Elastic-Plastic Analysis of Steel Damper by Using Degenerated Timoshenko Beam Element Based on the ASI Technique

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ABSTRACT

The finite beam element with the use of the adaptively shifted integration (ASI) technique is suitable for elastic-plastic analyses of framed structures, because a plastic hinge can be set at an appropriate position. In this paper the ASI technique is applied to a degenerated Timoshenko beam element that is generally used. The developed element is applied to the analyses of the steel damper for base isolation systems of large scale buildings.

1. INTRODUCTION

In elastic-plastic analyses of framed structures the evaluation of appropriate positions of plastic hinges is essential. However, a position of a plastic hinge is restricted in conventional finite element method (FEM) since stresses are evaluated at limited number of integration points. On the other hand, with the use of the adaptively shifted integration (ASI) technique, which is developed by Toi, one of authors, and his associates, a plastic hinge can be located at an arbitrary position. Also the technique can be easily implemented by slightly modifying conventional codes of FEM.

Although the ASI technique could be applied to both cubic Euler element and linear Timoshenko element, the latter is only treated in this paper. The original linear Timoshenko beam element based on the ASI technique was described by using stress resultants and general strains[4][5]. The ASI technique was also applied to a two dimensional layered beam element[7]. The layered element was used for the analyses of the crack propagation in framed structures made of brittle materials. Appropriate positions of cracks could be represented with the use of the ASI technique.

In this paper the ASI technique is applied to a three dimensional degenerated Timoshenko beam element with linear interpolation function. A feature of degenerated elements is that the development of a plastic region in a section can be represented since the element is similar to solid elements except for interpolation functions in which assumptions of the beam theory are introduced. Another feature is the element can easily be extent to the geometrically nonlinear element since nonlinear continuum mechanics can be directly applied.

The presented element is applied to elastic-plastic analyses of a cantilever and a steel damper for base isolation systems. The shape of the damper is like a coil of spring, and damping effect can be obtained due to the consumption of seismic energy by the plastic deformation[3]. Both infinitesimal displacement analyses and geometrically nonlinear analyses based on the total

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Lagrangian formulation are carried out by using the presented element.

2. A DEGENERATED LINEAR TIMOSHENKO BEAM ELEMENT BASED ON ASI TECHNIQUE

2.1 DEGENERATED TIMOSHENKO BEAM ELEMENT

Fig. 1 shows the degenerated Timoshenko beam element presented by Bathe[1] and developed by Dvorkin[2] to which the ASI technique is applied in this paper. The displacement field of the element is following the assumptions of Timoshenko beam theory. Those are (1) a straight line normal to the beam axis of initial configuration remains straight during the deformation, but not necessarily normal to the deformed axis, (2) the cross section of the beam is not deformed. Under these assumptions moderately thick beams can be treated since shear deformations are taken into account.

The ASI technique can be applied when the linear interpolation functions following the above-mentioned assumptions is employed with one-point quadrature. A position vector of a material point and a displacement vector can be represented as follows.

$$\mathbf{x}(s_1,s_2,s_3) = \sum_{n=1}^{2} N^n(s_1)\left(\mathbf{x}^n + \frac{a^n}{2} s_2\, {}^0\mathbf{V}_2^n + \frac{b^n}{2} s_3\, {}^0\mathbf{V}_3^n\right), \tag{1}$$

$$ {}^{t+\Delta t}_{\ \ t}\mathbf{u}(s_1,s_2,s_3) = \sum_{n=1}^{2} N^n(s_1)\left\{\mathbf{U}^n + \frac{a^n}{2} s_2\left({}^{t+\Delta t}\mathbf{V}_2^n - {}^t\mathbf{V}_2^n\right) + \frac{b^n}{2} s_3\left({}^{t+\Delta t}\mathbf{V}_3^n - {}^t\mathbf{V}_3^n\right)\right\}, \tag{2}$$

where

$$N^1(s_1) = \frac{1}{2}(1-s_1),\quad N^2(s_1) = \frac{1}{2}(1+s_1). \tag{3}$$

Although dimensions of the section at two nodes can be different in general, they must be constant when the ASI technique is applied (see the proof in Ref. [4]).

The total Lagrangian formulation is adopted for describing the geometrical nonlinearity. Following the formulations proposed by Dvorkin[2], finite rotations of directors $\mathbf{V}_2^n$ and $\mathbf{V}_3^n$ are taken into account when the directors are updated, and the tangent stiffness matrix which is consistent with the finite rotation is used.

When the element with circular section is treated, parameters (r,φ) in polar coordinate system are mapped to parameters (s_2,s_3) in natural coordinate system as shown in Fig. 2. For

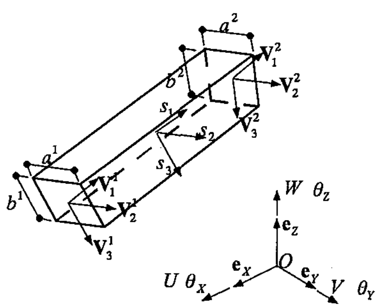


Fig. 1 Degenerated beam element

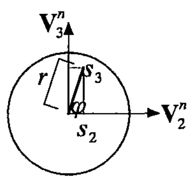


Fig. 2 A circular cross section

the numerical integration the trapezoidal rule is employed in circumferential direction and Simpson's rule in radial direction. Numerical experiments of elastic cantilevers with infinitesimal displacement show that three integration points in each direction are sufficient when a moment, a torsional moment or an axial load is applied independently. More integration points are needed in elastic-plastic analysis. In the latter examples cross sections are circular and the number of integration points in the radial direction is seven, and in the circumferential direction it is sixteen. The number of total integration points in a section is 112.

2.2 ELASTIC-PLASTIC CONSTITUTIVE EQUATIONS

When large displacements and rotations but small strain are assumed, constitutive equations for infinitesimal deformation can be employed for describing a relation between Green-Lagrange strain tensor and the 2nd Piola-Kirchhoff tensor, since these two tensors are invariant under rigid body rotations[1][6]. In the following equations stresses and strains are represented by the components of the 2nd Piola-Kirchhoff stress tensor, and that of Green-Lagrange strain tensor respectively. The total Lagrangian formulation is employed.

The elastic-plastic relation is introduced only in axial direction, and shear deformation is assumed to be elastic. A bilinear model considering kinematic hardening is employed for the relation in the axial direction such as

$$ {}^{t+\Delta t}\sigma_{11} = \frac{H'}{E+H'}\,{}^{t}\sigma_{11} + \frac{EH'}{E+H'}\Delta\varepsilon_{11} + \frac{E}{E+H'}\,{}^{t}\sigma_{Y11} \,, \qquad (4) $$

where ${}^{t}\sigma_{11}$ is the stress at time t, E is Young's modulus, H' is the strain hardening parameter, $\Delta\varepsilon_{11}$ is an increment of the strain and ${}^{t}\sigma_{Y11}$ is the yield stress at time t. Eq. (4) can be rewritten as

$$ \begin{aligned} {}^{t+\Delta t}\sigma_{11} &= \frac{H'}{E+H'}\left({}^{t}\sigma_{11} + E\Delta\varepsilon_{11}\right) + \frac{E}{E+H'}\,{}^{t}\sigma_{Y11} \\ &= \frac{H'}{E+H'}\,{}^{t+\Delta t}\sigma_{11}^{T} + \frac{E}{E+H'}\,{}^{t}\sigma_{Y11} \end{aligned} \,, \qquad (5) $$

where ${}^{t+\Delta t}\sigma_{11}^{T}$ is the elastic predictor of stress ${}^{t+\Delta t}\sigma_{11}$. If $\left|{}^{t+\Delta t}\sigma_{11}^{T}\right|$ is smaller than $\left|{}^{t}\sigma_{Y11}\right|$, unloading occurs and ${}^{t+\Delta t}\sigma_{11}$ is equal to ${}^{t+\Delta t}\sigma_{11}^{T}$.

2.3 OVERVIEW OF THE ASI TECHNIQUE

Toi, one of authors, showed the equivalence of approximated strain energy between the linear Timoshenko beam element using one-point (reduced) integration and the one dimensional Rigid Bodies-Spring Model (RBSM) considering the effect of lateral shear deformations. The condition of the equivalence is that the integration point of the beam element and the connecting point of corresponding RBSM are symmetrically placed at equal distances from the center of the element.

The equivalence means a linear Timoshenko element can be the RBSM whose connecting point is at an arbitrary position by simply shifting the integration point. Therefore a plastic hinge, which is represented by a plastic rotational spring of the RBSM, can be located at an arbitrary position in the beam element.

A deficiency of the beam element with a shifted integration point is the accuracy of elastic solutions become worse. The most accurate elastic solution is obtained when the integration point is at the center. For overcoming the deficiency Toi and Isobe presented adaptively shifted integration (ASI) technique. In the ASI technique a position of an integration point is adaptively

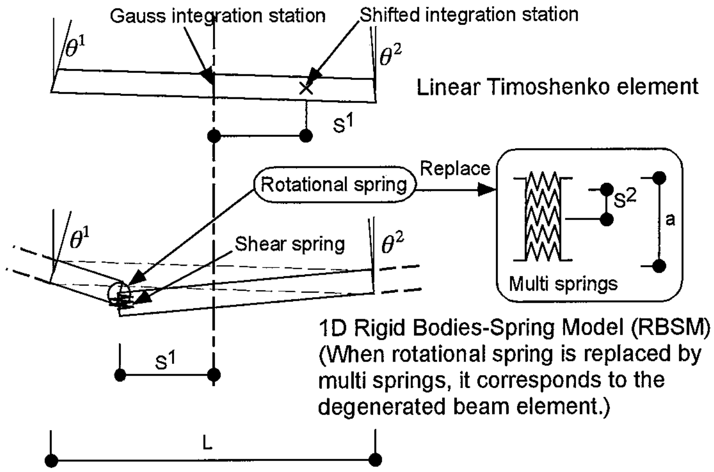


Fig. 3 Equivalence between the linear Timoshenko beam element and 1D RBSM

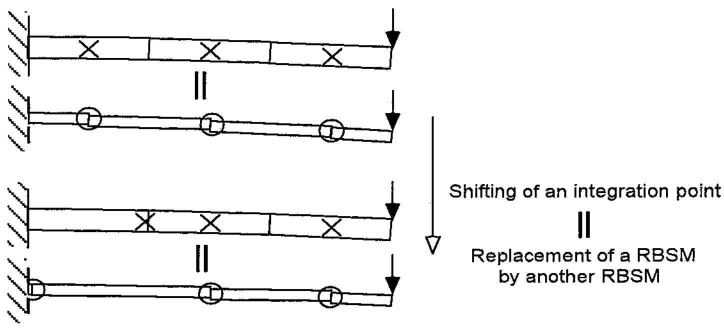


Fig. 4 Shifting of an integration point and Replacement of RBSMs

shifted when plastic deformation occurs, but it remains at the center in elastic deformation.

In geometrically nonlinear cases the RBSM and Timoshenko beam element with a shifted integration point may not be equivalent, especially when the deformation become large. However the same procedure of shifting is employed in this paper. The numerical experiments with a cantilever, which is presented in the next section, show the element with ASI technique behaves well when displacement and rotation are moderately large.

2.4 APPLICATION OF THE ASI TECHNIQUE TO DEGENERATED BEAM ELEMENT

2.4.1 Overview

The above-mentioned ASI technique can be applied to the degenerated Timoshenko beam element shown in the section 2.1. The original Dvorkin's element will be called a "conventional" element in this paper. The essential points in the application are (1) the method for the extrapolation of stresses at points other than integration points, and (2) the condition for adaptively shifting.

Note that in the degenerated element several quadrature points exist in a section and these points should be shifted simultaneously. This procedure is called "the shifting of the integration points" in this section. Also note that, as mentioned in section 2.3, the stresses recovered at the shifted integration points are really the stresses evaluated at the opposite side of the element.

2.4.2 Extrapolation of Stresses

When ASI technique is applied, the stresses at the points other than integration points should be computed for the evaluation of an appropriate position of a plastic hinge. However, such stresses cannot be directly recovered in the linear Timoshenko beam element since stresses are evaluated by the one-point integration. In the presented element the stresses are computed from the bending moments extrapolated by using shear stress resultants (see ref. [5]). For example a bending moment around a director $\mathbf{V}_2^n$ can be extrapolated as follows.

$$M_2 = M_{Q2} + Q_{Q13} l, \tag{6}$$

where M_{Q2} is the bending moment calculated from axial stresses at the integration points, M_2 is the bending moment at the distance l from the integration points and Q_{Q13} is the shear stress resultant. The parameter of length l is positive when the position of the extrapolated moment is placed in the right-hand of the integration points.

The stresses in the axial direction are computed by using eq. (6). By adding contributions of the bending moment around the director $\mathbf{V}_3^n$ and the axial stress resultant, which is the product of the sectional area A and the axial stress at the neutral axis of the integration position, the axial stress is evaluated as

$$\begin{aligned}\sigma_{11} &= \frac{N_{11}}{A} + \frac{M_2}{I_2}s_3 + \frac{M_3}{I_3}s_2 \\ &= \frac{N_{11}}{A} + \frac{M_{Q2}}{I_2}s_3 + \frac{Q_{Q13}l}{I_2}s_3 + \frac{M_{Q3}}{I_3}s_2 + \frac{Q_{Q12}l}{I_3}s_2 \\ &= \sigma_{Q11} + \frac{Q_{Q13}l}{I_2}s_3 + \frac{Q_{Q12}l}{I_3}s_2 \\ &= \sigma_{Q11} + \frac{A\tau_{mQ13}l}{I_2}s_3 + \frac{A\tau_{mQ12}l}{I_3}s_2\end{aligned} \tag{7}$$

where σ_{11} is the extrapolated axial stress at (s_2, s_3), σ_{Q11} is the axial stress at the integration point placed at (s_2, s_3), τ_{mQ12} and τ_{mQ13} are the shear stresses at the neutral axis of the integration station (the shear stresses that contribute to a torsional stress resultant can be excluded when the shear stress at the neutral axis is employed) and I_2 and I_3 are moments of inertia in two directions.

Eq. (7) should not be applied when the element is plastic. However, since in the presented scheme (described in the next subsection) the shifting is carried out before yielding occurs and just after all points in the element are unloaded, eq. (7) may be appropriate.

2.4.3 Conditions for the Shifting.

Determining when a plastic hinge takes place is not easy in the degenerated beam element, since material points in a section do not become plastic simultaneously. The following conditions for the shifting are used.

(1) The integration points can only be placed at three cross sections along the element. Those are the sections at the center, the left node and the right node.

(2) The integration points in a section are shifted simultaneously and they are always placed in a same section.

(3) Once the shifting is carried out the integration points are not shifted until all points in the

element are unloaded.

(4) The integration points are placed at the position where the number of yielded integration points is the largest. However, the yielded points at the surface are excluded in counting the number, since elastic deformation dominates when only a surface region is yielded, and in that case the plastic hinge should not be set.

(5) When a pure bending moment or an axial force is applied to a beam, it deforms homogeneously in the axial direction along the beam and the plastic hinge should not be set. Therefore when the difference among the numbers of yielded points in the three cross sections is small, the integration points are placed at the center.

(6) The shifting is carried out at the beginning of each incremental step before the stiffness matrix is computed. In the iteration process based on the Newton-Raphson method the integration points are not shifted.

A problem of the presented scheme is, when strain hardening is taken into account, yield stresses cannot be updated except for the integration points. If a plastic hinge occurs in different positions in an element during a successive analysis, the problem could affect solutions. In the following examples a yield stress corresponding to a position (s_2, s_3) assumed to be constant in an element.

3. ELASTIC-PLASTIC ANALYSES OF A CANTILEVER

For showing the validity of the presented element, the analyses of a cantilever under a point loading are carried out. Incremental displacement method is employed and the lateral displacement at the tip is prescribed. Material properties and dimensions of the model are shown in Table 1.

Results for two kinds of strain hardening parameter are shown in Fig. 5 and Fig. 6. Geometrical nonlinearity is not considered in both cases. The load-displacement curves obtained by using the conventional or the ASI technique with 1, 2, 4 or 40 elements are shown in each figure and compared.

The slope of the curves changes gradually, since the development of plastic regions could be represented by the degenerated beam element. When the model is subdivided into 40 elements the solutions obtained by two methods are almost the same, and they are considered to be converged. In Fig. 5 the solutions with the use of ASI technique converge rapidly, and a collapse load is evaluated even with one element model.

In Fig. 6, however, the results obtained by the ASI technique underestimate the collapse load. This is due to the inaccuracy of the elastic solution using the element with shifted integration points, that means the increases of yield stresses are not evaluated properly. The results suggest that when large strain hardening parameters are used, the ASI technique should not be applied.

Table 1 Dimensions and material properties of the cantilever

Length	200 cm
Diameter of a section	7cm
Young's modulus E	2.1x106 Kgf/cm^2
Strain hardening parameter H'	E/2100 or E/80
Initial yield stress	3000 Kgf/cm^2

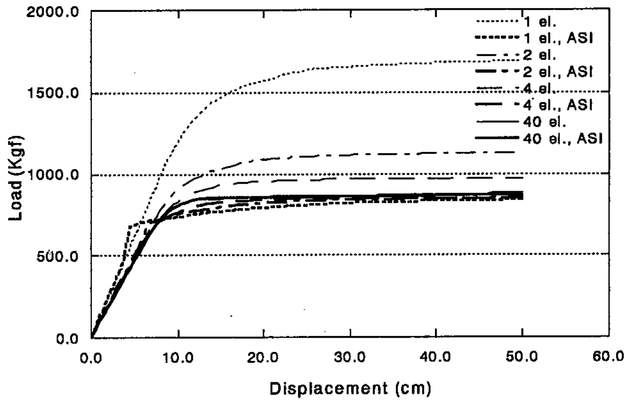


Fig. 5 Load-displacement curves of a cantilever ($H' = E/2100$, geometrically linear)

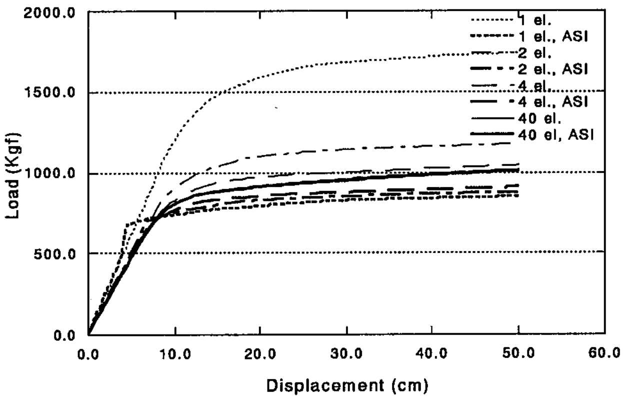
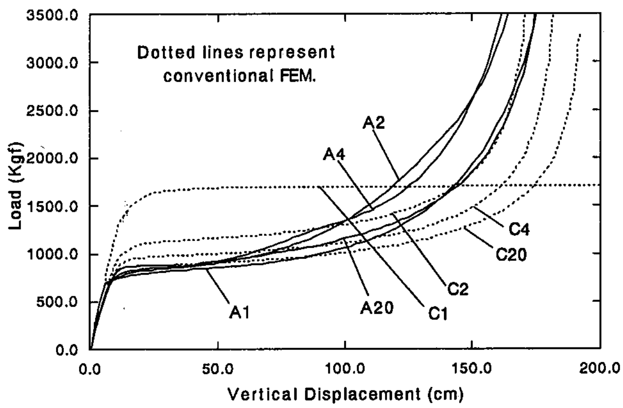


Fig. 6 Load-displacement curves of a cantilever ($H' = E/80$, geometrically linear)



"A20" means "ASI technique with 20 elements".
"C20" means "Conventional FEM with20 elements".

Fig. 7 Load-displacement curves of a cantilever (geometrically nonlinear)

Fig. 7 shows the results when geometrical nonlinearity is taken into account. When the lateral displacement is under about 50 cm (corresponding to the rotation of 15 degrees at the fixed boundary) the models with ASI technique behave well, after that, however, a very over-stiff solutions are obtained with the models. It is concluded that the presented element based on the Dvorkin's element could be applied to the cases with moderately large rotation.

4. ELASTIC-PLASTIC ANALYSES OF THE STEEL DAMPER FOR BASE ISOLATION SYSTEMS

Photo 1 is an overview of the steel dampers. A Plan and an elevation of the damper are shown in Fig. 8. As shown in Photo 1 four dampers are used for an isolating system, though analyses are carried out for one damper. Material properties and dimensions of the model are shown in Table 2. A small strain hardening parameter is employed. The displacement at the upper boundary in X or Y (see Fig. 8) direction is prescribed incrementally, the rotation around the Z axis at the upper boundary is set free, and the other degrees of freedom at the upper and the lower boundaries are constrained.

Fig. 9 shows the results without considering geometrical nonlinearity. The model is subdivided into 10 or 50 elements, and each model is analyzed by using the conventional and the ASI technique. When the 50 elements model is used, the difference between two methods is very little.

In Y direction, the obtained load-displacement curves are almost the same. The 10 elements model is sufficient for this direction. In X direction, the solution of the 10 elements model with ASI technique is almost identified with the solution of the 50 elements model when the displacement is smaller than about 15 cm. Even when the larger displacement is prescribed, although the solution underestimates the collapse load, it is better than that of conventional method with 10 elements.

Fig. 10 shows the results with geometrical nonlinearity. The results of the linear analysis with conventional 50 elements are also shown for comparison. In X direction the results almost coincide with the linear solutions. The behavior of the 10 elements model with the ASI technique is better than the model with conventional method. In Y direction, the hardening due to geometrical nonlinearity occurs. The solution of the 10 elements model with the ASI technique is better when the displacement is less than about 15 cm, but after that the solution is worse than that of the conventional method. As shown in previous section, the over-stiff due to the shifting may have occurred.

Table 2 Dimensions and material properties of the steel damper

Radius	26.5 cm
Height	17.0 cm
Diameter of a section	7cm
Young's modulus E	2.1x106 Kgf/cm^2
Strain hardening parameter H'	E/2100 or E/80
Initial yield stress	3000 Kgf/cm^2

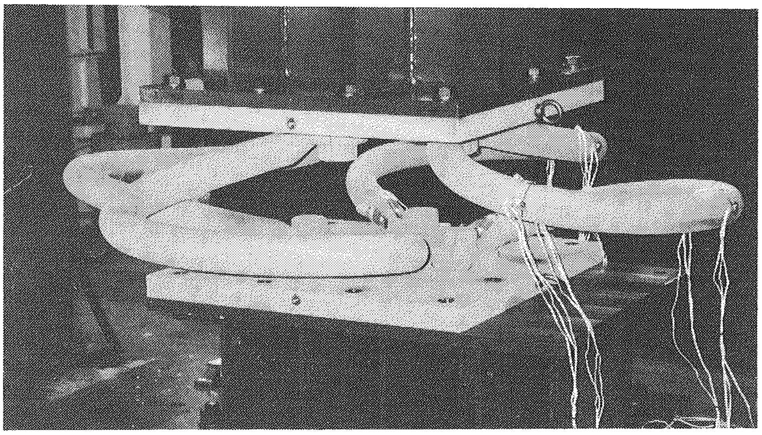


Photo 1 The steel damper for base isolation systems

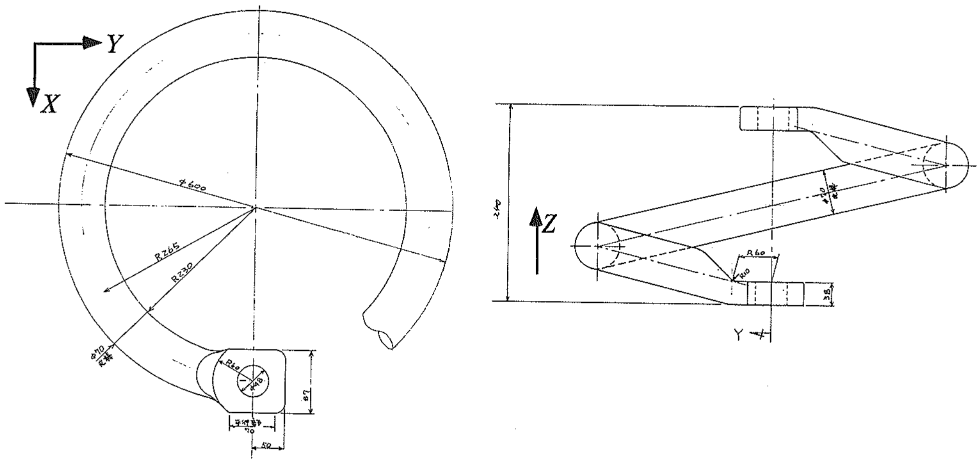


Fig. 8 Plan and section of the steel damper

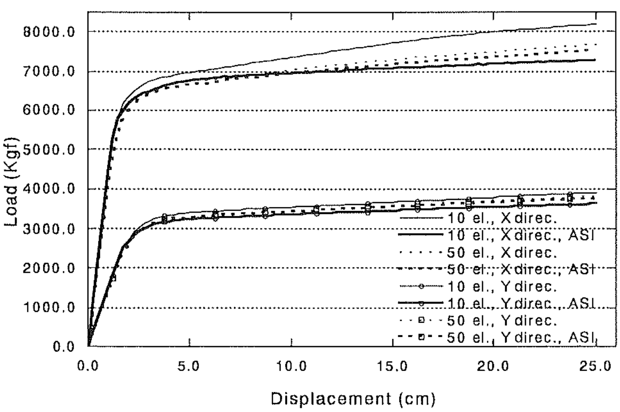


Fig. 9 Load-displacement curves of the steel damper (geometrically linear)

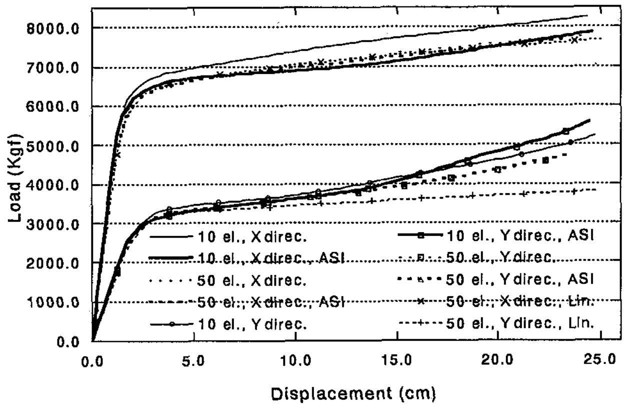


Fig. 10 Load-displacement curves of the steel damper (geometrically nonlinear)

5. CONCLUDING REMARKS

Adaptively shifted integration (ASI) technique is applied to the Dvorkin's degenerated Timoshenko beam element. Several basic equations for the application are shown, and a computational scheme is presented. Elastic-Plastic analyses of a cantilever and a steel damper for base isolating structures are carried out, and validity and restriction of the presented element are evaluated. In geometrically nonlinear analyses the presented element can treat moderately large rotations, however it should not apply to the problems with very large rotations.

REFERENCES

[1]Bathe, K. J. : Finite Element Procedures in Engineering Analysis, Prentice-Hall, 1982
[2]Dvorkin, E, et al. : On a Non-linear Formulation for Curved Timoshenko Beam Elements Considering Large Displacement/Rotation Increments, Int. J. for Num. Meth. in Engin., Vol. 26, pp. 1597-1613, 1988
[3]Hasegawa, H, et al : Experiment on Large-Capacity Steel Damper for Base Isolation System, Summaries of Technical Papers of Annual Meeting, AIJ, pp. 673-674, 1990 (in Japanese)
[4]Toi, Y. : Shifted Integration Technique in One-dimensional Plastic Collapse Analysis Using Linear and Cubic Finite Elements, Int. J. for Num. Meth. in Engin., Vol. 31, pp. 1537-1552, 1991
[5]Toi, Y. and Isobe, D. : Adaptively Shifted Integration Technique for Finite Element Collapse Analysis of Framed Structures, Int. J. for Num. Meth. in Engin., Vol. 36, pp. 2323-2339, 1993
[6]Hisada, T and Noguchi, H : Foundations and Applications of Nonlinear Finite Element Method, Maruzen, 1995 (in Japanese)
[7]Toi, Y and Saito, Y : Finite Element Collapse Analysis of Brittle Framed Structures by the Adaptively Shifted Integration Technique, Trans. of the Japan Soc. of Mech. Engin., Vol. 62, No. 604 (in printing), 1996 (in Japanese)