned circuit enables us to simulate the soil-structure interaction in model tests by using a structure model alone without a physical ground model. Since analog circuits lose hardly any time in producing output signals, they can be used in a shaking table test which is operated under relatively fast loading in earthquake simulation.

(3) When a less-damped flexible structure model is tested on a shaking table, unexpected noise amplification can cause serious problem in operating the shaking table. When the soil-structure interaction motion $\tilde{u}$ is given in the present system as a convolution of the transfer function *H* and the actual motion of the shaking table, the amplification factor is identical to $H/(1-H)$. This transfer function must be examined beforehand to avoid bringing the value of *H* close to 1.

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