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OPTIMUM DESIGN BASED STRUCTURAL RELIABILITY THEORY
- Decision making of the optimum shear force coefficient of structures -

Ki KOH[1), Koichi TAKANASHI[2)

§ 1. Introduction

From a point of view of necessitating a lot of studies on account of rational establishment and certain ground of Importance Factor proposed in the first draft on seismic load in Archtechural Institute of Japan, the authors theoritically investigated on making a decision of load factor of structures with deteriorating strength during the life time of structures(1). But in this paper, we used a physical vague variable of general strength of structures. Hanai(2) and Kanda(3) were using the same variable as well.

In the elastic design, such a treatment gives us an advantage in making it possible to assess the reliability function with the difference between the load effect and the structural resistance. However, in the ultimate limit state design of structures subjected to large amount of plastic deformation, the reliability function assesed by the above mentioned difference may not be generally accepted. Therefore, it is very important to find out how to define the ultimate limit states of structures and what criterion and variables we should use in evaluating the reliability function of structures. Especially, the variable included in the reliability function must be closely related with the construction cost in the analysis of optimum design based on reliability theory.

According to the investigation that the construction cost for change of design from the design with the seismic coefficient of 0.2 to 0.3 showed an increase of 10%(4), we can say that the seismic coefficient

1) Research Associate, Institute of Industrial Science, Univ. of Tokyo
2) Professor , Institute of Industrial Science, Univ. of Tokyo

of structures is closely related with the construction cost. Provided we represent the seismic coefficient causing the plastic deformation of structures by the yield seismic coefficient, the ultimate limit state of structures may be prescribed by the following three parameters, i.e., yield seismic coefficient, plastic obsorption capacity of structures and energy input to structures. This concept has been very familiar to earthquake engineers.

In this study, the strong part of earthquake motion will be modelled by the stationary random process. The reliability function of structures is expressed by the function of yield seismic coefficient, in which the earthquake load effect is the energy input absorbed by the cumulative plastic deformation, and the structural resistance is the cumulative plastic absorption capacity of structures.

From the economical point of view of minimizing the sum of the expected value of loss caused by disasters and the construction cost, this paper describes the method for estimating the optimum yield seismic coefficient of structures. For the present time, the shear force coefficient prescribed by earthpuake resistant design method is expressed by the product of the following parameters, i.e., structural characteristic coefficient, usage factor, and seismic zoning coefficient and so on, however, it is shown that the method for obtaining the coefficient by such a treatment may be irrational in view of the analytical results based on reliability theory.

§2. Reliability criterion

The energy balance equation of hysteretic structures subjected to the stationary random process is given by

$$\tilde{\omega}_e(t)+\tilde{\omega}_h(t)+\tilde{\omega}_p(t)=\tilde{\omega}_t(t) \qquad (2\text{-}1)$$

where $\tilde{\omega}_e(t)$: the energy input absorbed by elastic strain

$\tilde{\omega}_h$ (t): the energy input absorbed by viscous damping

$\tilde{\omega}_p$ (t): the energy input absorbed by cumulative plastic strain

$\tilde{\omega}_t$ (t): the total energy input to system

$\tilde{\omega}_t$ (t) in Eq.(2-1) is a monotonically increasing function of time. Provided that the energy absorption capacity of structures prescribed to the structural failure in advance, denoted by $\tilde{\omega}_r$, is greater than $\tilde{\omega}_p$, structures can survive.

Therefore, the criterion for safety of structures is given by

$$\tilde{\omega}_r > \tilde{\omega}_p \qquad (2\text{-}2)$$

When $\tilde{\omega}_r$ and $\tilde{\omega}_p$ are random variables, the safety of structures is equal to the occourrence probability of the event{ $\tilde{\omega}_r > \tilde{\omega}_p$ }, in which $\tilde{\omega}_r$ and $\tilde{\omega}_p$ show the structural resistance and the load effect in reliability analysis. The random variable in the following is shown by big letters.

§ 3. Statistics of energy input absorbed by cumulative plastic strain

The energy input at the end of earthquake ground motion is dissipated by the elastic strain. If we neglect it, the total energy input to system can be expressed as follows(11).

$$\omega_t = 0.5M\tilde{V}_e^2 \qquad (3\text{-}1)$$

where M : the mass , $\tilde{V}e$: the reduced velocity spectrum $\tilde{V}e$ for the energy input absorbed by elastic strain and plastic strain

Akiyama(11) got the conclusion that the reduced velosity spectrum $\tilde{V}e$ is almost equal to elastic velocity spectrum and is approximately shown by a bilinear relation with the natural period of the system. The energy input $\tilde{\omega}_p$ absorbed by cumulative plastic strain within $\tilde{V}e$ is given by

$$\tilde{\omega}_p = 0.5M\tilde{V}_p^2 \qquad (3\text{-}2)$$

where $\tilde{V}p$: the reduced velocity spectrum for the energy input absorbed by plastic strain

$\tilde{V}p$ is dependent on the amount of the input energy absorbed by viscous damping within the total energy input. However, $\tilde{V}p$ will be evaluated on the concept of reducing $\tilde{V}e$ with the structural characteristic coefficient Ds which shows the level of plastic deformation capacity of the structures. If we use the relation of the energy constant between elastic response and elasto-plastic response as shown in Fig. 1. the energy input absorbed by elastic strain is shown by $\tilde{\omega}_e = 0.5M\,(Ds\tilde{V}e)^2$, $\tilde{V}p$ can be got by the energy balance as follows.

$$\tilde{V}_p = \sqrt{(1 - D_s^2)}\ \tilde{V}_e \qquad (3\text{-}3)$$

The reduced velocity spectrum $\tilde{V}e$ is approximated by a bilinear relation with some amendment in short period zone for the velocity spectrum of elastic system(11). Substituting Eq.(3-3) into Eq(3-2), and using the first period of system T_1 ($= 2\pi / \omega_1$), we obtain the following equation.

$$\tilde{\omega}_p = \sqrt{(1-D_s^2)\,\xi}\ \tilde{\omega}_t \qquad (3\text{-}4)$$

where $\xi = 1$ $\qquad T_1 > Tg$

$\xi = (T_1 / Tg)^2$ $\qquad T_1 < Tg$

Tg : the period that bound between the long period zone and the short period zone

On the other hand, the energy input to elastic system subjucted to the staionary random process with the frequency characteristic of ground proposed by Kanai and Tajimi are random variables. The statistics of it is approximately shown as follows(8).

$$E\,[Wt] = S_y(\omega_1)\,\pi\, SoTd\, M \qquad (3\text{-}5)$$

$$V\,[Wt] = S_y(\omega_1)\,2\pi\, SoTd\, M\sigma_{\dot{x}}^2$$

where $$S_y(\omega_1) = \frac{\omega_g^4 + 4h_g^2 \omega_g^2 \omega_1^2}{(\omega_g^2 - \omega_1^2)^2 + 4h_g^2 \omega_g^2 \omega_1^2}$$

$E[Wt]$, $V[Wt]$: the expectation and variance of Wt

S_o : the power spectrum of earthquake ground motion

$\sigma_{\dot{x}}^2$: the variance of stationary velocity response of elastic system

ω_g, h_g : the predominent frequency and damping coefficient of the ground , T_d : the duration time

The statistic for the energy input to MDOF of elastic system can be approximately expressed by Eq(3-5) as well(appendix(A)). In this case, the parameter $\sigma_{\dot{x}}^2$ included in Eq.(3-5) must be described for the first mode of system. According to the conclusion on the reduced velocity spectrum obtained in ref.(11), the statistics of the system with viscous damping 10% in Eq.(3-5) is approximately equal to the total energy input of elasto-plastic system in consideration of the frequency characteristic of ground. The energy input absorbed by cumulative plastic strain is almost dissipated by the first mode, and the dissipation by the higher mode is small(12). Hence, $\tilde{\omega}_p$ will be dissipated by the cumulative plastic strain which is caused by the repeated horizontal load that is equivalent to the first mode of system. If we regard $\tilde{\omega}_p$ and $\tilde{\omega}_t$ in Eq.(3-4) as the energy input of system subjected to the stationary random process, those two variables become random variables. The statistics of the energy input is obtained from Eq.(3-4) and Eq.(3-5) as follows.

$$E[\tilde{W}p] = (1-D_s^2)\,\xi\,S_y(\omega_1)\,\pi\,S_oT_d\,M \qquad (3\text{-}6)$$

$$V[\tilde{W}p] = (1-D_s^2)^2\,\xi^2 S_y(\omega_1)\,2\pi\,S_oT_d\,M\sigma_{\dot{x}}^2$$

The statistics of Wp normalized by $Q_y\mathcal{X}_y$ ($Wp=\tilde{W}p/Q_y\mathcal{X}_y$, $Q_y=MgC_y$, Q_y =yield strength , $\mathcal{X}_y$= yield deformation, C_y = yield seismic coefficient) will be regarded as the statistics on the condition that C_y and S_o take some value c_y and s_o, provided that C_y and S_o are ramdom variables.

In this paper, we assume that only S_o is a random variable. Then the mean and mean square value of W_p on the condition that S_o takes on some value is given by

$$E[W_p \mid S_o = s_o] = (1-D_s^2)\xi S_y(\omega_1) \frac{\pi s_o T_d \omega_1^2}{g^2 C_y^2} \quad (3\text{-}7)$$

$$E[W_p^2 \mid S_o = s_o] = (1-D_s^2)^2 \xi^2 S_y(\omega_1) \frac{\pi^2 s_o^2 T_d \omega_1^3}{g^4 C_y^4} \left(\frac{1}{h} + \omega_1 T_d S_y(\omega_1)\right)$$

where C_y: the yield seismic coefficient h : viscous damping

Finally, the statistics of W_p is got by using the probabilistic procedure.

$$E[W_p] = (1-D_s^2)\xi S_y(\omega_1) \frac{\pi E[S_o] T_d \omega_1^2}{g^2 C_y^2} \quad (3\text{-}8)$$

$$V_{w_p} = \sqrt{\frac{1}{\omega_1 T_d h S_y(\omega_1)} (V_s^2 + 1) + V_s^2}$$

The statistics of the energy input showm in Eq.(3-8) is represented as the function of yield seismic coefficient. Since the denominator of the first term in the rigth-hand of Eq.(3-8) includes the duration time T_d, this term is not so large value for viscous damping h =10%. So, the effect of $S_y(\omega_1)$ on Eq(3-8) is small, and still more, since the value of $S_y(\omega_1)$ is small than 2, we assume that it takes 1 on the average. Then Eq(3-8) is shown by

$$V_{w_p} = \sqrt{\frac{1}{\omega_1 T_d h} (V_s^2 + 1) + V_s^2} \quad (3\text{-}9)$$

Eq.(3-9) shows that the coefficient of variation V_{w_p} is close to the coefficient of variation V_s for the input level of earthquake excitation. The reason is that $\tilde{\omega}_p$ and $\tilde{\omega}_t$ shown in Eq.(3-4) are related to the variables which depend on only Ds and T_1 under the excitation of the staionary random process.

§4. Probability of structural safety after t years

If we assume that the occurrence probability of an earthquake event is modelled by Possion process, the probability of structural safety after t years will be shown by the following equation , even if random variables in the structural system are included.

$$R(t)=\exp(-\int_0^t \lambda(\tau)\,d\tau) \tag{4-1}$$

where R(t) : probability of structural safety after t years

$\lambda(\tau)d\tau$: probability in which Wp exeeds the cumulative plastic deformation capacity Wr at time $(\tau, \tau+d\tau)$ under the condition that structure survives during the $(0, \tau)$, $\lambda(\tau)$ is called the failure rate

In this study, we regard Wr as a random variable, but the yield seismic coefficient is regarded as an independent variable, not random variable. It is necessary to estimate the failure rate considering the non-failure effect concerning the cumulative plastic deformation(1), however, we neglect this effect because of rare occurrence of structural failure. Then, the failure rate is shown by

$$\lambda(\tau)=\lambda_o p_f \tag{4-2}$$

λ_o : the expected occurrence rate of an earthquake event with larger input level than some that of the earthquake causing plastic deformation

p_f : the probability of structural failure in case that an earthquake event occured once

The probability of structural failure p_f in Eq.(4-2) is given by

$$p_f = \int \overline{F}w_p(\omega_r)\,p_{w_r}(\omega_r)\,d\omega_r \qquad (4\text{-}3)$$

where $\overline{F}_{w_p}(\omega_p) = 1 - F_{w_p}(\omega_r)$ ($F_{w_p}(\omega_p)$: the probability disribution of W_p) $p_{w_r}(\omega_r)$: the probability density of Wr

Generally, as the probability density function of Wp is described by Gamma distribution(6), the probability of structural failure will be obtained by the integral of the distribution function. The integral makes more complex expression. So, in order to avoid the integral expression, we will focus on the tail at the right-side of the gamma distribution on the concept that the large value of Wp generally causes the failure of structures. Since the probability distribution for the tail is regarded as that of the largest value of Wp, it will approaches to the type I asymptotic distribution. The reason is shown in Appendix (B). As shown in Fig. B-1 , good consistence is got in the part of small probability of failure. Hence , in this study, we assume that the distribution function of Wp is described by the double exponential distribution as follows.

$$F_{w_p}(\omega_p) = \exp(-\exp(-\alpha_1 \frac{\omega_p}{\overline{w}_p} + \alpha_2)) \qquad (4\text{-}4)$$

where $\alpha_1 = 1.28/V_{w_p}$, $\alpha_2 = \alpha_1(1 - 0.45V_{w_p})$

$\overline{X}$: the expectation of X

Eq. (4-4) is expressed by the function of the yield seismic coeffcient. Considering only the small part of the probability of failure, assuming a normal distribution as the density function of Wr, and obtaining p_f from Eq. (4-3), the probability of structural safety of Eq. (4-1) is shown by

$$R(t) = \exp(-\lambda_o p_f t) \qquad (4\text{-}5)$$

where $p_f = \exp(-\alpha_1\bar{\omega}(1-0.5\alpha_1\bar{\omega}\, V_{w_r}^2) + \alpha_2)$

$$\bar{\omega} = \frac{\bar{W}_r}{\bar{W}_p} = \frac{\bar{W}_r g^2 C_y^2}{(1-D_s^2)\xi S_y(\omega_1)\pi E[S_o]T_d\omega_1^2}$$

V_{w_r} : the coefficient of variation of W_r

$\bar{\omega}$ in Eq.(4-5) stands for the central safety factor.

§ 5. Optimum yield seismic coefficient of hysteretic structures

In the structural reliability theory, it is generally recognized to use probability as the only measure of structural safety. There is , however, no clue to how to decide the proper probability of safety to be given for our structures at present(2). In this study , we assume that it is decided from an economical point of view of minimizing the sum of the expected value of loss caused by disasters and the constuction cost (1),(2). The total loss is shown by

$$E[D] = \int_0^{t_u} W_p(t)\,[-\frac{\delta R(t)}{\delta t}]\,dt + C_s(C_y)$$

$$W_p(t) = \int_t^{\infty} p\,e^{-\gamma\tau}\,d\tau \qquad (5\text{-}1)$$

E [D] : the expected value of total loss

$W_p[t]$: loss in case that structures destroyed after t years

C (Cy) :the construction cost

p : the expected annual profit from structures

γ : the annual rate of profit

t_u : the expected life time of the structures

Substituting Eq(4-5) into Eq.(5-1) and neglecting the infinitesimal terms gives

$$E[D] = \frac{W_p(0)\lambda_o}{\gamma}\, p_f + C_s(C_y) \qquad (5\text{-}2)$$

The construction cost is asummed to be described as follows(Appendix(C).

$$C_s(C_y) = C_1 + C_2 C_y^2 \tag{5-3}$$

where C_1, C_2 : constant

As Eq.(5-2) is shown by the function of C_y, the yield seismic coefficient is obtained by the differential of E [D] about C_y^2 as follows.

$$\frac{\delta E(D)}{\delta C_y^2} = \frac{W_p(0)\lambda_o \alpha_1}{\gamma}(\alpha_1 \overline{\omega}_{opt} V_{W_r}^2 - 1)\frac{\overline{\omega}_{opt}}{C_{y\,opt}^2}$$

$$\exp(-\alpha_1 \overline{\omega}_{opt}(1-0.5\alpha_1 \overline{\omega}_{opt} V_{W_r}^2) + \alpha_2) + C_2 = 0 \tag{5-4}$$

where $\dfrac{\overline{\omega}_{opt}}{C_{y\,opt}^2} = \dfrac{\overline{W}_r g^2}{(1-D_s^2)\xi S_y(\omega_1)E[S_o]T_d \omega_1^2}$

$C_{y\,opt}$: the optimum yield seismic coefficient

As a special case, when the coefficient of variation V_{W_r} is equal to zero, Eq.(5-4) becomes the following equation.

$$\frac{W_p(0)\lambda_o \alpha_1}{\gamma}\exp(-\alpha_1 \overline{\omega}_{opt} + \alpha_2)\frac{\overline{\omega}_{opt}}{C_{y\,opt}^2} = C_2 \tag{5-5}$$

The optimum yield seismic equation got from Eq.(5-5) is shown by

$$C_{y\,opt}^2 = \frac{W_p(0)\lambda_o \alpha_1}{\gamma C_2}\exp(-\alpha_1 \overline{\omega}_{opt} + \alpha_2) \tag{5-6}$$

Eq.(5-6) stands for the relation between the optimum central safety factor and the optimum yield seismic coefficient.

Substituting Eq(5-6) into Eq.(4-5), the optimum value of the probability of safety in case of V_{W_r} =0 is shown by

$$P_r[W_r > W_p \ (C_y = C_{y\,opt})] = \exp\left(-\frac{\gamma V_{W_p} C_{y\,opt}^2 C_2 t}{1.28 W_p(0)\overline{\omega}_{opt}}\right) \tag{5-7}$$

From Eq.(5-7), we may understand the optimum probability of safety is independent of λo.

Next, in order to get the relationship between the optimum yield seismic coefficient of standard structures and that of non-standard structures, we introduce the following five equations.

$$1-\tilde{D}_s^2 = \alpha_p(1-D_s^2) \qquad \frac{\tilde{W}(0)}{\gamma} = I\frac{W(0)}{\gamma} \tag{5-8}$$
$$\tilde{\lambda}_o = Z\lambda_o \qquad \tilde{S}_y(\omega_1) = R\, S_y(\omega_1)$$
$$\tilde{C}_{yopt}^2 = C^2 C_{yopt}^2$$

where α_p :the coefficient to describe the rate of plastic deformation of non-standard structures to that of standard structures

I :the coefficient to describe the rate of impotance factor of non-standard structures to that of standard structures

Z :the coefficient to describe the rate of seismic zoning coefficient on non-standard structures to that of standard structures

R :the coefficient to describe the rate of vibrational characteristic coefficient of non-standard structures to that of standard structures

C :the coefficient to describe the rate of optimum yield seismic coefficient of non-standard structures to that of standard structures

Substituting Eq.(5-8) into Eq.(5-6) gives

$$C = \sqrt{\alpha_p R\ (1+\beta\ (\ln I + \ln Z - \ln \alpha_p - \ln R))} \tag{5-9}$$

where $\beta = V_{W_p} / (1.28\ \overline{\omega}_{opt})$

V_{W_p} in Eq.(5-9) is shown by Eq.(3-9). The coefficient of variation of input level Vs is about 0.3 or 0.4(8). As β is dependent on the duration time, natural period, and damping factor, it shows around 0.2 in case of T_d =10sec, h =0.1, and $\overline{\omega}_{opt}$ >2. Thus, each product of β and $\ln\alpha$ or $\ln R$ is small in comparision with 1. Finally, neglecting this two terms in Eq.(5-9) gives

$$C = \sqrt{\alpha_p R\,(1 + \ln I^{\beta} + \ln Z^{\beta})} \qquad (5\text{-}10)$$

from which we may understand that the coefficient C is independent of I and Z in case that the V_{W_p} is equal to zero. In the case of $V_{W_p} = 0$, it means that there is no probabilistic variation in the earthquake excitation, and W_p caused by the deterministic input like El Centro earthquake becomes deterministic. Hence, according to Eq.(2-2), the structures will be completely assured for larger cumulative plastic absorption capacity than plastic deformation caused by the earthquake excitation. It may support the intuitive judgement that the above-mentioned two factors are independent of the optimum yield seismic co-efficient in case that the safety of structures is evaluated by the deterministic excitation. Therefore, the coefficient Z and I affecting the probability of structural failure and the seriousness of the failure must be considered only in case that the structural failure is a probabilistic event.

The coefficient C in Eq.(5-10) stands for the value that reduced the optimum yield seismic coefficient for the inertia force distribution with the first mode to the seismic coefficient. As it is natural that optimum yield shear coefficient is expected to be near the maximum shear force coefficient of linear system(14), we can replace C_{yopt} with the products of Ai and Co, in which Ai stands for Ai-distribution and Co stands for the standard value of shear force coefficient. Then the distribution of shear force coefficient based on reliability theory is shown by

$$Ci = \sqrt{\alpha_p R\,(1 + \ln I^{\beta} + \ln Z^{\beta})}\; Ai\, Co \qquad (5\text{-}11)$$

where Ci: the shear force coefficient of i-story

For the present time, though the shear force coefficient prescribed by earthquake resistance method is expressed by the products of structural characteristic coefficient, usage factor, and seismic zoning coefficient and so on. Eq.(5-11) shows the different type of distribution. The second term in the rigth side of Eq.(5-11) stands for the increase of shear force coefficient to assign to the seriousness of failure, and the third term stands for the increase of that to assign to the probability of failure. On the contrary, those two factors will be regarded as the decrease of that in relation with the way of how to evaluate the standard shear force coefficient. The optimum shear force coefficient in Eq.(5-11)shows the expression multipling the sum of those two factors by the structural characteristic coefficient and the vibrational characteristic coefficient. Though the bold assumption and approximation are introduced in evaluating the probability of structural failure and the construction cost, Eq.(5-11) shows that the coefficient obtained from Eq.(5-11) may be more rational than the one producting each factor under the optimum design based on reliability theory. In this case, undoubtedly, it is necessary to re-evaluate those coefficient. The conclusion that the system adding the factor affecting the seriousness of failure and the factor affecting the probability of structural failure may be better than the one obtained by producting those two factors is already reported in ref.(2).

Finally, we would like to show the application limit for the principle of minimizing the expected value of total loss. In order to get relation between $W(0)/\gamma$ and C_{yopt} from Eq.(5-2), the differential of the first term in Eq.(5-2) about C_y^2 must be negative, because that of second ter mis positive. In that case, we can get the optimum value. It may be stands for the concept in which the optimum design with the minimum cost will be executed in case that the yield seismic coefficient is as small as possible. However, such a concept is not non-realistic and should not be used for the actural design of structures. Hence we can obtain the upper bound of the coefficient of variation for the cumulative plastic absorption capacity of structures on the condition that the differential of the first term in Eq.(5-4) is negative.

$$V_W < 0.884\sqrt{\frac{V_{W_P}}{\overline{\omega}_{opt}}} \qquad (5\text{-}12)$$

§ 6. Conclusion

By modelling the strong part of an earthquake with the stationary random process, we evaluated the probability of structural safety after t years.in which the energy input absorbed by the cumulative plastic strain is regarded as the earthquake load effect, and the cumulative plastic absorption capacity is regared as the structural resistance, and still more, we described the new method to estimte the optimum yield seismic coefficient or the optimum shear force coefficient from an economical point of view of minimizing the sum of expected value of loss caused by disasters and the constuction cost.

Appendix (A)

The expectation of the energy input to system subjected to white noise excitation is shown as follows.

$$E[W_t] = \pi S_o T_d \sum_{j=1}^{m} M_j \qquad (A\text{-}1)$$

where M_j : the effective mass of j-mode

If we neglect the interactive terms between each mode, the variance of the energy input is shown by

$$V[W_t] = 2\pi S_o T_d \sum_{j=1}^{m} M_j \sigma^2_{\dot{q}_j} \qquad (A\text{-}2)$$

where $\sigma^2_{\dot{q}_j}$: the variance of velocity response of j-mode

According to the conclusion that the total energy input to system mainly depends on the total mass and the first natural period of system, and considering the frequency characteristics of the ground, denoted by $S_y(\omega_1)$, the statistics of Wt is shown as follows(8).

$$E[Wt] = Sy(\omega_1)\pi SoTdM \quad (A\text{-}3)$$

$$V[Wt] = Sy(\omega_1)2\pi SoTdM\sigma^2_{\dot{q}_1}$$

Appendix (B)

We will focus on the tail at the right-side of the gamma distribution on the concept that the large value of Wp generally causes the failure of structures. Since the probability distribution fot the tail is regarded as that of the largest value of Wp, it is obtained by Cramer's method as follows(13).

First of all, we introduce the following variable.

$$\xi_n = n(1 - F_{w_n}(w_n)) \quad (B\text{-}1)$$

where w_n : the largest value of a random variable w with the gamma distribution

$$F_{w_n}(w_n) = \frac{\tilde{\Gamma}(\nu, \eta w_n)}{\Gamma(\nu)} \quad (B\text{-}2)$$

$\Gamma(\nu)$: Gamma distribution, $\tilde{\Gamma}(\nu, \eta w_n) = \int_0^{\eta w_n} u^{\nu-1} e^{-u} du$

$\eta = \nu / E[w]$, $\nu = 1/V_w^2$

$E[w]$, V_w :the expectation and the coefficient of variation of W

According to formulas of mathematics, $\tilde{\Gamma}(\nu, \eta w_n)$ in Eq.(B-2) is transformed as follows.

$$\tilde{\Gamma}(\nu, \eta w_n) = e^{-\eta w_n} \sum_{i=0}^{\infty} \frac{(\eta w_n)^{\nu+i}}{\nu(\nu+1)\cdots(\nu+i)} \tag{B-3}$$

The generality is not lost, if we assume that ν is a positive integer. Then, Eq.(B-3) is turned into

$$\tilde{\Gamma}(\nu, \eta w_n) = e^{-\eta w_n} \Gamma(\nu)[-1-\eta w_n-\cdots-\frac{(\eta w_n)^{\nu-1}}{(\nu-1)!} + \sum_{i=0}^{\infty} \frac{(\eta w_n)^{i}}{i!}] \tag{B-4}$$

where $\Gamma(\nu) = (\nu-1)!$

and still more, the coefficient k ($k < 1$) that satisfy the following equation existes.

$$e^{k\eta w_n} = 1 + \eta w_n + \cdots + \frac{(\eta w_n)^{\nu-1}}{(\nu-1)!} \tag{B-5}$$

Then, Eq.(B-4) becomes

$$\tilde{\Gamma}(\nu, \eta w_n) = \Gamma(\nu)[1 - e^{-(1-k)\eta w_n}] \tag{B-6}$$

Substituting Eq.(B-2) and Eq.(B-6) into Eq.(B-1) gives

$$\xi_n = e^{-(1-k)\eta w_n + \alpha_n} \tag{B-7}$$

where $\log n = \alpha_n$

Finally, according to Cramer's method, the probability distribution of Wn is described by the type I asymptotic distribution.

$$F_{W_n}(\omega) = \exp(-e^{-(1-k)\eta\omega + \alpha_n}) \tag{B-8}$$

The two parameters k, α_n in Eq.(B-8) are approximately obtained by replacing the mean and standard variation of Wn with E[w] and σ_W as follows(13).

$$k = 1 - 1.28V_W \quad , \quad \alpha_n = \frac{1.28}{V_W}(1 - 0.45V_W) \tag{B-9}$$

The numerical result of Eq.(B-2) and Eq.(B-8) is shown in Fig.(B-1) The broken line is Eq.(B-2). The solid line is Eq.(B-8). The difference between those two lines is very small.

Appendix (C)

As shown in 1, there is a report that the construction cost for the change of design from the working stress design with the seismic coefficient of 0.2 to 0.3 shows an increase of 10%(4). The following equation will be obtained from the above-mentioned conclusion, though it may be more or less rough.

$$\frac{\Delta C_s}{\Delta C_y} = C_s = \frac{\delta C_s}{\delta C_y} \qquad \text{(C-1)}$$

From Eq.(C-1), the following equation is obtained.

$$C_s\ (C_y) = C_o \exp\ (C_y) \qquad \text{(C-2)}$$

where C_o : constant

Eq.(C-2) is shown by the real line in Fig.(C-1). We assume that it is possible to approximate Eq.(C-2) by the linear function of C_y^2 ($C_s = C_1 + C_2 C_y^2$). The coefficient of C_1 and C_2 is decided by the least mean square method. The result is shown by the broken line.

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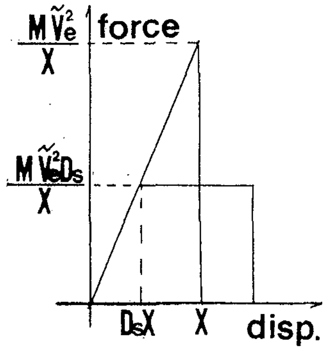

Fig. 1 The relation of energy constant between elastic response end elasto-plastic response

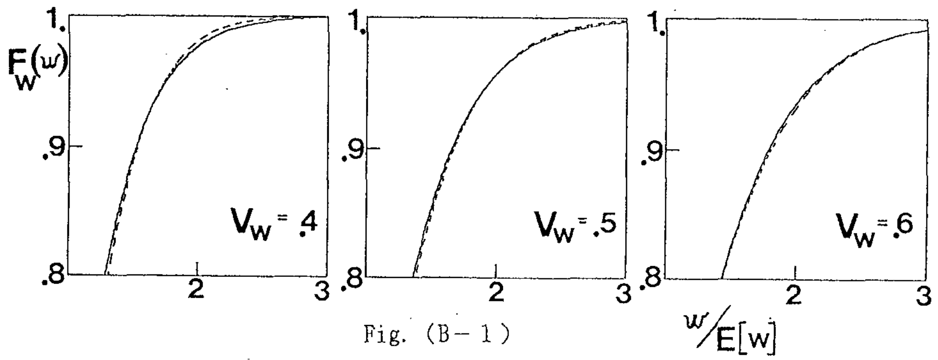

Fig. (B－1)

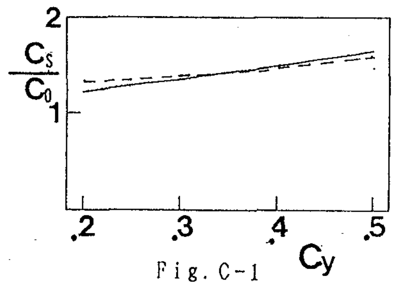

Fig. C－1