A New Plastic Design Method to Assure Target Collapse Mechanism under Extreme Earthquake Loading

by

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A plastic design method based on minimum-norm stress fields and the lower-bound theorem are proposed by one of the authors[1], by which designers' dissatisfaction due to the required resistance magnitudes could be minimized under various cases of load combination. This method provides an efficient preliminary design process aiming at sufficient load-carrying capacity without consideration of resulted collapse mechanism. In case of extreme earthquake loading, however, design concerns should be not only focused on the resistance magnitude but on the type of collapse mechanism as experienced in the building damage caused by the Great Hanshin-Awaji Earthquake. This paper presents how to proportion non-plasticized portions in the target collapse mechanism by use of minimum-norm pseudo-inverse techniques.

1. Introduction - Plastic Design Based on Minimum-Norm Stress Fields

One of the popular plastic design methods is to utilize the lower bound theorem in the limit analysis: when a statically admissible stress field is found for a certain load, this load becomes a lower bound of the true collapse load. So, if the member strength is proportioned to exceed a certain stress field in equilibrium with the design load, the collapse load of the proportioned structure exceeds the design load. The problem with this method is how to choose a stress field from infinite number of stress fields in equilibrium with the design load. A simple way is to make use of the elastic stress field, in other words, to make use of the member force vector denoted by $\{m\}$ that satisfies the following three conditions:

Equilibrium: $\{f\}=[C]\{m\}$ (1)
Compatibility: $\{\theta\}=[C]^T\{d\}$ (2)
Elasticity: $\{m\}=[K]\{\theta\}$ (3)

where $\{f\}$: design load vector, $[C]$: connectivity matrix, $\{m\}$: member force vector,
$\{\theta\}$: member deformation vector, $\{d\}$: deformation vector corresponding to $\{f\}$,
$[K]$: member stiffness matrix

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It is obvious that the latter two Eqs. (2) and (3) are unnecessary conditions from the standpoint of the lower bound theorem. They are used only to get a unique stress field. Additionally, the use of the elastic stress field takes no advantage of the redistribution of stress after yielding. As long as sufficient plastic deformability is provided in the members, Eqs. (2) and (3) can be modified by any other conditions convenient for design purpose. The use of an alternative stress field termed the minimum-norm (or least squares) stress field is proposed by one of the authors[1]. This stress field is defined by substituting the following condition for Eqs. (2) and (3):

To minimize $\{m\}^T\{m\}$ (Norm of member force) (4)

Or, in a more general form, To minimize $\{m\}^T[D]^T[D]\{m\}$ (5)

2. Plastic Design Process Without Target Collapse Mechanisms

We denote the minimum-norm member force vector by $\{m^*\}$ for a given external design load $\{f\}$. We assume that the connectivity matrix [C] with n rows by m columns has the properties:

$n \leqq m$ and $\mathrm{rank}[C]=n$

To obtain $\{m^*\}$ is to minimize the member force norm $\{m\}^T\{m\}$ under the equilibrium condition $\{f\}=[C]\{m\}$. This is identical to the minimization of the following function:

$$G= 0.5\{m\}^T\{m\}+\{\nu\}^T(\{f\} - [C]\{m\})$$

where ν : La grange's indeterminate coefficients

Partial derivative of G with respect to m_i shall be zero at $m_i=m^*_i$, and we obtain $\{m^*\}=[C]^T\{\nu\}$. Substituting this into the equilibrium equation, we can solve $\{\nu\}$ as follows:

$$\{f\}=[C]\{m^*\}=[C][C]^T\{\nu\}, \text{ and then } \{\nu\}=([C][C]^T)^{-1}\{f\}$$

Finally $\{m^*\}=[C^*]\{f\}$ where $[C^*]=[C]^T([C][C]^T)^{-1}$ (6)

The matrix [C*] is termed a 'minimum norm pseudo-inverse.' It should be noted that a linear transformation on $\{f\}$ gives the minimum norm stress field for $\{f\}$, that is, the following superposition is possible: the sum of $\{m_A^*\}$ and $\{m_B^*\}$, the minimum norm stress fields for two design loads $\{f_A\}$ and $\{f_B\}$, respectively, makes the minimum stress field for the design load $\{f_A+f_B\}$. This feature is similar to the allowable stress design procedure, and it is advantageous for the plastic design under several situations of multiple load combination.

3. Necessity of Target Collapse Mechanism Consideration

If the load-carrying capacity is everything to be considered in the ultimate limit state design of structures, we just apply the plastic design method mentioned above to several load situations. This is right for most cases of persisting loads, but not enough for a very severe seismic load effects. We have to rely on plastic energy absorption of structural members, and we should know which members can work as energy absorber, in other words, how the structural system will collapse. While the amount of seismic energy exerted into the buildings with similar mass and natural period can be regarded identical, the seriousness of structural damage strongly depends on the type of collapse mechanism during the earthquake. Photo 1 shows a flexibly jointed steel frame still standing after braking of its braces, and Photo 2 is the complete collapse at the intermediate story in the weak-column type building, both seen at Kobe City devastated by the Great Hanshin-Awaji Earthquake on January 17, 1995.

Photo 1 Overall collapse of flexibly jointed steel frame after brace braking

Photo 2 Local collapse at intermediate story of weak-column building

By assuming that only one collapse mechanism and one equivalent-static load pattern can represent the inelastic earthquake response, we can write the required base shear strength Q_P in the following form[2]:

$$Q_P = E_P / \{ \theta_{cr} \ H \ \Sigma \ p_i d_i \} \tag{7}$$

where E_P : accumulative plastic energy demand (load effect in term of energy)

θ_{cr} : accumulative plastic rotation capacity of plastic hinges in the collapse mechanism

H: the lowest story height in the collapse mechanism
pi: equivalent-static load pattern normalized by base shear
di: collapse mode shape normalized by the lowest non-zero plastic displacement

When a linearly changed load pattern is assumed, the value of Σ pidi is calculated for several types of collapse mechanism of a 3-story frame as follows:

-Overall collapse mechanism like Photo 1:	Σ pidi =2.33
-Local collapse at the intermediate story like Photo 2:	Σ pidi =0.83
-Local collapse at the first story (soft first story):	Σ pidi =1.00

This means that the required seismic strength for the soft first story collapse is twice, that of the intermediate story collapse is three times, greater than that of overall collapse case, if their structural members have identical rotation capacity. Thus, the type of collapse mechanism greatly influences on the energy dissipation capacity of a frame, and then it should be considered carefully in the ultimate limit state design against severe earthquake loading.

4. Concluding Remarks - Plastic Design Process With Target Collapse Mechanism

The collapse mechanism mentioned in the preceding section can be preferably specified by designer so that he can prepare enough energy absorption capacity for the plasticized portions in the mechanism. As concluding remarks, a model process in such a plastic design method is proposed as follows:

1) The target collapse mechanism is specified by designer.

2) The energy demand E_P is determined from mass, natural period, and the specified design earthquake properties.

3) Available plastic rotational capacity θ_{cr} of plasticized portion in the target mechanism is assigned, which depends on designer's choice.

4) The moment capacities $\{m_P\}$ at the plasticized portion is proportioned so that $\Sigma m_{Pi} \theta_{cr}$ exceeds the total energy demand E_P .

5) The external force {f} at the target collapse state is determined by applying the virtual work principle to the target mechanism, together with an equivalent-static load pattern and the moment capacities $\{m_P\}$ proportioned in the stage 4):

$$\{f\}=[A]\{m_P\}$$

where [A]: coefficient matrix depending on the target mechanism and the load pattern

As for the load pattern, the profile based on the fundamental mode of elastic vibration may be acceptable for most cases of low-rise buildings.

6) The moments at non-plasticized portions {mn} are in equilibrium with the required strength {f} (=[A]{mP}) as well as the moment capacities {mP} at the portions plasticized in the target state:

[A]{mP}-[CP]{mP}=[Cn]{mn}

where [CP], [Cn]: sub-matrices of original connectivity matrix [C] corresponding to plasticized and non-plasticised member forces, respectively.

7) The minimum norm solution for {mn} is denoted by {mn*} and given by:

{mn*}= [Cn*] ([A]-[CP]){mP} (9)

8) The moment capacities {mnP} for non-plasticized portions are proportioned so that they exceed {mn*}. This proportioning process assures that the target mechanism becomes the true one. When uncertainty about moment capacities and load patterns are considered, an appropriate magnification factor may be applied to {mn*} before proportioning. To specify such a magnification factor, 'resistance versus resistance reliability problem[3]' should be solved with a specified reliability level.

References

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