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Simple Approach for Evaluation of Earthquake Response of Irregularly Bounded Surface Layer

by

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1. Introduction

When there is a large change in mechanical impedance between a soft surface layer and its bedrock, the irregular shape of the surface layer affects much its seismic response. Since such underground structures as tunnels, deep foundations and so on follow closely displacement of the surrounding soil during an earthquake, a close investigation of motion of a surface layer will yield an important information for design of these structures.

Finite element method for three dimensional analysis would be one of the powerful techniques for this purpose. However, more complicated the soil profile and structures are, more tedious is the numerical calculation, and sometimes, it will be close to impossible when the number of unknown variables is too large.

The authors have developed a simple numerical model for this purpose[(1)~(4)]. The model is a horizontally spread two-dimensional finite element net with its nodes supported by springs of the Winkler type (Fig. 1). Numerical and experimental studies by the authors clarified that the present model can account rationally for an irregular shape and heterogeneity of a surface layer in frequency range including vibration modes of comparatively low order. It is, however, still necessary to review closely the assumptions adopted in the model so that the model can be properly used in practical cases.

2. Plane-Strain and Plane-Stress Hypothesis

For the sake of simplicity, a two-dimensional surface layer will be discussed here. The governing equations for a medium are as follows:

$$\frac{\partial^2 u}{\partial t^2} = V_p^2 \frac{\partial^2 u}{\partial x^2} + (V_p^2 - V_s^2)\frac{\partial^2 w}{\partial x \partial z} + V_s^2 \frac{\partial^2 u}{\partial z^2} \quad \cdots\cdots\cdots\cdots(1)$$

$$\frac{\partial^2 w}{\partial t^2} = V_p^2 \frac{\partial^2 w}{\partial z^2} + (V_p^2 - V_s^2)\frac{\partial^2 u}{\partial z \partial x} + V_s^2 \frac{\partial^2 w}{\partial x^2} \quad \cdots\cdots\cdots\cdots(2)$$

where, u, w = displacement components in x and z directions, t=time and V_p, V_s = longitudinal and shear wave velocities. In the above equations, vertical ground motion w is considered to be small when dynamic response of a wide-spread and horizontally layered surface with gentle change of depth is studied in low frequency range including its natural frequencies of low order.

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Thus, one of the practical approaches customarily used to simplify numerical procedures is to neglect vertical ground motion w, and this assumption yields the following equation:

$$\frac{\partial^2 u}{\partial t^2} = V_p^2 \frac{\partial^2 u}{\partial x^2} + V_s^2 \frac{\partial^2 u}{\partial z^2} \quad \text{..........(3)}$$

Though this assumption is efficient, it should be noted that vertical ground motion still has pronounced effect if the surface layer with a radical change of depth is studied. When a surface layer is not so thick in comparison with the wave length to be discussed, not the vertical motion w but the vertical normal stress σ_{zz} will be considerably small. Thus, substituting $\sigma_{zz}=0$ in the Hooke's law for a two-dimensional medium, the following equation can be obtained:

$$\sigma_{zz} = \lambda\varepsilon_v + 2\mu\,\varepsilon_{zz} = 0 \quad \text{.......(4)}$$

where λ, μ = Lame's constants, $\varepsilon_v = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$ = volumetric strain and $\varepsilon_{zz} = \frac{\partial w}{\partial z}$. Rearranging of the above equation yields:

$$\lambda \frac{\partial w}{\partial z} = -(\lambda + 2\mu)\frac{\partial u}{\partial x} \quad \text{........(5)}$$

Since eq. (5) holds good along the surface (in x direction), it is possible to differentiate eq. (5) with respect to x. Thus:

$$\frac{\partial^2 w}{\partial z \partial x} = -\frac{(\lambda + 2\mu)}{\lambda}\frac{\partial^2 u}{\partial x^2} \quad \text{..........(6)}$$

Substituting eq. (6) in eq. (1), we obtain:

$$\frac{\partial^2 u}{\partial t^2} = V_p^{*2} \frac{\partial^2 u}{\partial x^2} + V_s^2 \frac{\partial^2 u}{\partial z^2} \quad \text{..........(7)}$$

where, $V_p^* = \sqrt{\frac{\lambda^* + 2\mu}{\rho}}$ and $\lambda^* = \frac{\lambda\mu}{\lambda+2\mu}$. Eq. 7 has the same form as the eq. (3) except for the change of V_p into V_p^* . It is nothing but the plane stress assumption while the former is the plane strain one. To discuss the difference between these two hypotheses, natural frequencies of a simple rectangular surface layer as shown in Fig. 2 was calculated. Three sides of the layer except the surface are fixed. Using the simplified governing equation (3) or (7), The vibration mode $X_{nm}(x,z)$ and the corresponding frequency f_{nm} are respectively obtained as:

$$X_{nm}(x,z) = \sin \alpha_m z \sin \gamma_n (x+L) \quad \text{.......(8)}$$

$$f_{nm} = \frac{1}{2\pi} \sqrt{V_s^2 \alpha_m^2 + V_p^2 \gamma_n^2} \qquad \text{.......(9)}$$

where, $\alpha_m = \frac{2m-1}{2H}$ and $\gamma_n = \frac{n\pi}{2L}$ in which n and m are integer number. In the above eq. (9), V_p must be replaced by V_p^* when the plane-stress condition is assumed. With increase of L/H, eq. (9) converges f_0 ($= \frac{1}{2\pi} \frac{Vs}{4H}$), which is the fundamental natural frequency of the surface layer with an infinite spread. Eq. (9) divided by the f_0 is the following equation as:

$$f_{nm}/f_0 = \sqrt{1 + \left(\frac{H}{L} \frac{n}{2m-1} \frac{Vp}{Vs} \right)^2} \qquad \text{.......(10)}$$

Variations of the normalized frequency f_{nm}/f_0 with $(\frac{H}{L})$ are shown in Figs. 3(a),(b) together with the solutions by the Finite Element method. Poisson's ratios of the medium in Figs. 3(a) and 3(b) are 0.4375 and 0.49777, respectively. Marked difference can be observed between these two different assumptions, and it should be noted here that not the results under plane-strain hypothesis but those under plane-stress one agree well with those by FEM.

when a three-dimensional medium is studied, the longitudinal wave velocity V_p^* in a plane-stress plate in the model is expressed as:

$$V_p^* = \sqrt{\frac{\lambda^* + 2\mu}{\rho}} \qquad \text{........(11)}$$

where, $\lambda^* = \frac{2\lambda\mu}{\lambda+2\mu}$. Fig. 4 shows the rigorous solution of frequency-domain rocking stiffness of a cylindrical massless caisson embedded in a surface layer with an infinite spread (by Tajimi, H.[5]). The solutions by the proposed model with wave-transmitting boundaries (Fig. 5) are also shown in this figure. Since the rigorous solution was obtained assuming that the vertical ground motion is negligibly small, very good agreement was seen assuming the plane-strain hypothesis. Though this agreement validates the present approach itself, it does not assure us that the plane-strain hypothesis yields a good approximation. And the marked difference between these plate conditions urges us to review the customarily used hypothesis in this field.

3. Vibration Mode along Depth

Another important assumptions in the proposed method is the shape of vibration mode along depth. Several numerical and experimental studies revealed that it is appropriate and efficient to use the fundamental vibration mode of soil columns which is being assembled into a surface layer as shown in Fig. 1. It would be, however, necessary to describe more irregular shape of vibration in the model not only when a surface layer is excited in fairly high frequency, but also when the change of depth and inhomogeneity of the surface layer is considerably radical.

Increase of the number of slices along depth would be one efficient means for this purpose. It is, however, desirable to minimize the number of division as fewer as possible for the practical use. Thus, the transfer matrix approach was adopted here in the time domain analysis assuming that deformation of each element (Fig.6) is expressed in a polynomial form with the least number of terms required to define the boundary conditions of this slice. That is:

$$u_i(t,z) = C_{3,t}z^3 + C_{2,t}z^2 + C_{1,t}z + C_{0,t} \qquad \text{..........(12)}$$

where, $C_{3,t}$, $C_{2,t}$, $C_{1,t}z$ and $C_{0,t}$ = unknown constants at time t. Given the displacements at both ends of the slice, these unknown constants are so determined as to satisfy the governing equation (3) in plane-strain case or eq. (7) in plane-stress case. Thus the following transfer matrix in the time domain can be obtained:

$$\begin{Bmatrix} u_i(t+\Delta t,H) \\ S_i(t,H) \end{Bmatrix} = \begin{bmatrix} -2 & -6\,R_z^2\,\frac{H}{\mu} \\ -\frac{\mu}{2R_z^2 H} & -2 \end{bmatrix} \begin{Bmatrix} u_i(t+\Delta t,0) \\ S_i(t,0) \end{Bmatrix} + \begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} \qquad \text{.......(13)}$$

where, $b_1 = (6R_z^2 + 2)\,u_i(t,H) - (6R_z^2 - 4)\,u_i(t,0) - u_i(t-\Delta t,H)$

$$- u_i(t-\Delta t,0) + R_x^2\,(u_{i+1}(t,H) - 2\,u_i(t,H) + u_{i-1}(t,H))$$

$$+ 2\,R_x^2\,(u_{i+1}(t,0) - 2\,u_i(t,0) + u_{i-1}(t,0))$$

$$b_2 = \frac{3\mu}{H}\,(u_i(t,H) - u_i(t,0)) + \frac{\mu}{R_z^2 H}\,u_i(t,0) - \frac{\mu}{2R_z^2 H}\,u_i(t-\Delta t,0)$$

$$+ \frac{\mu}{2R_z^2 H}\,R_x^2\,(u_{i+1}(t,0) - 2\,u_i(t,0) + u_{i-1}(t,0))$$

$R_z^2 = (V_s\,\frac{\Delta t}{H})^2$, $R_x^2 = (V_p\,\frac{\Delta t}{\Delta x})^2$, Δt= time increment.

A surface layer as shown in Fig. 7 is excited sinusoidally with the frequency of 1.5 Hz which coincides with the fundamental natural frequency of a soil column (shear beam) having the same depth as that of the shallow part of this layer. Figs. 8(a) - (c) shows the time histories of the tremor on the whole ground surface in different numbers of soil slice, while Fig. 8(d) show the rigorous time history obtained using the finite difference method. In this figure, n_{z1} and n_{z2} are vertical slice numbers in shallow and deep parts of the surface layer, respectively, and n_x is the number of lateral division. There is no marked difference of tremor between the above numerical results on the shallow part, while the shape of the surface tremor is getting closer to the rigorous solution in the deep part. Fig. 9 shows the time histories of the ground tremor along the depth at the middle of the deep part of the layer together with the rigorous solution by the finite difference method. It is obvious that the marked improvement of the vibration mode with a little increase of slice number n_{z2} resulted in the above-mentioned improvement shown in Fig. 8. Though the boundary shape of the surface layer in this study is rather unreal, these figures 8 and 9 show that it is necessary to divide the medium into vertical slices when earthquake response of a surface layer with a radical change of depth is studied in fairly high frequency range, so that an irregular vibration mode can be expressed well.

3. Conclusions

Two important assumption adopted in the quasi-three (or two) dimensional ground model were discussed in this paper. The concluding remarks are summarized as:

(1) Treating the surface plate linking soil columns in the proposed model as a plane stress plate, effect of vertical ground motion due to a radical change in its shape is efficiently taken into account without analyzing the vertical motion.

(2) It is sometimes necessary to divide the medium into vertical slices so that an irregular vibration mode can be expressed well when a surface layer with an irregular shape of boundary is studied. And this number can be reduced if an appropriate shape function expressing displacement within a slice is adopted.

References

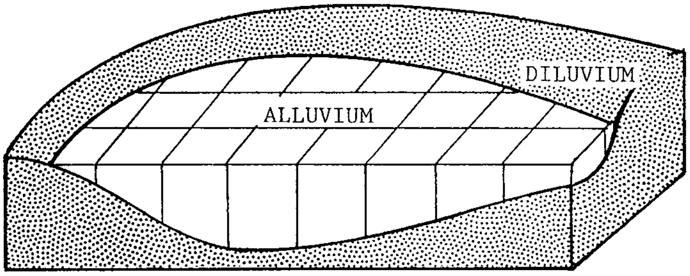
(1) Tamura C. and T. Suzuki: A Quasi-Three-Dimensional Ground Model for Earthquake Response Analysis of Underground Structures -- Construction of Ground Model --, "SEISAN-KENKYU" (Monthly Jour., Inst. of Industrial Science, Univ. of Tokyo), Vol. 39, No.1, pp.37-40, (1987).

(2) Tamura C. and T. Suzuki: Proposal of a Mathematical Model for Earthquake Response Analysis of Irregularly Bounded Surface Layer, Proc., 9th World Conference on Earthquake Engineering, Vol. II, pp.665-670, (1988).

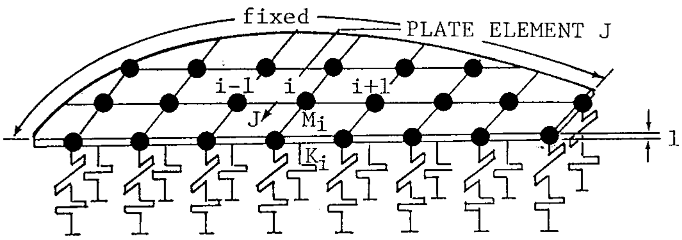
(3) Tamura C. and K. Konagai: Simple Approach for Evaluation of Dynamic Stiffness of Embedded Structure, Proc., 9th World Conference on Earthquake Engineering, Vol. III, pp.373-378, (1988).

(4) Tamura C. and K. Konagai: On Numerical Analysis on Dynamic Characteristics of Irregularly Bounded Surface Layer using Quasi-Three Dimensional Ground Model, Proc., National Symposium on Effects of Surface Geology on Seismic Motion, pp.311-318, (1989).

(5) Tajimi H.: Dynamic Analysis of a Structure Embedded in an Elastic Stratum, Proc., 4th World Conference on Earthquake Engineering, Vol. A-6, pp.54-69, (1969).



(a) Drowned Valley surrounded by Diluvium



(b) Alluvial Ground Model

Fig. 1 Quasi-three-dimensional ground model

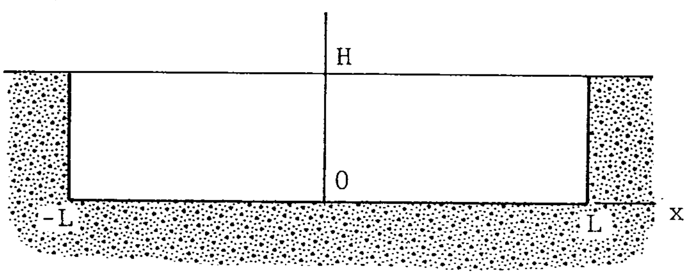


Fig.2 Rectangular surface layer

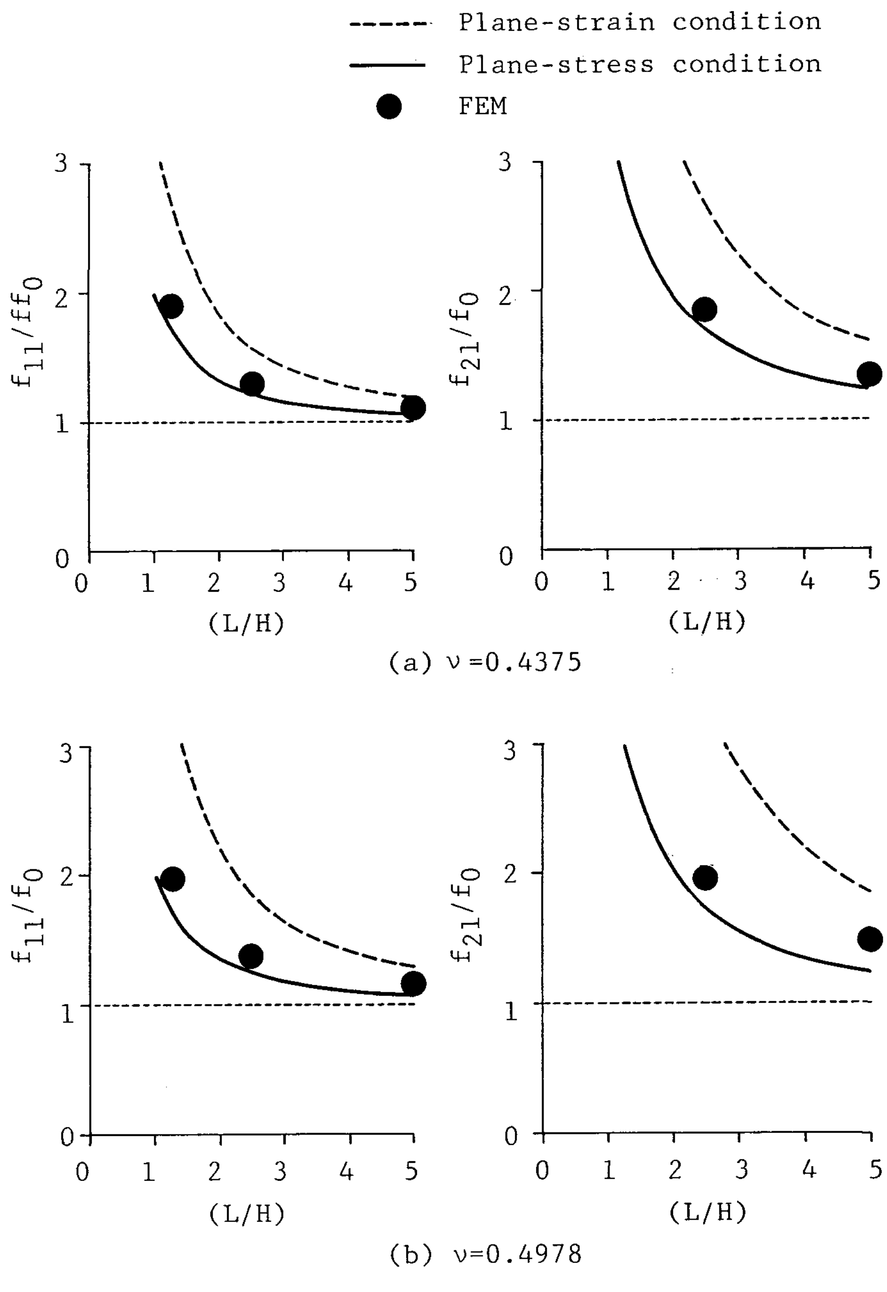


Fig.3 Variation of natural frequency with aspect ratio

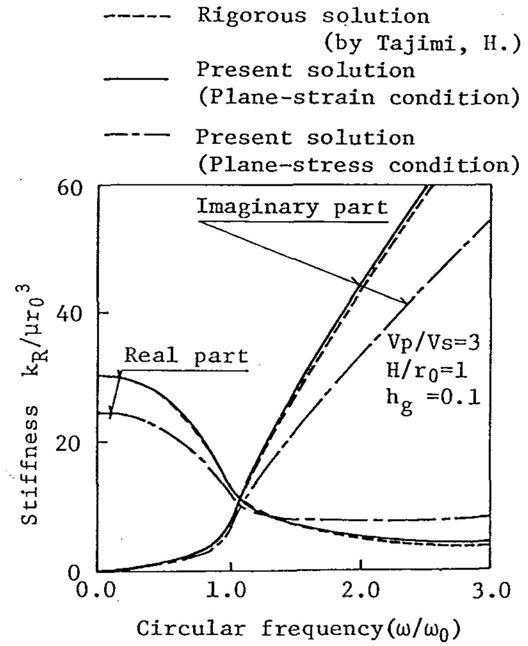


Fig.4 Variation of stiffness of embedded structure with frequency

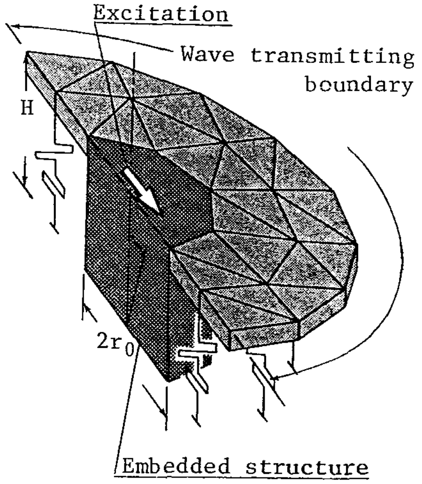


Fig.5 Model of embedded structure

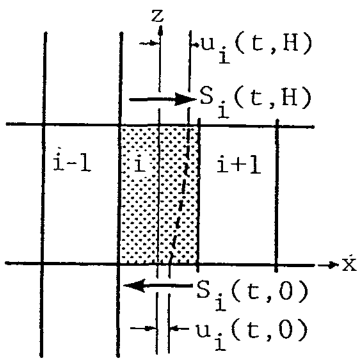
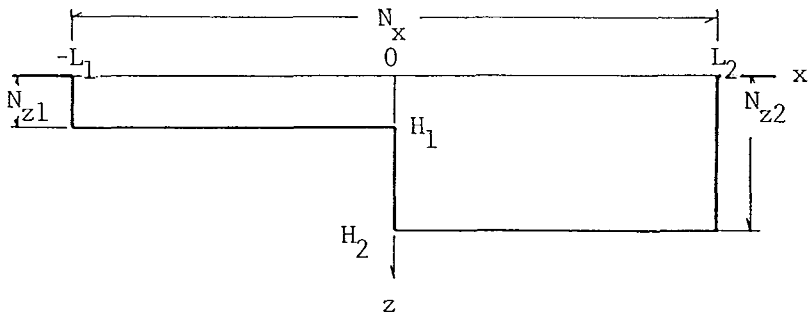


Fig.6 Shear forces and displacement on segment



V_s = 120 m/s H_1 = 20 m

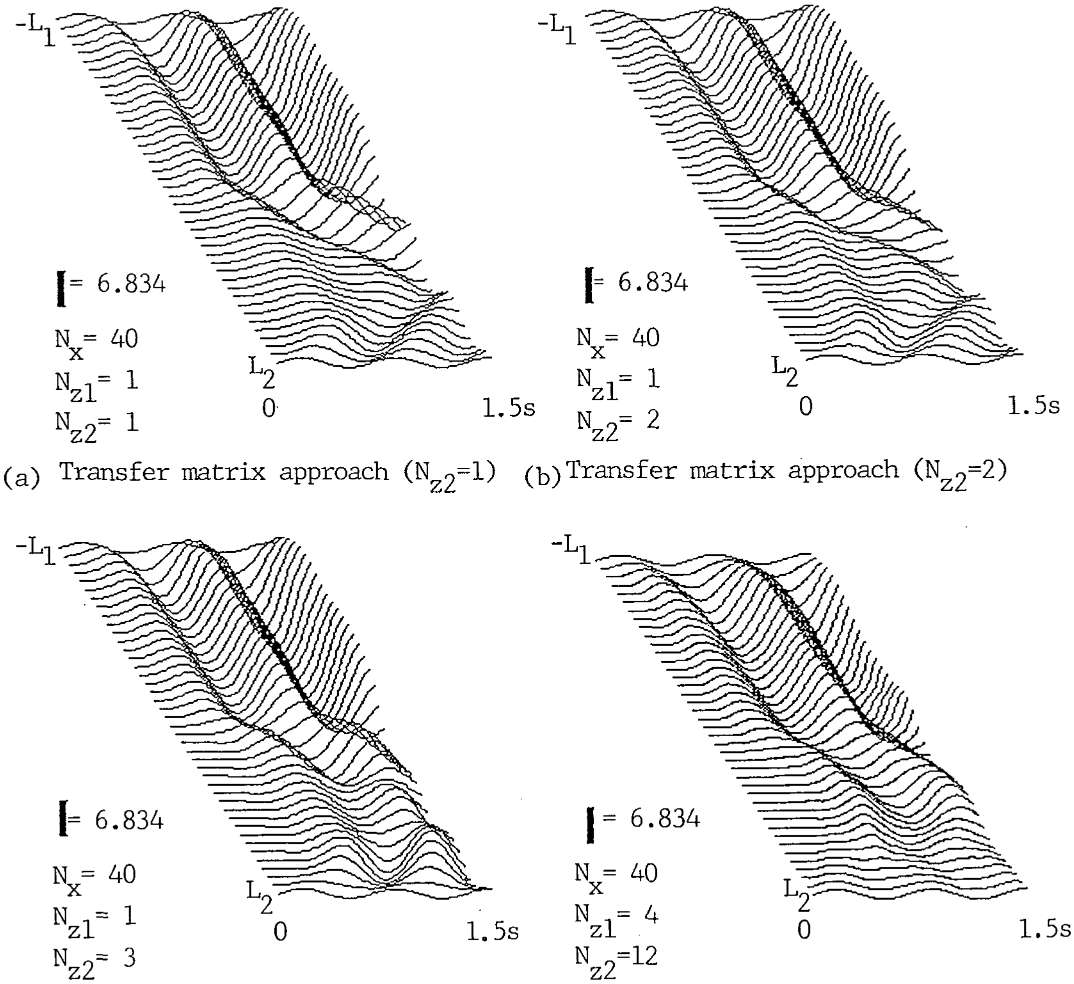
V_p = 600 m/s H_2 = 60 m

ρ = 1.6 t/m^3 L_1 = L_2 = 400 m

exciting frequency = 1.5 Hz

N_x, N_{z1}, N_{z2}: Number ob slices

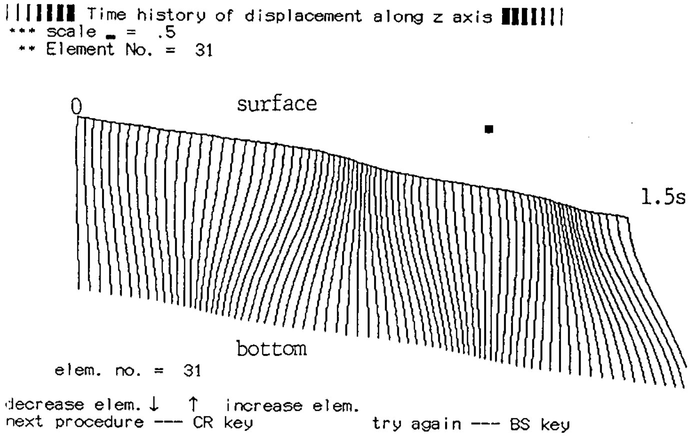
Fig.7 Surface layer with a radical change of depth



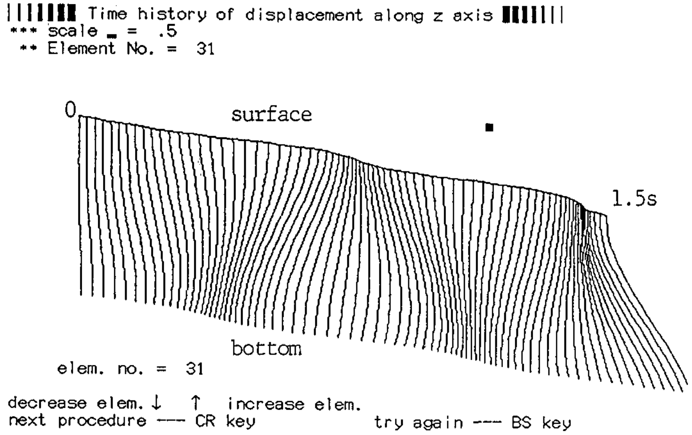
(a) Transfer matrix approach (N_{z2}=1) (b) Transfer matrix approach (N_{z2}=2)

(c) Transfer matrix approach (N_{z2}=3) (d) Finite difference method (N_{z2}=12)

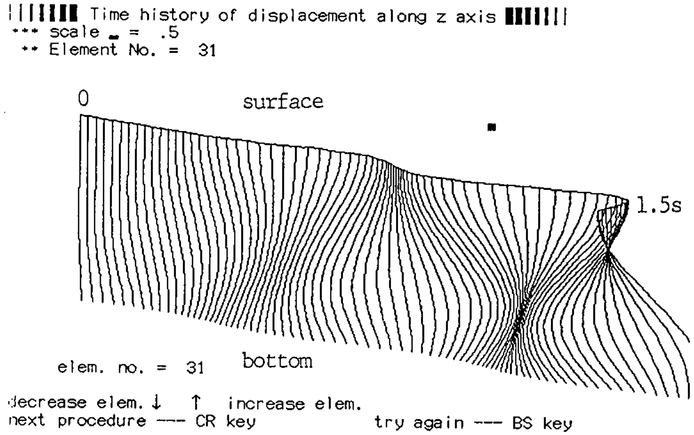
Fig.8 Time history of tremor on ground surface



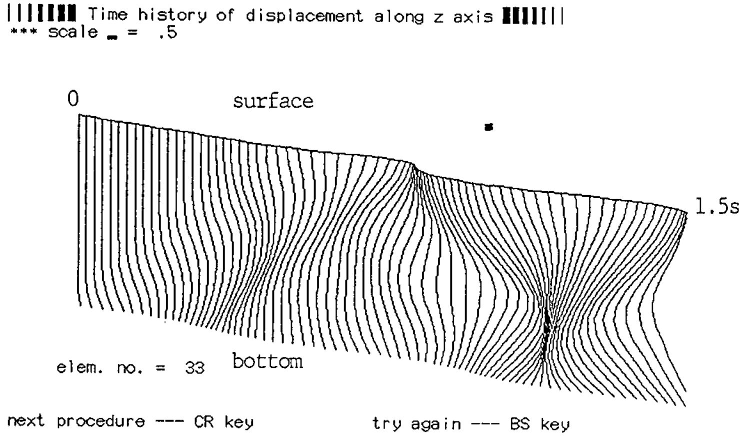
(a) Transfer matrix approach (N_{z2}=1)



(b) Transfer matrix approach (N_{z2}=2)



(c) Transfer matrix approach (N_{z2}=3)



(d) Finite difference method (N_{z2}=12)

Fig.9 Time history of ground tremor along depth