An Analytical Approach to Energy Response of Hysteretic Structures Consisted of Uncertainty Materials

by

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§1. Introduction

Since the Second-Moment Method is establisched by Cornell,Lind and Ang, lots of probabilistic investigations on the effects of uncertainty concerning the input ground motion have been studied(1),(2),(3),(4). In recent years, studies have been carried out not only onthe effects of uncertainty concerning the input ground motion, but alsoon the effects of variability in material property(5),(6),(7),(8),(9).

Sues et al(5) introduced probabilistic concept into the structural response by quantification of variability in loads and resistances. Additionally, the authors examined potential inaccuracies in assumptions concerning hazard analysis.This led to lifetime damage probability as an assessment of structural safety. O'Connor and Ellingwood(6) investigated the statistics of the simple nonlinear systems subjected to seismic excitation using numerical analysis. In their study, account was taken of variations in earthquake ground motion as well as uncertainties in structural properties such as mass, stiffness, damping and yield displacement of the systems. It was concluded that the variability in structural properties had a decisive influence on the inelastic response.

Kuwamura(7) investigated the effect of variations of yield stress on the ductility of steel frames. This study led to the conclusion that steel can exhibit high ductility ratio, provided the ratio of yield stress to tensile strength is low. The randomness in structural members' yield strength on the structural systems' ductility was examined by kuwamura and kato(8). The structural system considered was a six-

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story three span plane frame, designed based on the weakbeam-strong column concept. The authors showed, utilizing standard Monte Carlo simulation, that the randomness in members' yield strength has a significant influence on the failure mechanism and consequently on the ductility of the system, both for static and dynamic behaviour. In particular, a high coefficient of variation of the members' yield strength causes a higher risk of local failure. They concluded that the earthquake motion randomness has much less effect on preventing the occurrence of the dynamic overall failure mechanism than on the randomness in members' yield strength when the coefficient of the yield strength is greater than 2.5% to 5.0%. Alexopoulous, Elnashai and Chryssanthopoulos(9) investigated the effect of random yield strength on seismic design parameters of steel frames. A simple portal frame was considered and designed according to code characteristic values and verified by nonlinear transient dynamic analysis. The influence of the yield stress variability on several response parameter, including the degree of correlation between beam and column material properties, was assessed through a Monte Carlo simulation study. The main conclusion from the investigation was that random variability in yield strength and its degree of spatial correlation with a particular structure have a significant effect on a number of response parameters used in seismic design of steel frame, such as load carrying capacity, energy absorption, and even for variation with code-defined limits.

Next, discussions are briefly done on the methodology for the structural dynamic analysis with uncertainty factors.

Generally, uncertainty analysis in the dynamic response of structures are devided into the following two kinds; one is the case in which structural properties(mass, stiffness and damping) is deterministic, and input is nondeterministic, and in which input ground motion is usually modeled as white noise or nonwhite noise, and the other is the case in which structural properties are random variables and input motion is deterministic. There are mainly four methods for analyzing the former case, namely (1) analytical method by differential equations (2) power balance method (3) Equivalent linearization method and (4) Monte Carlo simulation.

On the other hand, there are two methods for analyzing the latter problem. One is an expansion method by taylor series expansion about the mean values, but it is effective only in case that the response value is obtained in completely closed form with random variables. Anather is perturbation method, which it is very familiar method as well as expansion method.However,it is almost solely limited to linear problem except for some nonlinear problems.

Generally, the variability of the resistance in structures depends on the variability of material strength and section shape in structural members. In this paper, only influence of the material strength on the dynamic energy response is studied.

The objectives of this investigation are as follows

- To evaluate the statistics of energy response of structures consisted of uncertainty materials subjected to earthquake excitation
- To investigate the effect of random variability in material property on the probability of safety for structures

§ 2. Probabilistic Modelling for Hysteretic Restoring Force of Structures Consisted of Uncertainty Materials

First of all, we should describe modelling stress-strain relationship of mild steel. Under the monotonic loading, an experimental results show that the uniaxial behavior, in its virgin state, isessentially linear up to the yield point, and the yield plateau is followed by strain hardening. To model exactly stress-strain relationship of steel member with such a material characteristics is not only hard, but also unpractical treatment. The bilinear model shown in Fig.1 allows the strain hardening and it is one of the acceptable models in the sense of engineering, though it is not exactly consistent with the experimentally obtained material behavior. Here, we have adopted the bilinear model.

The bilinear model is described by two material parameters, ie yield stress and strain hardening, which are random variables. Yield stress variability has been extensively studied in the UK(11) and Japan(12). By theses studies, two conclusions that a normal distribution can be used for describing a random yield stress variability(12) and a lognormal distribution too is avaiable(11), have been reached. In this study, we have chosen the normal distribution because of its simplicity and used a mean yield stress of 2.8t/cm^2 and a coefficient of variation of 0.069(11).

On the other hand, random variability of strain hardening has so rarely been studied, that there is no information on the distribution of strain hardening. Hence, in this initial study, an uniform distribution is used to account for the random variation of strain hardening.

The probabilistic distribution of material models used are shown below.

$$\sigma_y = N(2.8t/cm^2, 0.193t/cm^2) \quad (2\text{-}1)$$

$$\mu = U(0.007E, 0.012E)$$

where σ_y:yield stress μ :strain hardening

N(X, Y) :normal distribution(X:expectation, Y:standard deviation)

U(X, Y) :uniform distribution(X:upper, Y:lower)

E : modulus of elasticity

The full plastic bending moment M_p is calculated by

$$M_p = Z_p \sigma_y \quad (2\text{-}2)$$

where Z_p : plastic section modulus

and from Fig.3, the following equation is obtained.

$$Q_y = \frac{2}{H} Z_p \sigma_y \quad (2\text{-}3)$$

where Q_y : yield shear force

The shear force in Fig.3 stands for the restoring force of the structure. As a probabilistic hysteretic model of stress-strain relationship is described by bilinear model, and the model of hysteretic restoring force is expressed by the same bilinear model with two random variables, ie yield force and stiffness after yield as shown in Fig. 2.

The statistics of random variable in the probabilistic hysteretic restoring force model is as follows.

$$
\begin{aligned}
&E[Q_y] = \frac{2}{H} Z_p E[\sigma_y] \\
&C.O.V.[Q_y] = C.O.V.[\sigma_y] \\
&E[\alpha] = E[\mu] \qquad\qquad (2\text{-}4) \\
&C.O.V.[\alpha] = C.O.V.[\mu]
\end{aligned}
$$

where $E[X]$: expectation of X

$C.O.V.[X]$: coefficient of variation of X

Eq. (2-4) are approximate evaluation formulas to compute the statistics of parameters including in the restoring force model from the statistics of yield stress and strain hardening of uncertainty steel materials.

§ 3. Frequency Response Analysis of Inelastic MDF System Subjeted to Earthquake Excitation

It is a key point to define how to describe the hysteretic restoring force of nonlinear term in the dynamic equation, in finding the analytical solution of elasto-plastic response analysis for the structures with uncertainty materials. Suzuki and Minai(13) carried out studies to get an analytical solution for structures under the nondeterministic excitation, expressing the restoring force by an incremental force form written in differential equation.

An analytical approach tried here is based on the approximate method expressed the hysteretic restoring force by a first term of Fourie series expansion. The dynamic equation in time domain for jth-story in the nth-story frame can be written in the following form.

$$\sum_{k=j}^{n} m_k \sum_{\ell=1}^{k} \ddot{X}_\ell + D_j \dot{X}_j + F_j(X_j, t) = -\sum_{k=j}^{n} m_k \ddot{y}(t) \qquad (3\text{-}1)$$

where m_j : mass of jth-story

K_j : stiffness of jth-story

D_j : $2hK_j/\omega_1$

h : viscous damping , ω_1: 1st natural frequency

$F_j(X_j, t)$: hysteretic restoring force with deterministic value $\tilde{Q}_j$, $\tilde{\alpha}_j$

X_j: relative displacement, $\ddot{y}(t)$: deterministic input

First of all, we focus on getting the resolution of Eq.(3-1), in case of harmonic exciation input $\ddot{Y}(\omega)\exp(i\omega t)$ with ω − component included in earthquake exciation. If we adopt the above mentioned approximation method by Fourier series expansion and assume $X(\omega)\exp(i\omega t)$ as the relative displacement of jth-story , the bilinear restoring force model is ex-pressed by ellipse hysteretic characteristic as shown in Eq.(3-2). (Complex coordinate system is adopted in order to simplify the computing by FFT)

$$F_j(X_j, t) = X(\omega)(c_j - i s_j)\exp(i\omega t) \qquad (3\text{-}2)$$

where $i^2 = -1$

$$c_j = \frac{K_j}{\pi X_j(\omega)} \int_0^{2\pi} F_j(X_j, \omega t) \cos\omega t \, d(\omega t)$$

$$s_j = \frac{K_j}{\pi X_j(\omega)} \int_0^{2\pi} F_j(X_j, \omega t) \sin\omega t \, d(\omega t)$$

c_j, s_j in case of bilinear model

$$c_j(\tilde{u}_j) = \frac{K_j}{\pi}\left(\theta_j - \frac{1}{2}\sin 2\theta_j\right)(1-\tilde{\alpha}_j) + K_j\tilde{\alpha}_j \quad \cdots\cdots \quad \tilde{u}_j > 1$$

$$c_j(\tilde{u}_j) = K_j \quad \cdots\cdots\cdots\cdots \quad \tilde{u}_j \leq 1$$

$$s_j(\tilde{u}_j) = -\frac{K_j}{\pi}(1-\tilde{\alpha}_j)\sin^2\theta_j \quad \cdots\cdots \quad \tilde{u}_j > 1$$

$$S_j(\tilde{u}_j) = 0 \quad \cdots\cdots\cdots\cdots \quad \tilde{u}_j \leq 1$$

$$\theta_j = \cos^{-1}\left(1 - \frac{2}{\tilde{u}_j}\right) \quad \cdots\cdots \quad \tilde{u}_j > 1$$

$\theta_j = \pi$ $\qquad\cdots\cdots\cdots\cdots\ \tilde{u}_j \leqq 1$

$$\tilde{u}_j = \frac{K_j X_j}{\tilde{Q}_{yj}}$$

$\tilde{Q}_{yj}$, $\tilde{\alpha}_j$: some fixed value of random variable Q_{yj}, α_j

By substituting $X_j(\omega)\exp(i\omega t)$ and complex stiffness Eq.(3-2) into Eq.(3-1), Eq.(3-1) will be transformed to linear equation in frequency domain. As a result, Eq.(3-1)is rewitten in matrix form as follows.

$$[\mathbb{K}(\omega, \tilde{Q}_{yj}, \tilde{\alpha}_j, c_j(\tilde{u}_j), s_j(\tilde{u}_j), D_j)]\ \{X\} = \{Y\} \qquad (3\text{-}3)$$

where $\{X\}$: $\{X_1(\omega), X_2(\omega), \cdots, X_n(\omega)\}^T$

$\{Y\}$: $-(\sum_{k=1}^{n} m_k, \cdots, \sum_{k=j}^{n} m_k, \cdots, m_n)^T \ddot{Y}(\omega)$

$\ddot{Y}(\omega)$: Fourier Transform of $\ddot{y}(t)$

$\mathbb{K}(\omega)$: stiffness matrix of ω-component in frequency domain

EQ.(3-3) is linear equation with complex function. In case of harmonic excitation input, we are able to get the relative displacement X_j as the resolution of Eq.(3-3). However, in case of earthquake exciation input, we are not able to get the relative displacement in a unique meaning due to the various frequency component included in an earthquake exciation. Therefore, in this paper, we assume that $\tilde{u}_j$ is regarded as parameters in order to evaluate the equivalent stiffness and the equivalent damping for hysteretic structures. If $\tilde{u}_j$ may be evaluated in some way, the coefficient in Eq.(3-3) becomes known variables, hence, we can resolve Eq.(3-3) for the earthquake excitation. Namely, the frequency analysis for hysteretic structures is carried out by superposing each frequency response of the structure forced by harmonic excitation with a frequency component included in an earthquake excitation. The method to define $\tilde{u}_j$ is shown in the next section.

§ 4. An Expression Equation of Tatal Energy Input for Structures

First of all, we will show the method to express the total energy input for structures in frequency domain.

An earthquake total energy input of W(t) jth-story in n-story framed structures in time domain is given by

$$W_j(t) = -\sum_{i=j}^{n} m_i \int_0^t \ddot{y}(\tau)\dot{X}_j(\tau)\,d\tau \tag{4-1}$$

where t: duration time

Since the velosity response obtained from Eq.(3-3) is described in frequency domain, it is convenient to convert Eq.(4-1) into frequency domain. Therefore, we replace fixed values $\tilde{Q}_{yj}$, $\tilde{\alpha}_j$ in Eq.(3-2) by random variables Q_{yj}, α_j and convert Eq.(4-1) into frequency domain, according to a general form of Parseval's formula(14),(16). Then Eq.(4-1) is rewritten as

$$W_j(t) = -\frac{1}{2\pi}\sum_{i=j}^{n} m_i \,\mathrm{Real}\left\{\int_{-\infty}^{\infty} \ddot{Y}(\omega)\dot{X}_j^*(\omega, Q_{yi}, \alpha_i, \cdots, \tilde{u}_j)\,d\omega\right\} \tag{4-2}$$

where * : conjugate complex

As we have described an analytical closed form of total energy input with random variables as shown in Eq.(4-2), the statistics of total energy input is got by using the probabilistic procedures.

Next, we will show the method to get the unknown values $\tilde{u}_j$. We assume that the variability of energy response of the structures with random hysteretic restoring force due to uncertainty material can be measured by the deviation from the energy response obtained under the condition that the structures with restoring force represented by mean values of random variables subjected to deterministic earthquake excitation. $\tilde{u}_j$ means the displacement amplitude in the case of harmonic excitation input with one frequency component, but $\tilde{u}_j$ does not have a clear physical meaning in the case of earthquake excitation input with lots of frequency components. So, in this study, we assume that $\tilde{u}_j$ is so decided that Eq. (4-3) has specifided values. According to the first order approximation method, it is natural that $\tilde{u}_j$ is decided by the condition that energy respone of structures with restoring force represented by mean values of random variables as shown in Eq. (4-3) has specified values.

$$\overline{W}_j(t) = -\frac{1}{2\pi}\sum_{i=j}^{n} m_i \ \mathrm{Real}\left\{\int_{-\infty}^{\infty} \ddot{Y}(\omega)\dot{X}_j^*(\omega, \overline{Q}_{yi}, \overline{\alpha}_i, \cdots, \tilde{u}_j)\, d\omega\right\} \qquad (4\text{-}3)$$

where $\overline{X}$:mean value of X

If the numerical response analysis for structures with restoring force represented by mean values of random variables is acceptable only once, we can regard the total energy input $\overline{W}_{js}$ obtained from the numerical analysis as specified values. Consequently, we can get $\tilde{u}_j$ from the condition that $\overline{W}_j$ in Eq. (4-3) is equal to $\overline{W}_{js}$ .The iterative procedure is needed in computing it. Here, the following equation is used.

$$|\overline{W}_{js} - \overline{W}_j| \leqq 0.005\ \overline{Q}_{yj}^2 K_j \qquad (4\text{-}4)$$

Subsituting $\tilde{u}_j^0$ obtained from computing Eq. (4-4) into Eq. (3-2), we get

$$X_j = \tilde{u}_j^0 \ \frac{\overline{Q}_{yj}^2}{K_j} \qquad (4\text{-}5)$$

where $\tilde{u}_j^0$: known value of $\tilde{u}_j$

then, θ_j in Eq. (3-2) is updated as follows

$$\theta_j = \cos^{-1}\left(1 - \frac{2}{\tilde{u}_j^0 \cdot Q_{yj}/\overline{Q}_{yj}}\right) \qquad (4\text{-}6)$$

As Eq(3-3) is defined as the function with the closed form including only random variables, the statistics of total energy input is obtained by using probabilistic procedures. In case of $\tilde{u}^0{}_j \cdot Q_{yj}/\overline{Q}_{yj} \leqq 1$, the dynamic response of structures becomes elastic.

§ 5. The Statistics of Energy Response of Structures

The energy input in Eq.(4-1) is the total energy input of structures under earthquake loading. Generally, the total energy input is divided into three terms as

$$\widetilde{W}_j = \widetilde{W}_{ej} + \widetilde{W}_{dj} + \widetilde{W}_{pj} \qquad (5\text{-}1)$$

where $\widetilde{W}_{ej}(t)$: the sum of the nondimensional energy input absorbed by elastic vibration in jth-story

$\widetilde{W}_{dj}(t)$: the sum of the nondimensional energy input absorbed by viscous damping in jth-story

$\widetilde{W}_{pj}(t)$: the sum of the nondimensional energy input absorbed by cumulative plastic deformation in jth-story

All terms in Eq.(5-1) are random variables. The statistics of $\widetilde{W}_j$ in Eq.(5-1) is as follows

$$E[\widetilde{W}_j] = -\frac{K_j}{2\pi}\sum_{i=j}^{n} m_j \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \mathrm{Real}\left\{\int_{-\infty}^{\infty} \ddot{Y}(\omega)\dot{X}_j^*(\omega, Q_{y1}, \alpha_1, \cdots, \tilde{u}_j^0)\, d\omega\right\}$$

$$\cdot\frac{1}{Q_{yj}^2}\cdot p(Q_{y1}, \alpha_1 \cdots)\, dQ_{y1}\, d\alpha_1 \cdots \qquad (5\text{-}2)$$

$$E[\widetilde{W}_j^2] = \frac{K_j^2}{4\pi^2}\left\{\sum_{i=j}^{n} m_i\right\}^2 \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \mathrm{Real}\left\{\int_{-\infty}^{\infty} \ddot{Y}(\omega)\dot{X}_j^*(\omega, Q_{y1}, \alpha_1, \cdots, \tilde{u}_j^0)\, d\omega\right\}^2$$

$$\cdot\frac{1}{Q_{yj}^4}\cdot p(Q_{y1}, \alpha_1 \cdots)\, dQ_{y1}\, d\alpha_1 \cdots$$

$$V[\widetilde{W}_j] = E[\widetilde{W}_j^2] - E^2[\widetilde{W}_j]$$

where $p(Q_{y1}, \alpha_1 \cdots)$: joint probability density function of $Q_{y1}, \alpha_1 \cdots$

$E[X], E[X^2], V[X]$: expectation, mean square and variance of X

We are able to get the exact statistics of $\widetilde{W}_j$. However, to get exactly the statistics of $\widetilde{W}_{pj}$ is extremely difficult. So, we will propose the approximate method to speculate the statistics of $\widetilde{W}_{pj}$ directly from Eq.(5

-1) by using the numerical results for structures with restoring force represented by mean values of random variables. The expectation of Eq. (5-1) is

$$E[\tilde{W}_j] = E[\tilde{W}_{ej}] + E[\tilde{W}_{dj}] + E[\tilde{W}_{pj}] \qquad (5\text{-}3)$$

The mean square value of the difference between Eq.(5-1) and Eq.(5-3) gives approximately

$$V[\tilde{W}_j] \simeq V[\tilde{W}_{ej}] + V[\tilde{W}_{dj}] + V[\tilde{W}_{pj}] \qquad (5\text{-}4)$$

Eq.(5-4) includes the assumption that each correlation coefficient between random variables $\tilde{W}_j - E[\tilde{W}_{ej}]$, $\tilde{W}_{dj} - E[\tilde{W}_{dj}]$ and $\tilde{W}_{pj} - E[\tilde{W}_{pj}]$ is small. Since the left side of Eq.(5-3) and Eq.(5-4) are known, it is possible to speculate approximately the statistics of the right sides of Eq.(5-3)and Eq.(5-4) as follows.

$$E[\tilde{W}_{pj}] = r_j E[\tilde{W}_j] \qquad (5\text{-}5)$$

where $r_j = \tilde{W}_{pjs} / \tilde{W}_{js}$, $\tilde{W}_{pjs}$, $\tilde{W}_{js}$:the nondimensional total energy input and nondimensional energy input absorbed by cumulative plastic deformation for the structures with restoring force represented by mean values of random variables

The ratio of $V[\tilde{W}_{pj}]$ to $V[\tilde{W}_j]$ is unknown either, and we can not get it from numerical analysis. If it is assumed that the above-mensioned ratio is available for Eq.(5-4) as well, $V[\tilde{W}_{pj}]$ is given as the following.

$$V[\tilde{W}_{pj}] = r_j V[\tilde{W}_j] \qquad (5\text{-}6)$$

The reasonableness of such an assumption will be investigated by comparision between analytical results and Monte Carlo Simulation.

§ 6. First Order Approximation of Energy Response

Eq. (5-2) in the previous section is the expression to evaluate exactly the energy input $\tilde{W}_j$. It is very hard to compute the value in case of multi-degree of freedom system because of the execution of multi-integral computation, thougth it is not impossible. Therefore, a simple way to evaluate it is required.

The energy input defined by Eq. (5-2) includes the random variables Q_{yj}, α_j ($j=1,\cdots,n$). As we can regard Eq. (5-2) as a function with random variables, the approximate method by Taylor expansion is available.

The first order approximation of Eq. (5-2) gives

$$E[W_j] \simeq -\frac{1}{2\pi}\sum_{i=j}^{n} m_i \Gamma_j$$

$$V[W_j] \simeq \frac{1}{4\pi^2}\left\{\sum_{i=j}^{n} m_i\right\}^2 \sum_{\iota=1}^{2n}\sum_{k=1}^{2n} \frac{\partial \Gamma_j}{\partial Z_\iota}\frac{\partial \Gamma_j}{\partial Z_k} \mathrm{COV}[Z_\iota, Z_k] \qquad (6\text{-}1)$$

where $\Gamma_j = \mathrm{Real}\left\{\int_{-\infty}^{\infty} \ddot{Y}(\omega)\dot{X}_j^*(\omega,\bar{Z}_1,\bar{Z}_2,\cdots,\tilde{u}_j^o)\,d\omega\right\}$

Z_k : random variable ($Z_1=Q_{y1}$, $Z_2=\alpha_1$, $\cdots$)

$\mathrm{COV}(Z_\iota, Z_k)$: Covariance function of Z_K

To evaluate Eq. (6-1) and Eq. (6-2), the differential of the velosity component $\dot{X}_j^*$ about Z_K is needed. Differentiating Eq. (3-3) about Z_K directly and subsituting it into Eq. (3-3) gives

$$\frac{\partial \{\dot{X}_j^*\}}{\partial Z_k} = \mathrm{conjg}\left\{-i\,\omega[K]^{-1}\frac{\partial [K]}{\partial Z_k}[K]^{-1}\{Y\}\right\} \qquad (6\text{-}2)$$

By using Eq. (6-1) and Eq. (6-2), we can get the first order approximation for the statistics of total energy input. The nondimensional total energy input $\tilde{W}_j$ obtained by first order approximation is as follows.

$$E[\tilde{W}_j] = \frac{E[W_j]\,K_j}{E^2[Q_{yj}]} \qquad (6\text{-}3)$$

$$C.O.V.[\widetilde{W}_J] = \sqrt{C.O.V.[W_J] + 4 \cdot C.O.V.[Q_{yJ}]}$$

The statistics of $\widetilde{W}_{PJ}$ is

$$E[\widetilde{W}_{PJ}] = r_J E[\widetilde{W}_J]$$

$$C.O.V.[\widetilde{W}_{PJ}] = \frac{1}{\sqrt{r_J}} \cdot C.O.V.[\widetilde{W}_J] \qquad (6-4)$$

§ 7. Comparion between Analytical Results and Monte Calro Simulation

So far we have described the way to evaluate analytically the energy response of the structures with the uncertainty hysteretic restoring force subjected to earthquake exciation. We proposed two analytical methods, one is the exact method as shown in Eq.(5-2), and the other is first order approximation method as shown in Eq.(6-1).

In this section, we investigate the comparision between analytical results and simulation results. The following two model have been chosen to compare the results.

(1) Model A (hysteretic restoring force model : Fig.5)

$E[Q_{y1}]=36.6t$, $C.O.V.[Q_{y1}]=0.069$

K(stiffness)=20.5t/cm ,h(damping)=2%

(2) Model B (hysteretic restoring force model : Fig.6)

$E[\alpha_1]=0.0095$, $C.O.V.[\alpha_1]=0.149$, $\rho(Q_{y1}, \alpha_1)=0$ (correlation coefficient)

other values are the same as those in Model A

T(Period): 0.4sec, 0.5sec

Earthquake motion:El Centro Earthquake(max. $\ddot{y}(t)$=346gal, duration=20sec)

The results are shown in Tables 1 and 2. Method(1) stands for the results by the exact method, and Method(2) stands for the results by the first order approximation. Simulations show the results obtained by 100 trials. The results by method(1) is very close to the simulation results and good consistence is obtained. However, from the point of view of practical application, this method will be limited to SDOF system on account of increase of computing time and complicated computing techniques due to its being multi-integral. The results by method(2) show generally small values about 3% for mean and about 20% for coefficient of variation. Although coefficient of variation by Method(2) is pretty small, this method would be acceptable in the sense of engineering, from the point of view that computing is very easy. Because of the numerical analysis of energy response of structures with mean valuesonce, good consistence for mean value of energy response is got.

From the above results, it is concluded that the total input energy for structures under earthquake motion will be approximately obtained by superposing each energy input for linear system with restoring force Eq. (3-2) forced by harmonic excitation with a frequency component included in an earthquake ground motion.

Next, we will investigate the reasonableness of assumption used in Eq. (5-6). We introduce the following two parameters.

$$r=\frac{\sum_{j=1}^{m}\tilde{W}_{p1sj}/m}{\sum_{j=1}^{m}\tilde{W}_{1sj}/m}\quad,\quad \tilde{r}=\frac{\sum_{j=1}^{m}\left(\tilde{W}_{p1sj}-\sum_{i=1}^{m}\tilde{W}_{p1si}/m\right)^2\Big/m}{\sum_{j=1}^{m}\left(\tilde{W}_{1sj}-\sum_{i=1}^{m}\tilde{W}_{1si}/m\right)^2\Big/m} \tag{7-1}$$

where $\tilde{W}_{1sj}$: nondimentional total energy input obtained by simulation

$\tilde{W}_{P1sj}$: nondimensional energy input absorbed by cumulative plastic deformation obtained by simulation

Fig.7 stands for the relationship between ratio $\tilde{r}$ of r and $\tilde{W}_{1s}$, in which Δ is results for C.O.V.$[Q_{y1}]$=0.069, o is results for C.O.V.$[Q_{y1}]$ =0.138. The ratio shows very close to 1. Therefore the assumption used in Eq. (5-6) is reasonable.

Fig.8 shows the probability density function of $\tilde{W}_1$, $\tilde{W}_{P1}$ for model A. The density function is approximated by Gamma distribution. The real lines show results obtained by Method(1), and the broken lines show results by Method(2).

The influence of the shift of the vibration centor due to inelastic displacement in hysteretic structures on the velosity response is very small(20). Therefore, in this paper we have evaluated the energy input by the equivalent linear system , neglecting that influence. In the case that yield level of structures is considerably small in comparison with the acceleration level of earthquake motion, we can not get the equivalent linear system, but from the comparison between the analytical results and simulation results, we can get the results for acceleration level bringing about the ductility foctor 8-9 in structures with period 0.2sec, and the ductility factor 6-7 in structures with period 0.5sec.

§ 8. Probability of Safety for the Structures with Uncertainty Hysteretic Restoring Force

The unexpected low yield strength due to uncertainty material variability will cause remarkable effects on the probability of safety of the structures during the response time of the structures subjected to earthquake excitation. Therefore, it is very important to clarify quantitatively the effects of the uncertainty material variability on the probability of safety.

Generally, the damage of structures is in proportion to the quantity of the energy input absorbed by cumulative plastic deformation and there is a strong relationship between the energy input absorbed by cumulative plastic deformation and the failure or damage of structures. Based on the above-mentioned concept concerning the failure of the structures, we define the criterion for the probability of safety in jth-story of n-story framed structures as

$$R_j(t) = P_r[\widetilde{W}_R > \widetilde{W}_{Pj}(t)] \qquad (8\text{-}1)$$

where $R_j(t)$: the probability of safety in jth-story of n-story frame structures during th time(0, t)

$\widetilde{W}_R$: the cumulative plastic absorption capacity for structures (random variable)

This problem is well-known as the first-passage problem, and it is extremely difficult to compute exactly the probability distribution function of the probability density fuction of the first-passage time. Therefore, studies on the approximate analytical solution of the first passage problem have been carried out(13),(19). All of them are studied concerning the evaluation of the probability of safety for the structures subjected to nondeterministic excitation, ie white noise or non-white noise and so on.

In this study, we show an approximate analytical method to get the probability of safety in case that the structures with uncertainty restoring force is subjected to deterministic excitation, ie EL Centro Eartgquake. The probability of safety for such a structure is obtained as a function of the random variables. Hence, first of all, we define the conditional hazard function on the condition that all of random variables take on some values(15).

$$\lambda_j(\tau|Q_{y1},\alpha_1,\cdots) = -\frac{1}{R_j(\tau|Q_{y1},\alpha_1,\cdots)}\frac{\partial R_j(\tau|Q_{y1},\alpha_1,\cdots)}{\partial\tau} \quad (8\text{-}2)$$

where $R_j(\tau \mid Q_{y1}, \alpha_1, \cdots)$: Conditional probability of safety on the condition that random variable $Q_{y1}, \alpha_1, \cdots$, take on some values at the duration time $(0, \tau)$

$\lambda_j(\tau \mid Q_{y1}, \alpha_1, \cdots)d\tau$: probability which $\widetilde{W}_{Pj}(t)$ of jth-story exceeds the cumulative plastic deformation capacity $\widetilde{W}_R$ at the time $(\tau, \tau+d\tau)$ under the condition that random variables take on some values

Then, if λ_j is small, the probability of safety structures during the time (0, t) is given as

$$R_j(t) = e^{-\int_0^t \lambda_j(\tau)d\tau} \qquad (8\text{-}3)$$

where $\lambda_j(\tau)=E[\cdots E[\lambda_j(\tau)|Q_{y1},\alpha_1,\cdots]]$

According to the heuristic assumtion((15), we can get the expectation of Eq.(8-3)

$$\lambda_j(\tau)= -\frac{\int_{-\infty}^{\infty}\cdots\int \frac{\partial R_j(\tau|Q_{y1},\alpha_1,\cdots)}{\partial\tau} p(Q_{y1},\alpha_1,\cdots)dQ_{y1},\alpha_1,\cdots}{\int_{-\infty}^{\infty}\cdots\int R_j(\tau|Q_{y1},\alpha_1,\cdots)\ p(Q_{y1},\alpha_1,\cdots)dQ_{y1},\alpha_1,\cdots} \qquad (8\text{-}4)$$

Substituting Eq.(8-2) into Eq.(8-4) gives

$$\lambda_j(\tau) = \frac{\int_{-\infty}^{\infty}\cdots\int \lambda_j(\tau|Q_{y1},\alpha_1,\cdots)R_j(\tau|Q_{y1},\alpha_1,\cdots)p(Q_{y1},\alpha_1,\cdots)dQ_{y1},\alpha_1,\cdots}{\int_{-\infty}^{\infty}\cdots\int R_j(\tau|Q_{y1},\alpha_1,\cdots)\ p(Q_{y1},\alpha_1,\cdots)dQ_{y1},\alpha_1,\cdots} \qquad (8\text{-}5)$$

According to Bayes theorem, the following equation is obtained.

$$\frac{R_j(\tau|\ _{y1},\alpha_1,\cdots)p(Q_{y1},\alpha_1,\cdots)}{\int_{-\infty}^{\infty}\cdots\int R_j(\tau|Q_{y1},\alpha_1,\cdots)\ p(Q_{y1},\alpha_1,\cdots)dQ_{y1},\alpha_1,\cdots} = p(Q_{y1},\alpha_1,\cdots|\tau) \qquad (8\text{-}6)$$

where $p\ (Q_{y1},\alpha_1,\cdots|\ \tau)$: joint density function of $Q_{y1},\alpha_1,\cdots$ in residnal strength after response time τ

After all, the probability of safety for the structures with uncertainty hysteretic restoring force is given

$$R_j(t) = e^{-\int_0^t \lambda_j(\tau)d\tau}$$

where

$$\lambda_j(\tau)=\int_{-\infty}^{\infty}\cdots\int \lambda_j(\tau|Q_{y1},\alpha_1,\cdots)p(Q_{y1},\alpha_1,\cdots|\tau)dQ_{y1},\alpha_1,\cdots \qquad (8\text{-}7)$$

Eq.(8-7) is almost the exact expression for the probability of safety.

However, it is very difficult to evaluate exactly Eq.(8-7). Hence, we introduce an approximate concept as follows. In a denominator of Eq.(8-5), we assume that conditional probability of safety $R_j(\tau \mid Q_{y1}, \alpha_1, \cdots)$ during time$(0, \tau)$ is approximated by the probability of safety at the time τ, then $R_j(\tau \mid Q_{y1}, \alpha_1, \cdots)$ becomes

$$R_j(\tau \mid Q_{y1}, \alpha_1, \cdots) \simeq 1 - \int_0^\infty F_{\tilde{W}_R}(W)\, p_{\tilde{W}_{pj}}(W, \tau \mid Q_{y1}, \alpha_1, \cdots)\, dW \qquad (8\text{-}8)$$

where $F_{\tilde{W}_R}(W)$: probability distribution of $\tilde{W}_R$

$p_{\tilde{W}_{pj}}(W, \tau \mid Q_{y1}, \cdots)$: probability density fuction of $\tilde{W}_{pj}$ on the condition that $Q_{y1}, \alpha_1, \cdots$ take on some values

It is an approximte method similar to Markov's approximation. When $Q_{y1}, \alpha_1, \cdots (j=1, \cdots, n)$ take deterministic values, the cumulative plastic deformation $\tilde{W}_{pj}$ takes constant value at the response time . Hence, $p_{\tilde{W}_{pj}}(W, \tau \mid Q_{y1}, \cdots)$ is given

$$p_{\tilde{W}_{pj}}(W, \tau \mid Q_{y1}, \cdots) = \delta(W(\tau) - \tilde{W}^{o}_{pj}) \qquad (8\text{-}9)$$

where $\delta(w)$: delta function

$\tilde{W}^{o}_{pj}(Q_{y1}, \alpha_1, \cdots)$:energy input absorbed by cumulative plastic deformation at the response time t on the condition that $Q_{y1}, \alpha_1, \cdots$ take some values

Substituing Eq(8-9) into Eq.(8-8) gives

$$R_j(\tau \mid Q_{y1}, \alpha_1, \cdots) = 1 - F_{\tilde{W}_R}(\tilde{W}^{o}_{pj}) \qquad (8\text{-}10)$$

The probability of safety $R_j(t)$ will be written in the form.

$$R_j(t) = e^{-\lambda_j(t)} \qquad (8\text{-}11)$$

$$\text{where } \lambda_j(t) = \frac{\int_{-\infty}^{\infty}\cdots\int F_{\tilde{W}_R}(\tilde{W}^{o}_{pj}, t)\; p(Q_{y1}, \alpha_1, \cdots)\, dQ_{y1}\, d\alpha_1 \cdots}{1 - \int_{-\infty}^{\infty}\cdots\int F_{\tilde{W}_R}(\tilde{W}^{o}_{pj}, t)\; p(Q_{y1}, \alpha_1, \cdots)\, dQ_{y1}\, d\alpha_1 \cdots}$$

Kuwamura ,Galanbos(17) have evaluated approximately the probability of failure for maximum energy input which will be expected at the site of the building during lifetime, in case of using similar reliability criterion. Eq.(8-11) has merits that the probability of safety is directly evaluated from the probability distribution of $\widetilde{W}_R$ and the probability density function of random variable without computing the probability density function of energy response $\widetilde{W}_{PJ}$. If there is a variability of energy input due to uncertainty about earthquake ground motion, it will be possible to evaluate directly from Eq.(8-12) by regarding $\widetilde{W}^{o}_{PJ}$ as parameters including the effect of them.

§ 9. Parameter studies

In this section, parameter studies are carried out by using the Method (2). The structural models are 4 cases as follows.

(1) Case S-1 (SDOF system, restoring force:Fig.6)

$m=0.1tsec^2/cm$, $h=2\%$

$E[\alpha_1]=0.0095$, $C.O.V.[\alpha_1]=0.149$

For T=0.1sec - 0.2sec, $E[Q_{y1}]=36.4t$, $C.O.V.[Q_{y1}]=0.069$

For T=0.3sec - 0.8sec, $E[Q_{y1}]=28.0t$, $C.O.V.[Q_{y1}]=0.069$

(2) Case S-2 (SDOF system, restoring force:Fig.6)

$m=0.1tsec^2/cm$, $h=2\%$

$E[\alpha_1]=0.0095$, $C.O.V.[\alpha_1]=0.149$

For T=0.1sec - 0.2sec, $E[Q_{y1}]=36.4t$, $C.O.V.[Q_{y1}]=0.138$

For T=0.3sec - 0.8sec, $E[Q_{y1}]=28.0t$, $C.O.V.[Q_{y1}]=0.138$

(3) Case T-1 (TDOF system, restoring force(1st fr.,2nd fr.):Fig.6)

$m_1=0.1tsec^2/cm$, $m_2=0.075tsec^2/cm$, $K_1=K_2$, $h=2\%$

$E[\alpha_1]=E[\alpha_2]=0.0095$, $C.O.V.[\alpha_1]=C.O.V.[\alpha_2]=0.149$, $\rho(\alpha_1, \alpha_2)=0$

For T=0.1sec - 0.3sec(1st period)

$E[Q_{y1}]=54.6t$, $E[Q_{y2}]=33.6t$, $C.O.V.[Q_{y1}]=C.O.V.[Q_{y2}]=0.069$,

$\rho(Q_{y1}, Q_{y2})=0$

For T=0.4sec - 0.8sec(1st period)

$E[Q_{y1}]$=36.6t, $E[Q_{y2}]$=22.4t, C.O.V.$[Q_{y1}]$=C.O.V.$[Q_{y2}]$=0.069,

$\rho(Q_{y1}, Q_{y2})=0$

(4) Case T-2 (TDOF system, restoring force(1st fr., 2nd fr.):Fig.6)

$\rho(Q_{y1}, Q_{y2})=0.7$, Other values are the same as those of Case T-1

Earthquake motion : EL Centro earthquake

Ductility factors, maximum and pernament displacement, energy absorption, etc have been identified as important response parameters in the evaluation of structural performance under earthquake loading. In this study, we will focus on energy absorption from among the above-mentioned parameters. Therefore, the following two energy parameters are considered.

(1) total energy input for structures under earthquake loading($\widetilde{W}_J$)

(2) the sum of the energy input absorbed by cumulative plastic deformation for structures under earthquake loading($\widetilde{W}_{PJ}$)

Fig.9 shows the results in Case S-1. The mean value shows a little smaller value than simulation results as well as the outcome shown in Tables 1 and 2. The difference is within about 5% except 11% in T=0.1sec, and coefficient of variation shows a small value a little over 20% on average. Fig.10 shows the results in Case S-2. The differences between analytical results and simulation results become larger. The analytical results show about 10% small value for the mean and about 30%-40% small value for coefficient of variation. The coefficient of variation of $\widetilde{W}_{PJ}$ became a little over 3 times as large as that of yield force in the structures from the results in Figs.9 and 10. In this way, the first order approximation brings on a considerable underestimated results in case of the big variability of yield force. Therefore, we have to notice for the application of the first order approximation for uncertainty analysis of the structures with the big variability of yield force. Fig.11 and Fig.12 show the results for two degree of freedom system. Fig.11 stands for correlation coefficient $\rho(Q_{y1}, Q_{y2})=0.0$ and Fig.12 for

$\rho(Q_{y1}, Q_{y2})=0.7$. To analyze two DOF system, it is necessary to carry out iterative computing with two unknown variables $\tilde{u}_1, \tilde{u}_2$. The total energy input in the part of 1st-story of the 2nd-story frame structures get to large, and the total energy input in the part of 2nd-story get to small. So, unknown variable $\tilde{u}_2$ was smaller than 1 except in the case of T=0.2sec and 0.3sec. Hence, we could not carry out the exact iterative computing. However, as the total energy input in the part of 2nd-story is small and close to W_{2s} obtained by simulation, we approximately regard this value as the results got from Eq.(4-4). It is one of the demerits for the proposed method, but it will be dissolved by using a weight fuction

$f(\tilde{u}_1)$ instead of $\tilde{u}_1$. The mean values in Figs.11 and 12 show smaller values than simulation as well as SDF system.
The difference is about 5% smaller except in the case of T=0.1sec and the coefficient of variation shows about 20%-100% small value. We could not analytically clarify the effect of correlation between Q_{y1} and Q_{y2} on total energy input for structures because of the above-mensioned reason. But we could get the following results from simulation. The mean of $\tilde{W}_{P1}$ in $\rho(Q_{y1}, Q_{y2})=0.7$ shows about 3% large value in comparison with the value in $\rho(Q_{y1}, Q_{y2})=0.0$. On the contrary, the coefficient of variation of $\tilde{W}_{P1}$ in case of $\rho(Q_{y1}, Q_{y2})= 0.7$ shows about 6% small in comparision with the value in $\rho(Q_{y1}, Q_{y2})=0.0$

Fig.13 shows the time history of $\tilde{W}_{P1}$ for the structures with deterministic elasto-plastic restoring force without uncertainty factor. If there is no variability of yield force in the structures, the structures will be completely assured for larger cumulative plastic absorption capacity than the value expected for structures. However, if there is a variability of yield force, the safety of structures is not assured.

Fig.14 shows the effect of variability of yield force in the restoring force on the probability of safety for structures. Fig.14(a) shows the effect of mean value $E[\widetilde{W}_R]$ on the probability of safety $R(t)$ in case of C.O.V.$[Q_{y1}]$=0.069 and C.O.V.$[\widetilde{W}_R]$=0.069, and Fig.14(b) shows the effect of mean value on it in C.O.V.$[Q_{y1}]$=0.138 and C.O.V.$[\widetilde{W}_R]$=0.138, in which normal distribution is assumed for describing the random cumulative plastic absorption capacity. The figure shows that the mean of cumulative plastic deformation capacity is reqiured about 1.5 times of $\widetilde{W}_{P1}$ in order to assure the probability of safety of over 95% in case of C.O.V.$[Q_{y1}]$=0.069 and C.O.V.$[\widetilde{W}_R]$=0.069. The effect of variability in yield force on the probability of safety, in which Eq.(8-1) is used as a safety criterion for structures, is remarkably large.

§. 10 Conclusions

The analytical approach to clarify the effect of uncertainty hysteretic restoring force on energy response for structures under earthquake loading have been investigated . The results obtained in this study are summarized as follows.

(1) The studies on the variability of dynamic response for structures with random hysteretic restoring force have been carried out by numerical computing method based on Monte Carlo simulation. However, it is possible to evaluate analytically the variability of energy respone for structures with random hystertetic restoring force by executing once a deterministic numerical analysis for structures with restoring force represented by mean values of random variables.

(2) The total energy input for structures under earthquake loading will be approximately obtained by superposing each energy input for linear system with restoring force Eq.(3-2) forced by harmonic excitation with a frequency component including in an earthquake excitation.

(3) The coefficient of variation for energy input absorbed by cumula-

tive plastic deformation in structures with restoring force becomes a little over 3 times as large as that of yield force.

(4) The effect of hysteretic restoring force with variability in yield force on the probability of safety is remarkably large. The mean of cumulative plastic deformation capacity will be required about 1.5 times of the value to be expected under earthquake loading, in order to assure the probability of safety of over 95% for C.O.V.[Q_{y1}]=0.069 and C.O.V.[$\widetilde{W}_R$]=0.069.

(5) Finally, for the case of two DOF system, we could not carry out the exact iterative computation. This problem will be dissolved by using a weight function, which is one of the further research concerning this study.

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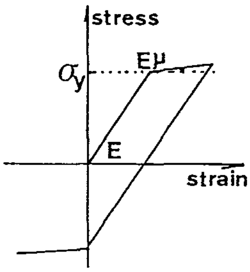


Fig. 1 Stress-Strain Model

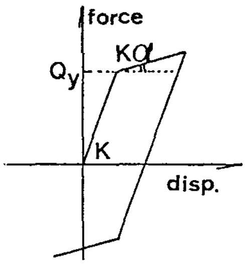


Fig. 2 Hysteretic Restoring Force Model

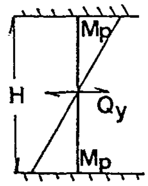


Fig. 3 Column

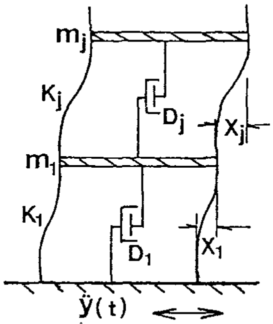


Fig. 4 MDOF Systems

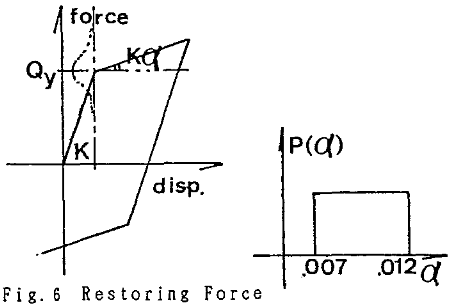


Fig. 6 Restoring Force Model (Q_y, α: Random Variable)

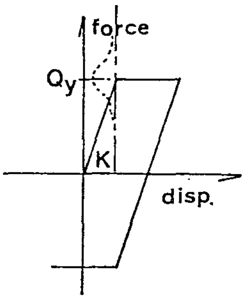


Fig. 5 Restoring Force Model(Q_y :Random Variable)

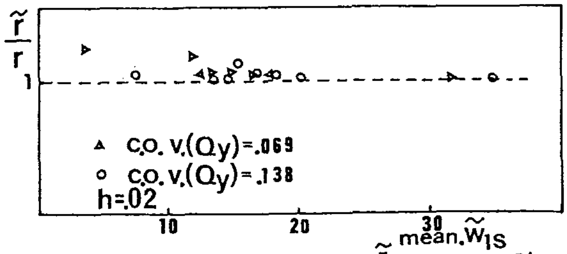


Fig. 7 Relationship Between $\frac{\tilde{r}}{r}$ and mean. $\tilde{W}_{1S}$

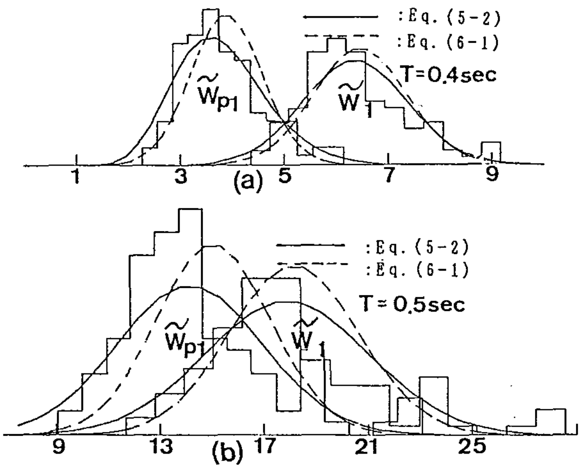


Fig. 8 Probability density function

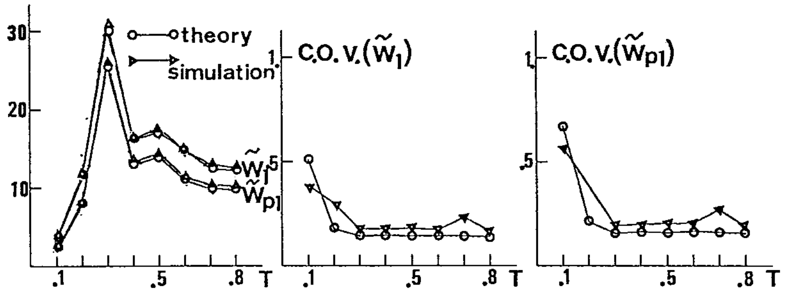


Fig. 9 Comparison with Simulation for Case S-1

(C.O. V.(Q_{y1}) = 0.069, C.O. V.(α_1) = 0.149)

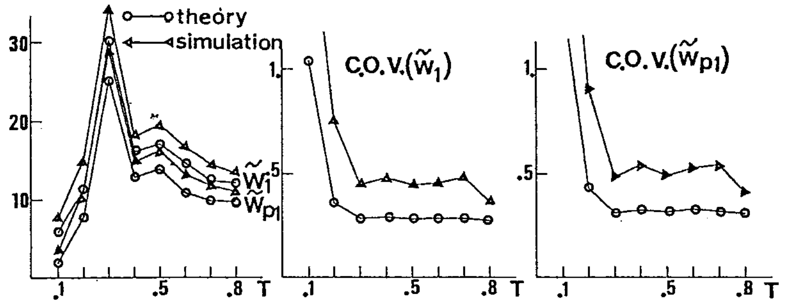


Fig. 10 Comparison with Simulation for Case S-2

(C.O. V.(Q_{y1}) = 0.138, C.O. V.(α_1) = 0.149)

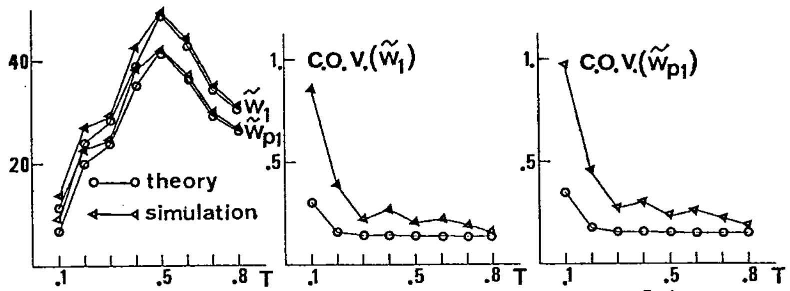


Fig. 11 Comparison with Simulation for Case T-1

(C.O. V.(Q_{y1}) = C.O. V.(Q_{y2}) = 0.069, ρ(Q_{y1}, Q_{y2}) = 0.,
C.O. V.(α_1) = C.O. V.(α_2) = 0.149, ρ(α_1, α_2) = 0.)

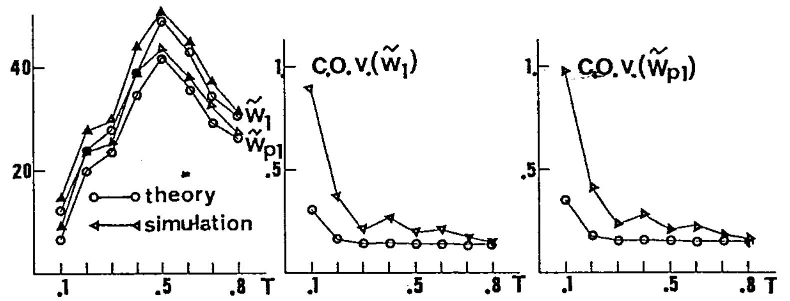


Fig. 12 Comparison with Simulation for Case T-2

(C.O. V.(Q_{y1}) = C.O. V.(Q_{y2}) = 0.069, ρ(Q_{y1}, Q_{y2}) = 0.7
, C.O. V.(α_1) = C.O. V.(α_2) = 0.149, ρ(α_1, α_2) = 0.)

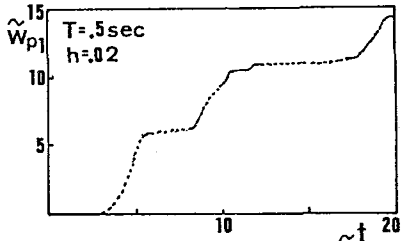
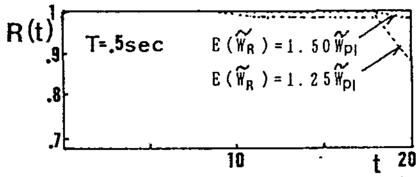
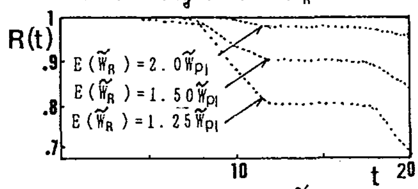


Fig. 13 Time History of $\tilde{W}_{pl}$



(a) C.O.V.(Q_y)=C.O.V.($\tilde{W}_R$)=0.069



(b) C.O.V.(Q_y)=C.O.V.($\tilde{W}_R$)=0.138

Fig. 14 Effect of Variability in Yield Force on the Probability of safety

	$\tilde{W}_l$		$\tilde{W}_{pl}$	
	mean	c.o.v.	mean	c.o.v.
Method(1)	6.37	0.180	3.79	0.233
Method(2)	6.22	0.143	3.70	0.186
Simulation	6.36	0.157	3.81	0.205

(T=0.4sec)

	$\tilde{W}_l$		$\tilde{W}_{pl}$	
	mean	c.o.v.	mean	c.o.v.
Method(1)	17.88	0.175	14.15	0.197
Methoh(2)	17.47	0.141	13.83	0.158
Simulaton	18.07	0.185	14.35	0.197

(T=0.5sec)

表 1 Comparision between analytical results and Monte Carlo Simulation (Model A)

	$\tilde{W}_l$		$\tilde{W}_{pl}$	
	mean	c.o.v.	mean	c.o.v.
Method(1)	6.39	0.181	3.78	0.235
Method(2)	6.23	0.143	3.68	0.186
simulation	6.36	0.157	3.81	0.204

(T=0.4sec)

	$\tilde{W}_l$		$\tilde{W}_{pl}$	
	mean	c.o.v.	mean	c.o.v.
Method(1)	17.87	0.176	14.11	0.198
Method(2)	17.49	0.140	13.81	0.158
Simulation	17.95	0.186	14.23	0.199

(T=0.5sec)

表 2 Comparision between analytical results and Monte Carlo Simulation (Model B)