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ENERGY INPUT RATE SPECTRA OF EARTHQUAKE GROUND MOTIONS

by

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1. INTRODUCTION

It is well recognized that the amount of energy absorption is one of important factors in the seismic damage assessment of structural system and elements. Most of past research efforts have been made to evaluate final amounts of energy absorption or energy input exerted by earthquakes, and they have achieved energy-based design methods(Ref.1). Furthermore, to assess the structural damage process more precisely, it is important to know not only final values but also more detailed information about seismic damage potential which exerts energy, in other words, when, how long, how much, and at which frequencies, such a potential is contained in the ground motion. Therefore, non-stationary characteristics of seismic waves shall be studied carefully. The goal of this paper is to propose a rigorous tool of non-stationary spectral analysis for the evaluation of the seismic damage potential.

2. ENERGY INPUT RATE SPECTRUM

To handle non-stationary signals, many researchers in various engineering fields have developed various techniques of non-stationary spectral analysis suitable for their purposes: for example, Mark's physical spectrum, Wigner distribution, Page's instantaneous power spectrum, multifilter techniques, and so on. Among them, the multi-filter spectrum proposed by Kameda (Ref. 2) is specifically related to the energy responses of linear oscillators subjected to a ground motion. Kameda's spectral value at every moment indicates the sum of the kinetic energy and the potential energy which are held by the viscously damped linear oscillator with respect to its undamped natural frequency. On the other hand, a new spectrum termed energy input rate spectrum proposed here (abbreviated as "EIR spectrum") indicates the power of the effective excitation force acting on the viscously damped linear oscillator per unit mass. In other words, the EIR spectrum indicates the energy input rate per unit mass exerted into the oscilla-tor with respect to its undamped natural frequency. The difference between these two formulations is schematically illustrated in Fig. 1.

The EIR spectrum has close relationship also with Page's instanta-neous power spectrum (Ref. 3). The recent study on the final value of the energy input (Ref. 4) developed interesting formulae to indicate the energy input in frequency-domain. According to this study, the energy input is obtained by integrating the Fourier square amplitudes of the ground acceleration together with the weighting function termed "energy admittance" as follows:

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$$e_I = -\int \dot{x}(\tau)\,\ddot{y}(\tau)\,d\tau \qquad \text{(Time-domain integral)} \quad (1)$$

$$= \int W(u)\left(\frac{1}{2\pi}|\ddot{Y}(u)|^2\right)du \qquad \text{(Frequency-domain integral)} \quad (2)$$

with $$W(u) = -\mathrm{Real}\,[H(u)] \qquad \text{(Energy Admittance)} \quad (3)$$

where e_I : energy input per unit mass
$\dot{x}(\tau)$: response velocity
$\ddot{y}(\tau)$: ground acceleration
τ : time, u : circular frequency
$\ddot{Y}(u)$: Fourier transformation of $\ddot{y}(\tau)$
$H(u)$: transfer function to obtain $\dot{x}$ from $\ddot{y}$

It is straightforward to obtain the energy input up to the time t:

$$e_I(t) = \int W(u)\left(\frac{1}{2\pi}|\ddot{Y}_r(u,t)|^2\right)du \quad (4)$$

where $\left(\frac{1}{2\pi}|\ddot{Y}_r(u,t)|^2\right)$ is called running Fourier square amplitude spectrum (up to the time t) of the ground acceleration.

Here we *denote* the natural frequency of the oscillator by ω and the damping constant by h. By differentiating eq. (4) with respect to the time t, we obtain:

$$S_E(t,\omega,h) = \int W(u,\omega,h)\,\frac{\partial}{\partial t}\left(\frac{1}{2\pi}|\ddot{Y}_r(u,t)|^2\right)du \quad (5)$$

where $\frac{\partial}{\partial t}\left(\frac{1}{2\pi}|\ddot{Y}_r(u,t)|^2\right)$: Page's instantaneous power spectrum(Ref. 3).

Thus the EIR spectrum is interpreted as an instantaneous power spectrum smoothed along the frequency axis by the set of spectral windows which are *identical* to the set of energy admittances for viscously damped linear oscillators with various natural frequencies and the specified damping constant. Especially in the case of no damping, the EIR spectrum is identical to Page's instantaneous power spectrum, for the following energy admittance function asymptotically becomes Dirac's delta function *as the damping constant* approaches to zero.

$$W(u,\omega,h) = \frac{2h\omega u^2}{(u^2-\omega^2)^2+4h^2\omega^2u^2} \rightarrow \pi/2\cdot\{\delta(u-\omega)+\delta(u+\omega)\} \quad (h\rightarrow 0) \quad (6)$$

In the actual calculation of the EIR spectrum, time histories of velocity responses are calculated in time-domain for the set of viscously damped linear oscillators with various natural frequencies and the specified damping constant, and they are then multiplied by the negative sign times ground acceleration history:

$$S_E(t,\omega,h) = -\dot{x}(t,\omega,h)\,\ddot{y}(t) \quad (7)$$

The EIR spectrum sometimes includes negative spectral values. The physical meaning of such negative values is that the response velocity of the oscillator opposes in the direction of the effective excitation force, and this phenomenon is not strange at all in the sense of deterministic earthquake responses. If a number of sample waves were available, and if ensemble average could be applied to these sample waves, such negative spectral values would disappear. When only one sample wave is given, moving time average techniques can possibly reduce such negative spectral values if desired. In the following examples, the smaller value of three times natural period and 1.5 seconds is adopted for the data length in the moving average.

3. EXAMPLES

Example 1 Figs. 2 (a) to (f) show Kameda's multifilter spectra and the EIR spectra calculated for the N-S component of El Centro (May 1940) with three different damping constants. It is seen that the oscillator damping affects both the smoothing effects along the frequency-axis and the time-lag effects in Kameda's spectra, while such time-lag effects are not induced by the oscillator damping in the EIR spectra.

Example 2 A non-linear oscillator with hysteresis can be used instead of the viscously damped linear oscillator if desired. Fig. 3 shows the EIR spectrum for the N-S component of El Centro calculated with elastic-perfectly plastic oscillators, the yield shear forces of which are set to 30 percent of elastic peak responses. It is seen that the pattern of the damage potential is similar to the pattern in the linear case shown in Fig. 2 (c) except the fact that the damage potential near the first shock is increased for the high frequency region.

Example 3 A long-term observation project on a scaled steel braced frame with intentionally reduced seismic strength is on-going since 1983 in Chiba, Japan (Ref. 5). Figs. 4 (a) to (d) show inelastic responses of the frame to the two moderate earthquakes observed on October 4, 1985 and June 24, 1986. The two earthquakes caused almost the same peak acceleration of ground shaking (about 70 cm/s/s), but the hysteretic behaviors were considerably different. To characterize these two ground shakings, the EIR spectra are calculated for the base acceleration records. The results are shown in Figs. 5 (a) and (b). It is seen that the 1986 excitation has long lasting damage potential enough to supply ample energy into the frame, while the 1985 excitation has a pulse-like damage potential. Additionally, the response velocity histories observed are used as the input signals for the calculation of the EIR spectra. As shown in Figs. 5 (c) and (d), the change of response frequency due to yielding can be well visualized by the same technique used in the evaluation of the seismic damage potential.

4. CONCLUDING REMARKS

A new concept of non-stationary spectral analysis termed "energy input rate spectrum (EIR spectrum)" is introduced to express the rate of the energy input fed by the earthquake ground motion. Features of this spectrum are demonstrated theoretically and by examples as well:

(1) EIR spectrum is mathematically equivalent to Page's instantaneous power spectrum smoothed along the frequency axis.
(2) Fourier transformation techniques are not needed to calculate the EIR spectrum. Velocity responses of viscously damped linear oscillators

are calculated in time-domain and multiplied by the negative sign times original signal. Non-linear oscillator can be used if desired.
(3) Damping constant in EIR spectrum influences only on the smoothness along the frequency axis, and does not induce the time-lag effects.

This spectrum is suitable to express the frequency-time distribution of the seismic damage potential. Furthermore, it has several new features summarized above, and it can be generally applied to the analysis of non-stationary signals.

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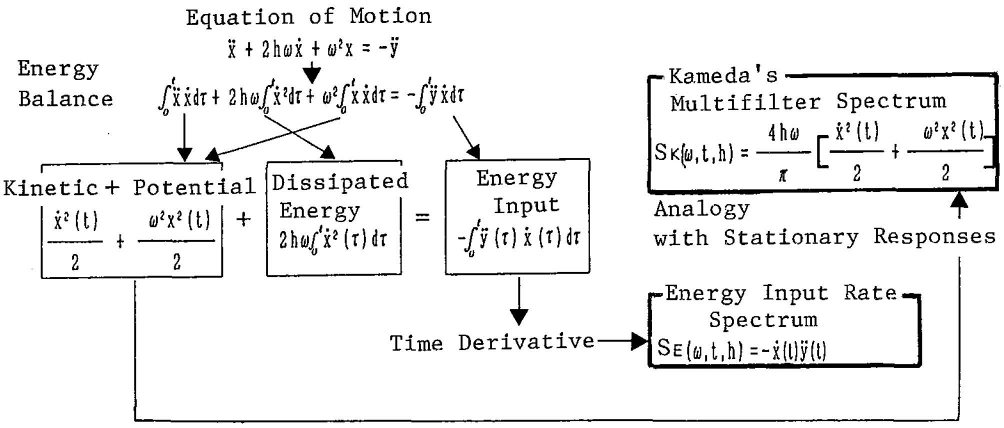


Fig. 1 Formulations of Two Non-stationary Spectra

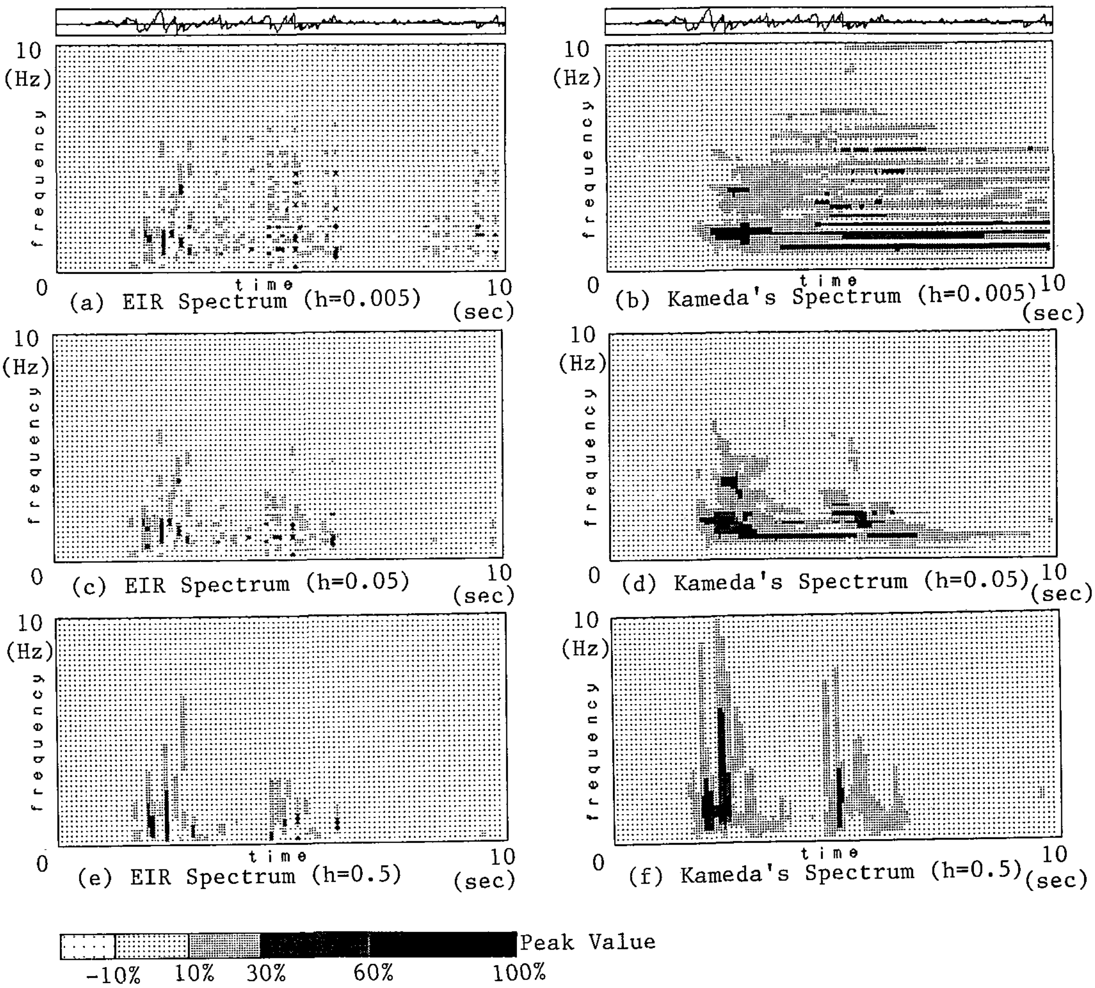


Fig. 2 EIR Spectra and Kameda's Spectra with Three Damping Constants
(El Centro NS, May 1940)

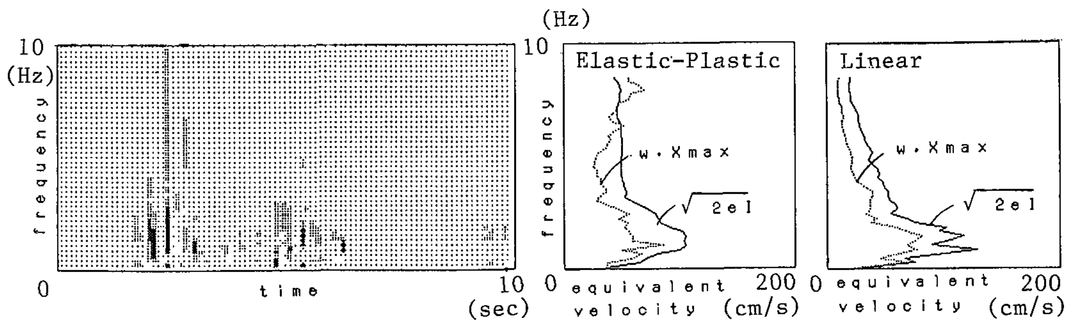
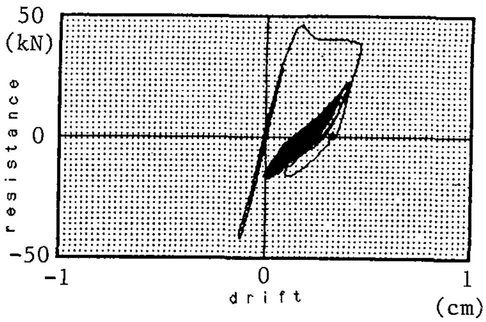
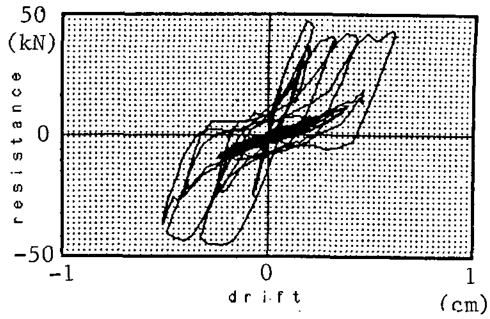


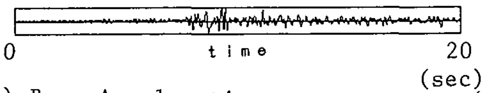
Fig. 3 EIR Spectrum with Elastic-Plastic Oscillators
(El Centro NS, May 1940; Yield Strength = 0.3*Elastic Response; h=0.05)



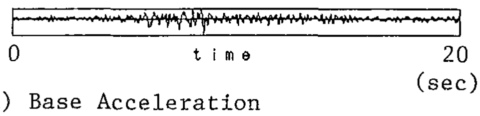
(a) Hysteresis Loop at First Story (Oct. 4, 1985)



(b) Hysteresis Loop at First Story (June 24, 1986)

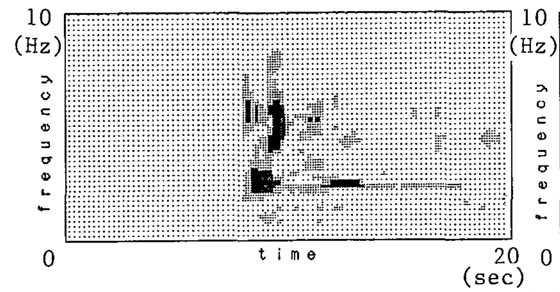


(c) Base Acceleration (Oct. 4, 1985; Peak:71cm/s/s)

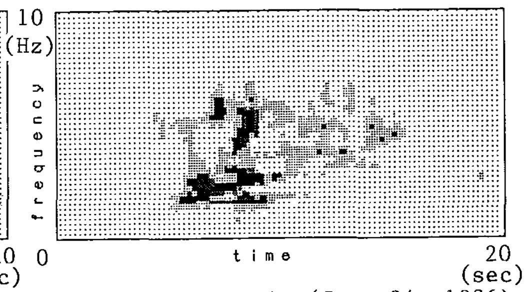


(d) Base Acceleration (June 24, 1986; Peak: 74cm/s/s)

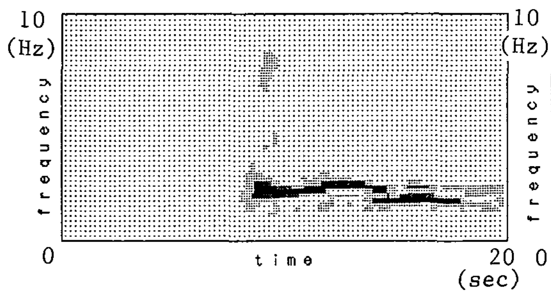
Fig. 4 Inelastic Responses of Steel Braced Frame Observed during Recent Two Earthquakes



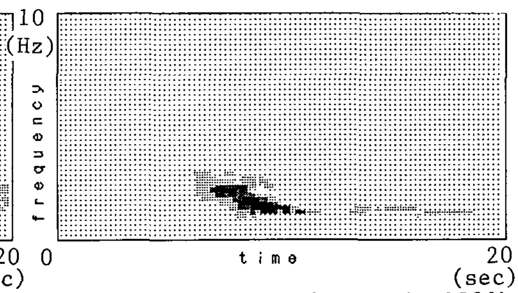
(a) Base Acceleration(Oct.4, 1985)



(b) Base Acceleration(June 24, 1986)



(c) Response Velocity(Oct.4, 1985)



(d) Response Velocity(June 24, 1986)

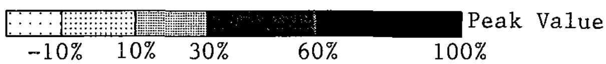


Fig. 5 EIR Spectra(h=0.05) of Base Acceleration and Response Velocity Observed during Recent Two Earthquakes