### ADAPTIVELY SHIFTED INTEGRATION TECHNIQUE FOR NONLINEAR FINITE ELEMENT ANALYSIS OF LARGE-SCALE FRAMED STRUCTURES

by

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#### 1 Introduction

The relations between locations of numerical integration points in the linear and the cubic beam elements and those of the occurrence of plastic hinges were first found out by Toi [1], by considering the equivalence conditions between the strain energy approximations of the finite elements and the physical models (the rigid bodies-spring models) in which the locations of stress evaluations and plastic hinge formations are explicitly given. These relations were effectively used in the adaptively shifted integration technique (abbreviated to the ASI technique) for the plastic collapse analysis of framed structures [2]. In the present study, the ASI tecnique with the cubic element is applied to the geometrically nonlinear, elasto-plastic analysis as well as the geometrically linear, plastic collapse analysis in order to reduce a computational cost for large-scale framed structures.

# 2 Adaptively Shifted Integration Technique

The relation between the locations of numerical integration points  $(s_i)$  and those of occurrence of plastic hinges  $(r_i)$  in the cubic element  $(-1 \le r_i, s_i \le 1)$  is expressed by the following equation [1]:

 $r_i = \mp \frac{1}{3s_2} \quad (i = 1, 2; s_1 = -s_2)$  (1)

In the ASI technique,  $s_i$  are placed on the Gaussian integration points  $(s_i = \mp 1/\sqrt{3})$  in an elastically deformed element, and they are automatically shifted to the new positions according to the equation above immediately after the occurrence of a plastic hinge on either end of the element  $(r_i = \mp 1)$ . Assuming that the fully-plastic section has first

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occurred at the left end of the element, the incremental stiffness matrix and the initial stress matrix for the element at the following incremental steps are expressed as

$$[k] = \frac{L}{2} \left\{ ([B_0(-\frac{1}{3})]^t + [B_L(-\frac{1}{3})]^t)[D_p(-1)] \left( [B_0(-\frac{1}{3})] + [B_L(-\frac{1}{3})] \right) + ([B_0(\frac{1}{3})]^t + [B_L(\frac{1}{3})]^t)[D_e(1)] \left( [B_0(\frac{1}{3})] + [B_L(\frac{1}{3})] \right) \right\}$$
(2)

$$[k_G] = \frac{L}{2} \left\{ [G(-\frac{1}{3})]^t [S(-1)] [G(-\frac{1}{3})] + [G(\frac{1}{3})]^t [S(1)] [G(\frac{1}{3})] \right\}$$
(3)

in which the parenthesized figures indicate the locations of integration points in  $[B_0]$ ,  $[B_L]$ , [G] and those of the points at which stresses are evaluated in  $[D_e]$ ,  $[D_p]$ , [S], respectively. The matrix  $[D_e(1)]$  is replaced with  $[D_p(1)]$  as the fully-plastic section has also occurred at the right end of the element.

## 3 Numerical Examples

By using the ASI technique, sufficiently accurate solutions can be obtained by oneelement idealization per member in the geometrically linear, plastic collapse analysis as shown in Fig. 1. In geometrically nonlinear analysis, the members which are judged to buckle during the course of calculation are automatically resubdivided into four elements in order to maintain the computational accuracy especially in the post-buckling range. Fig. 2 is an example of the buckling collapse analysis in which the resubdivision into four elements was conducted at the level of the buckling stress for the member multiplied by the factor B.

The ASI technique has been applied to the elasto-plastic collapse analysis of a large-scale framed structure subjected to earthquake loading as shown in Fig. 3, which is now under construction in Tokyo bay area. The number of members and member joints are 1630 and 546, respectively. Table 1 gives an applied static load in the horizontal direction due to earthquake. Figures 4 and 5 show the results of the geometrically linear and nonlinear analyses, respectively. The maximum load factor obtained by the geometrically nonlinear analysis is 2.34, which is slightly smaller than that (2.46) given by the geometrically linear analysis because of the occurrence of member buckling. Both calculations started with one-element idealization per member. However, 60 members were subdivided into 4 elements during the course of the geometrically nonlinear analysis. The computing time on the EWS (Sun SPARCstation 10) was 188 minutes (86 incremental steps) for the geometrically linear analysis and 364 minutes for the geometrically nonlinear analysis.

# 4 Concluding Remarks

The ASI technique for the plastic collapse analysis of framed structures has been applied to the geometrically nonlinear, elasto-plastic analysis as well as the geometrically linear, plastic collapse analysis of a large-scale framed structure. The highest computational efficiency has been achieved in both analyses. The present technique can be easily implemented in the existing frame codes using linear Timoshenko or cubic beam elements. The theoretical basis of the ASI technique including various numerical examples is given in [1-5]. The authors appreciate Mr. Harada of KUBOTA corporation who has offered the structural data of the large-scale frame analyzed in the present report.

#### References

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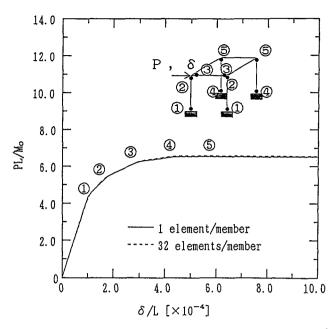


Fig.1 Plastic collapse analysis of a simple frame

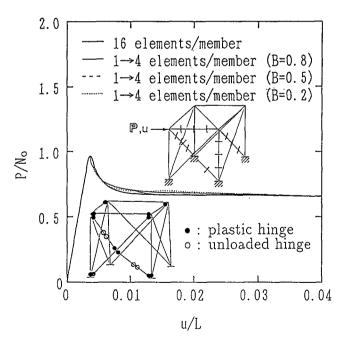


Fig.2 Elasto-plastic buckling analysis of a simple frame

Table 1: Horizontal load due to earthquake

Floor No.	Load (ton)
9th floor	0.0
8th floor	2061.7
7th floor	1668.8
6th floor	983.0
5th floor	411.0
4th floor	54.5
3rd floor	39.0
2nd floor	37.1
1st floor	40.7
Total	5295.8

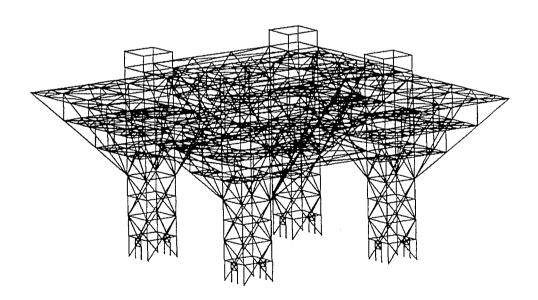
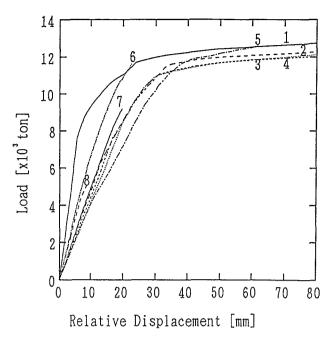
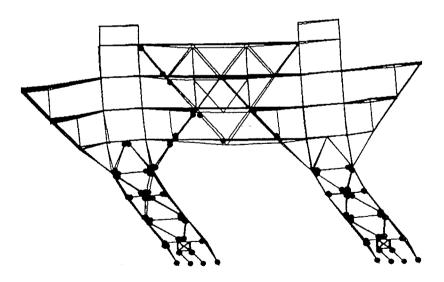


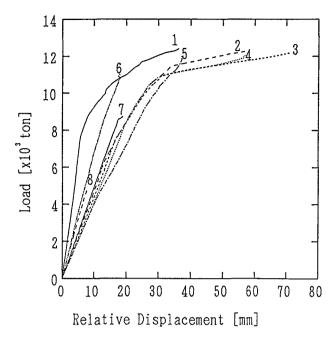
Fig.3 Large-scale frame



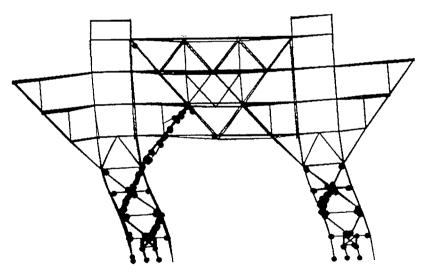
(a) shear force vs. interstory displacement



(b) deformed configuration (load factor = 2.46)
Fig.4 Plastic collapse analysis of a large-scale frame



(a) shear force vs. interstory displacement



(b) deformed configuration (load factor = 2.34)

Fig.5 Elasto-plastic buckling analysis of a large-scale frame