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EARTHQUAKE RESPONSES OF STRUCTURES WITH SLIDING FLOOR LOADS

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SUMMARY

Sliding of floor loads will occur during earthquake excitation in the case that the floor loads are not fixed on the floor. This causes considerable changes in response behavior of the frame. A theoretical approach and a numerical analysis, which are verified by the experiments, reveal how the response amplified factor is changing according to the variation of a friction coefficient and how the story shear force response reduces their values due to the sliding of the floor load.

INTRODUCTION

The seismic force is determined in proportion to the sum of dead and live loads in usual earthquake resistant design procedure. Sliding of floor loads will occur during earthquake excitations in the case that the floor loads are not fixed on the floor. The sliding of the floor loads may considerably change the frame response. Moreover, in some cases the floor loads collide against the loads nearby and the movement of loads is restrained by stoppers provided on the floor. These behaviors will also influence on the frame response. In this paper, firstly, rigorous solutions are derived from the basic equation of motion for a frame model with a block weight unfixed on the floor of the frame. Secondly, for analyses of rather complex frames with sliding blocks, a numerical method is proposed. These analyses are carried out by computer. The preciseness of computed results is proved by the experimental results which have been obtained by shaking table tests on a simple one-story frame model and a three-story frame model with a sliding block on each floor. Finally, response behavior of frames with sliding floor loads during earthquake is presented with the results of the computer analyses.

FUNDAMENTAL BEHAVIOR

A simple structural model as shown in Fig.1 was taken for a rigorous analysis. The equations of motion for this system can be expressed as

$$M(\ddot{X}+\ddot{Y})+C\dot{X}+KX-sgn(\dot{U})F = 0 \qquad (1.1)$$

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$$m(\ddot{U}+\ddot{X}+\ddot{Y})+sgn(\dot{U})F = 0 \tag{1.2}$$

where M : The mass of the frame
m : The mass of a block on the frame
$\ddot{X}$: Acceleration of the frame
$\dot{X}$: Velocity of the frame
X : Displacement of the frame
$\ddot{U}$: Acceleration of a block relative to the frame
$\dot{U}$: Velocity of a block relative to the frame
C : Damping coefficient of the frame
K : Stiffness of the frame
F : Friction force between the floor of the frame and the block
sgn( ): Signum function

In the Eqs.(1.1) and (1.2), the dynamic friction coefficient is assumed to be equal to the static friction coefficient.

In a sinusoidal excitation, if we consider only the period in which $\dot{U}\leqq 0$, Eqs.(1.1) and (1.2) can be expressed in the following forms:

$$\ddot{X}_R+2\omega_n h_n \dot{X}_R+\omega_n^2 X_R = -A\cos(\omega t+\varphi) \tag{1.3}$$

$$\ddot{U}-2\omega_n h_n \dot{X}_R+\omega_n^2 X_R-\frac{F}{m} = 0 \tag{1.4}$$

where $X_R=X+\frac{F}{K}$

$\omega_n^2=\frac{K}{M}$

$2\omega_n h_n=\frac{C}{M}$

$\ddot{Y}=A\cos(\omega t+\varphi)$

The general solutions of Eqs.(1.3) and (1.4) are

$$X_R = e^{-h_n\omega_n t}\{C_1\cos(\omega_n\sqrt{1-h_n^2}\cdot t)+C_2\sin(\omega_n\sqrt{1-h_n^2}\cdot t)\}$$
$$-\frac{A}{\omega_n^2}\cdot\frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+\frac{4h_n^2\omega^2}{\omega_n^2}}}\cos(\omega t+\varphi-\psi) \tag{1.5}$$

$$U = C_3+C_4 t+\frac{F}{2m}t^2+\frac{A}{\omega^2}\cos(\omega t+\varphi)-X_R \tag{1.6}$$

where $\psi = \tan^{-1}\left(\frac{2h_n\omega_n\omega}{\omega_n^2-\omega^2}\right)$

The integral constants in the above equations are determined according to the initial conditions. Two cases are considered here as the initial conditions :

For the case where no halt exists during a half period,

$$t=0\ :\ U=U_0\ ,\ \dot{U}=0\ ,\ X_R=X_0+\frac{F}{K}\ ,\ \dot{X}_R=\dot{X}_0\ ,\quad \ddot{X}+\ddot{Y}\geqq\mu\cdot g \tag{1.7}$$

$$t=\frac{\pi}{\omega}\ :\ U=-U_0\ ,\ \dot{U}=0\ ,\ X_R=-X_0+\frac{F}{K}\ ,\ \dot{X}_R=-\dot{X}_0 \tag{1.8}$$

where U_0: Maximum displacement of the block relative to the frame
X_0: Displacement of the frame at the time t=0
$\dot{X}_0$: Velocity of the frame at the time t=0

For the case where a halt exists within a half period, say, at $t=\alpha\frac{\pi}{\omega}(0<\alpha<1)$

$$t=0\ :\ U=U_0\ ,\ \dot{U}=0\ ,\ X_R=X_0+\frac{F}{K}\ ,\ \dot{X}_R=\dot{X}_0\ ,\quad \ddot{X}+\ddot{Y}=\frac{F}{m} \tag{1.9}$$

$$t=\alpha\frac{\pi}{\omega}\ :\ U=-U_0\ ,\ \dot{U}=0\ ,\ X_R=X_1+\frac{F}{K}\ ,\ \dot{X}_R=\dot{X}_1 \tag{1.10}$$

where X_1: Displacement of the frame at the time $t=\alpha\frac{\pi}{\omega}$
$\dot{X}_1$: Velocity of the frame at the time $t=\alpha\frac{\pi}{\omega}$

The equation of motion is derived when there is no occurrence of sliding.

$$\ddot{X}+2\omega_n h_n\zeta^2\dot{X}+\omega_n^2\zeta^2X=-A\cos(\omega t+\varphi) \tag{1.11}$$

where $\zeta=\sqrt{\frac{M}{M+m}}$

The solution of Eq.(1.11) is:

$$X=e^{-h_n\omega_n\zeta^2t}\{E_1\cos(\sqrt{1-h_n^2\zeta^2}\cdot\omega_n\zeta t)+E_2\sin(\sqrt{1-h_n^2\zeta^2}\cdot\omega_n\zeta t)\}$$
$$-\frac{A}{\omega_n^2\zeta^2}\cdot\frac{1}{H_2}\cos(\omega t+\varphi-\psi_2) \tag{1.12}$$

where
$$H_2=\sqrt{(1-\frac{\omega^2}{\omega_n^2\zeta^2})^2+(\frac{2h_n\omega}{\omega_n})^2}$$
$$\psi_2=\tan^{-1}(\frac{2h_n\omega_n\omega\zeta^2}{\omega_n\zeta^2-\omega^2})$$

The initial conditions for this case are:

$$t=\alpha\frac{\pi}{\omega}\ :\quad X=X_1\ ,\ \dot{X}=\dot{X}_1 \tag{1.13}$$

$$t=\frac{\pi}{\omega}\ :\quad X=-X_0\ ,\ \dot{X}=-\dot{X}_0\ ,\ \ddot{X}+\ddot{Y}=-\frac{F}{m} \tag{1.14}$$

As an example of the analysis mentioned above, the response amplified factors were obtained for various friction coefficients. The results of the analyses are shown in Fig.2. The natural frequencies in the abscissa were calculated for frames with fixed blocks. The fundamental response behavior of the system is known by this figure.

NUMERICAL ANALYSIS

A numerical analysis was developed in order to analyze response behavior of complex structural systems under random vibration such as earthquake excitations. A structural model for the analysis described here is shown in Fig. 3. The frame structure is idealized as a lumped mass system. A lumped mass is imagined at each floor level, where a block idealized as a sliding floor load is placed. Friction behavior between the floor and the block is characterized by specified friction coefficients μ_{si} for the static and μ_{di} for the dynamic. Two stoppers are installed at each floor. The space distances between the stoppers and the block can be adjusted according to a prescribed analysis.

The equation of motion describing the response behavior of the frame system and the blocks on the floors are shown in Eqs.(2.1) and (2.2).

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+\{R\}-\{F\}=-[M]\{1\}\ddot{Y} \tag{2.1}$$

$$[m]\{\ddot{U}+\ddot{X}\}+\{F\}=-[m]\{1\}\ddot{Y} \tag{2.2}$$

where $[M]$ is a mass matrix of the floor masses, $[m]$ a mass matrix of the blocks, $[C]$ a damping coefficient matrix, $\{F\}$ the friction force vector, $\{R\}$ the restoring force vector $=[K]\{X\}$ for an elastic system, $[K]$ the elastic stiffness matrix, and $\{\ddot{X}\}$,$\{\dot{X}\}$,$\{X\}$ are the response acceleration, the response velocity and the response displacement, respectively. Finally, $\ddot{Y}$ means a scholar value of the ground acceleration. By using the central difference method, the response values $\{X_n\}$ and $\{U_n\}$ at the time $t=n\cdot\Delta t$, where n is the number of time steps and Δt is the time increment, can be represented as:

$$\{X_n\} = 2\{X_{n-1}\}-\{X_{n-2}\}+\{\ddot{X}_{n-1}\}\Delta t^2 \tag{2.3}$$

$$\{U_n\} = \{U_{n-1}\}+\{\dot{U}_{n-\frac{1}{2}}\}\Delta t \tag{2.4}$$

Then,

$$\{\ddot{X}_n\} = \left([M]+\frac{\Delta t}{2}[C]\right)^{-1}\cdot\left(\{F_n\}-[M]\{1\}\ddot{Y} -\frac{1}{\Delta t}[C]\{X_n-X_{n-1}\}-\{R_n\}\right) \tag{2.5}$$

$$\{\ddot{U}_n\} = -\left(\{1\}\ddot{Y}_n+\{\ddot{X}_n\}+[m]^{-1}\{F_n\}\right) \tag{2.6}$$

$$\{\dot{U}_{n+\frac{1}{2}}\} = \{\dot{U}_{n-\frac{1}{2}}\}+\{\ddot{U}_n\}\Delta t \tag{2.7}$$

The element F_i of the vector $\{F_n\}$, that is, the friction force at the0 i-th floor, is expressed as:

Ⅰ. $F_i=-m_i(\ddot{X}_i+\ddot{Y})$ for the halt state (2.8)

Ⅱ. $F_i=sgn(\dot{U}_i)\cdot m_i\cdot\mu_d\cdot g$ for the slide state (2.9)

The change of the states is recognized by:

$|\ddot{X}_i+\ddot{Y}| > \mu_{si}\cdot g$ *for* Ⅰ→Ⅱ (2.10)

$$\dot{U}_i=0 \quad and \quad |\ddot{X}_i+\ddot{Y}| \leq \mu_{si}\cdot g \qquad for \qquad \text{II} \rightarrow \text{I} \tag{2.11}$$

The collision of the block against the stoppers is also considered simply. The collision is assumed to take place in an infinitely short time so that the friction force, the restoring force and the damping force do not affect the repulsion force. Then the repulsion force can be calculated by the assumption of a simple repulsion of two bodies. Fig.4 shows such a case. The two bodies change their velocities before and after the collision, namely, from C_1 to $-V_1$ for the body with the mass m_1 and from $-C_2$ to V_2 for the body with the mass m_2. V_1 and V_2 can be calculated by the following equations:

$$V_1 = \frac{m_1C_1+m_2C_2}{m_1+m_2}(1+\varepsilon)-\varepsilon C_1 \tag{2.12}$$

$$V_2 = \frac{m_1C_1+m_2C_2}{m_1+m_2}(1+\varepsilon)-\varepsilon C_2 \tag{2.13}$$

where ε is the repulsion coefficient.

Applying these to the frame with the sliding blocks, the velocity change of the frame and the block can be calculated by:

$$\dot{X}_e = \dot{X}_s+\frac{m}{M+m}\dot{U}_s(1+\varepsilon) \tag{2.14}$$

$$\dot{U}_e = -\varepsilon\dot{U}_s \tag{2.15}$$

where $\dot{X}_s$ and $\dot{X}_e$ are the horizontal velocities of the frame before and after the collision, respectively, and $\dot{U}_s$ and $\dot{U}_e$ are the velocities of the block before and after the collision.

The calculation procedures are thus summarized:

1) The response displacement of the frame and the relative displacement of the block to the floor at the time $t=n\cdot\Delta t$ are calculated by Eqs.(2.3), (2.4) with the values at the previous step. If collision is detected from the calculated relative displacement of the block to the floor, the required calculation is carried out.
2) According to the calculated value $\{X_n\}$, the restoring force vector at the time $t=n\cdot\Delta t$, $\{R_n\}$ is evaluated.
3) The values of $\{\dot{X}_n\}$, $\{\dot{U}_n\}$ and $\{\dot{U}_{n+\frac{1}{2}}\}$ are calculated by Eqs.(2.5),(2.6) and (2.7). In this calculation, the friction value $\{F_n\}$ is assumed.
4) By the velocity condition at the time t=n.dt, the actual friction force $\{F_n\}'$ can be determined. $\{F_n\}'$ must be compared with the previously assumed $\{F_n\}$. In case the coincidence between these two values is not obtained, then the above procedure must be repeated until the coincidence is attained.

VERIFICATION BY EXPERIMENTS

Shaking table tests on a single story frame model and a three story frame model were carried out, in order to verify the adequateness of the numerical analysis described previously and to examine the preciseness of the calculation results. Two types of the shaking table tests were done; resonance tests and response tests to earthquake waveform excitations. The structural models are shown in Fig.5. The weights of the floors and the blocks are summarized with the friction coefficients presented in Table 1. The natural frequencies and the damping ratios are shown in Table 2.

The results of the resonance tests are shown in Figs. 6 and 7. In Fig. 6, the same behavior as in Fig.2 can be observed. Moreover, the second peak in the low frequency range could be detected. Existence of the second peak has been anticipated in the past literature.

The experimental results are compared with calculation results which have been obtained by the previous numerical analysis in Figs.8 and 9. Agreement in these two sets of values is pretty good. It can be said that the adequateness of the numerical analysis is fully verified by the experiments.

The space distance between the block and the stoppers influences on the response displacement of the frame. Figs.10 and 11 show that the displacement reduces its value as the space distance increases. The space distances can be determined from these data in design procedure.

SOME RESULTS FOR DESIGN

It was observed that the response story shears of the frames were considerably reduced in case the blocks on the floors slide during vibration. From the viewpoint of frame design, this evidence encourages designers to estimate design seismic loads smaller than the values in actual design procedure. In another expression, design live loads can be estimated as being lower than the actual values. Several analyses were carried out to examine this evidence. In Figs.12 and 13, calculated story shear forces are expressed against the fundamental period of frames in the cases of various friction coefficients and weight ratios of blocks and floors. In these two figures, the values in the vertical axis show the equivalent response accelerations defined as:

$$A_i = \ddot{X}_i + \ddot{Y} + \frac{\ddot{U}_i m_i}{M_i + m_i} \qquad (4.1)$$

where A_i is the equivalent response acceleration of the i-th floor, $\ddot{X}_i$ the response acceleration of the i-th floor, $\ddot{Y}$ the ground acceleration, $\ddot{U}_i$ the acceleration of the block relative to the i-th floor, and M_i and m_i are the masses of the i-th floor and the block on it, respectively. If the total mass (M_i+m_i) is multiplied by A_i, the outcome shows the story shear actually occurred during the vibration.

From these spectra for the waveforms of acceleration recorded in past earthquakes, the equivalent value A_i is considerably reduced in smaller period range, say, less than 1.2 second. The reduction rate, however, is strongly dependent on the values of friction coefficients and the ratios of the masses. The design acceleration values, therefore, must be carefully determined with reference to the analytical data.

CONCLUDING REMARKS

Concludingly, the results obtained in this paper can be summarized as follows:

1) Vibration behaviors of a frame with sliding live loads were rigorously analyzed for sinusoidal excitations. Fundamental properties of the frame were known through the analyses.
2) A numerical analysis method was developed in order to obtain various response behaviors of frames with sliding live loads under earthquake excitations.
3) The numerical analysis was verified by shaking table tests on a single story and a three story frame model. The analytical results coincided the experimental results very well.
4) It was examined that response story shear forces of frames reduce their values considerably in case live loads on the floors slide during vibration. This evidence suggests that the live loads can be estimated lower than the actual values in seismic design procedures.

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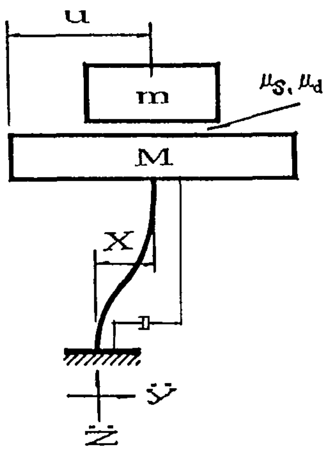


Fig.1 Single story frame model

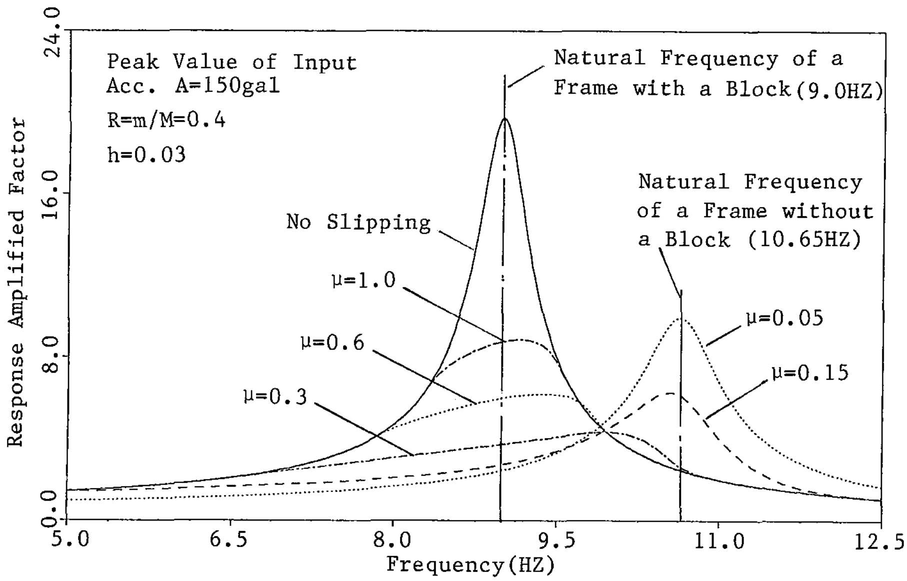


Fig.2 Response amplified factor

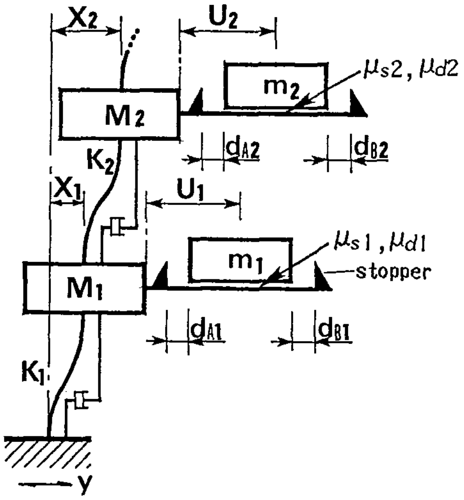


Fig.3 Frame model for numerical analyses

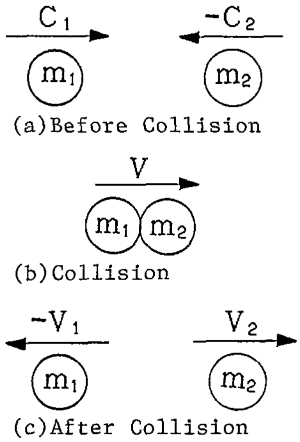
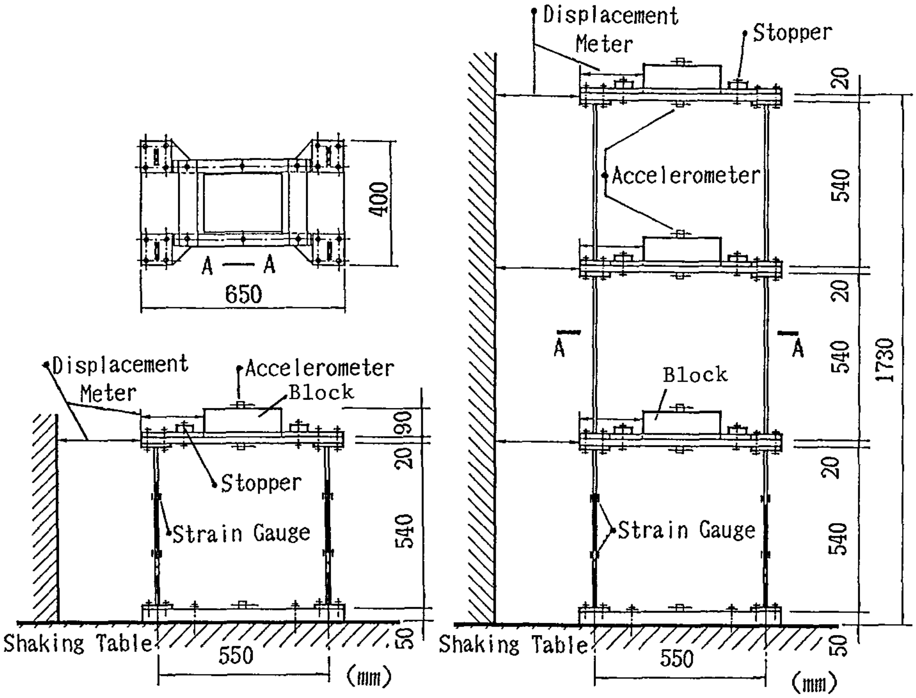


Fig.4 Simple repulsion of two bodies



(a) A single story frame

(b) A three story frame

Fig.5 Structural models for experiments

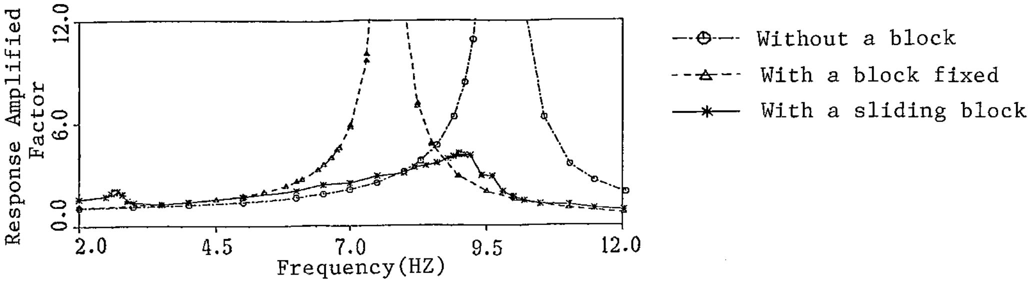
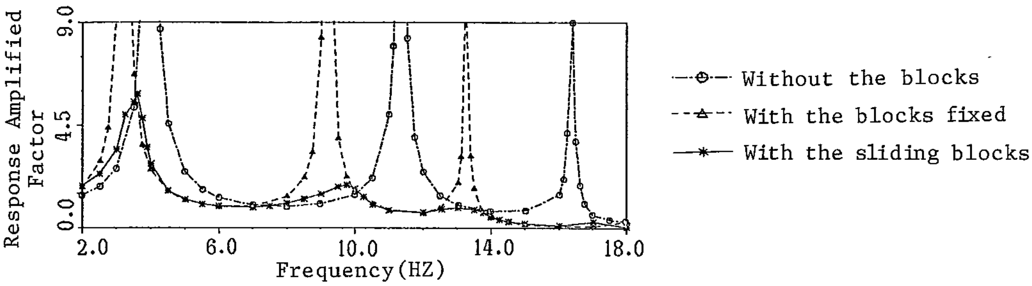
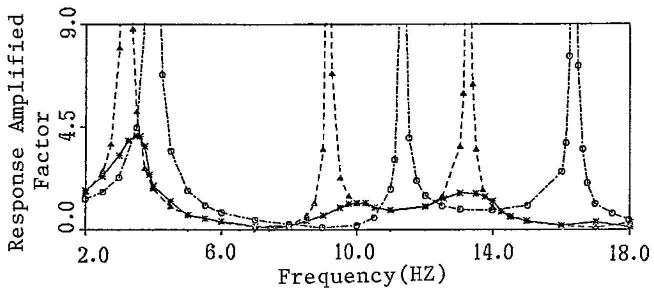


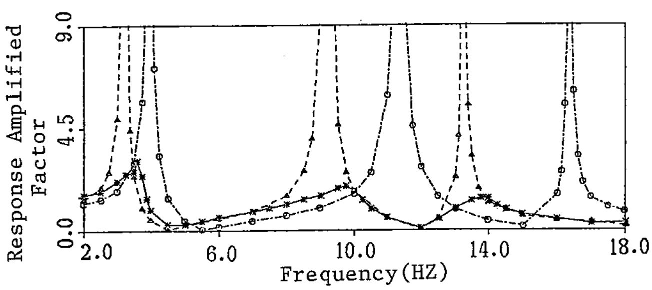
Fig.6 Response amplified factor of a single story frame for sinusoidal acceleration with the peak value of 150gal



(a) Top Floor

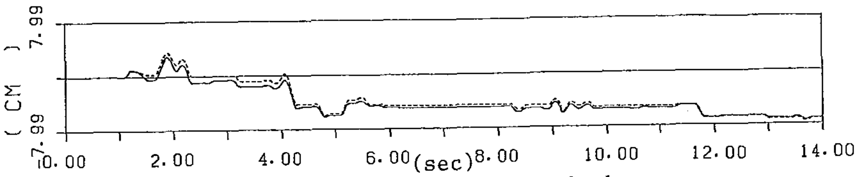


(b) Second Floor

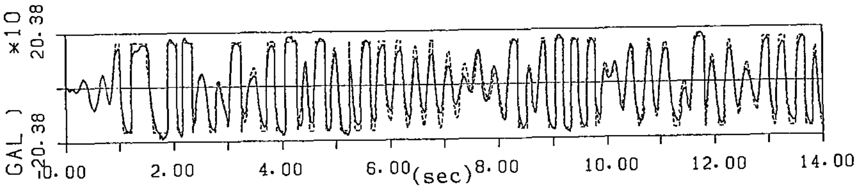


(c) First Floor

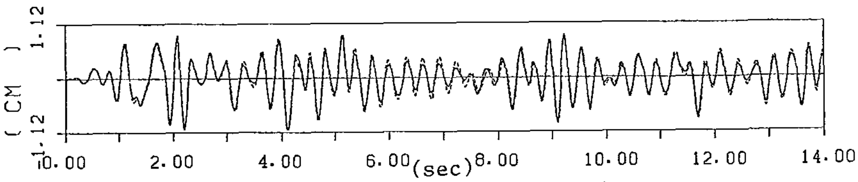
Fig.7 Response amplitude factor of a three story frame for sinusoidal acceleration with the peak value of 80gal



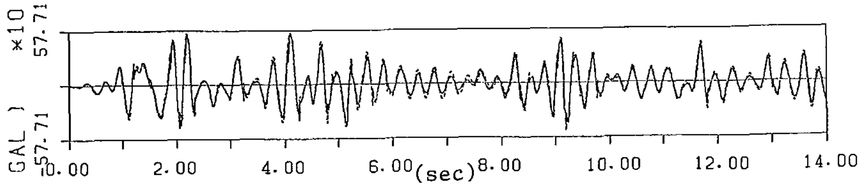
(a) Slide Displacement of a Block



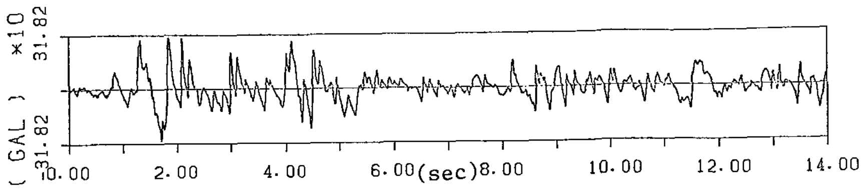
(b) Response Acceleration of a Block



(c) Response Displacement of a Floor



(d) Response Acceleration of a Floor



(e) Input Acceleration

Solid : Experiment
Dashed : Calculation

Fig.8 A single story frame subjected to El Centro(NS) with the peak value of 300gal

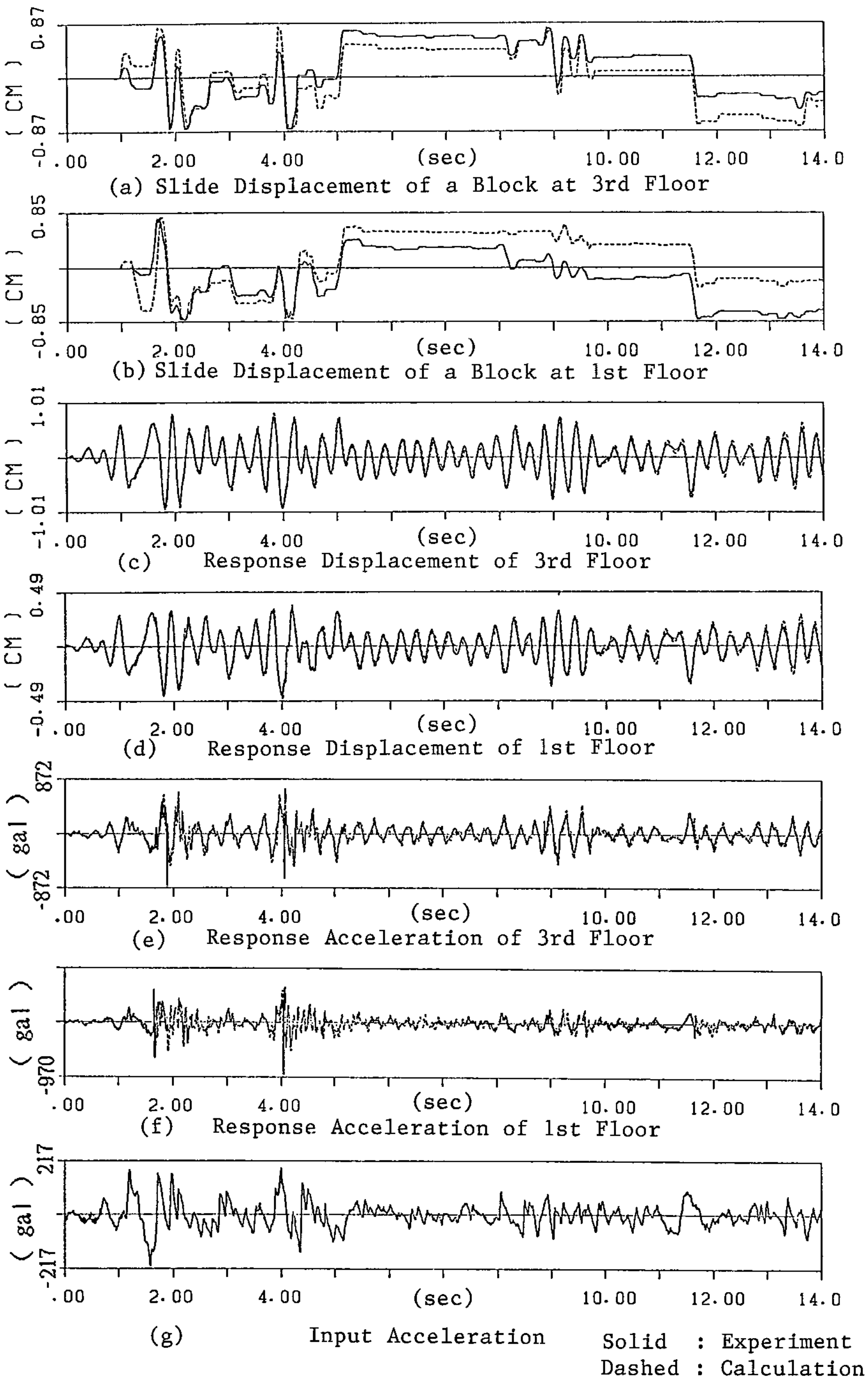


Fig.9 A three story frame subjected to El Centro(NS) with the peak value of 200gal

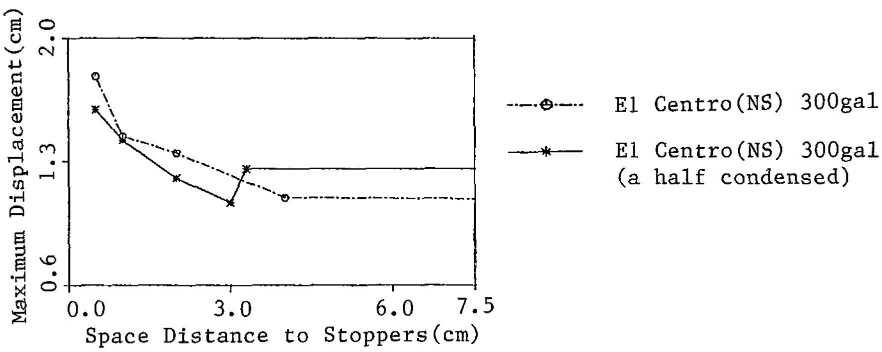


Fig.10 Maximum response story displacement vs. space distance at a single story frame

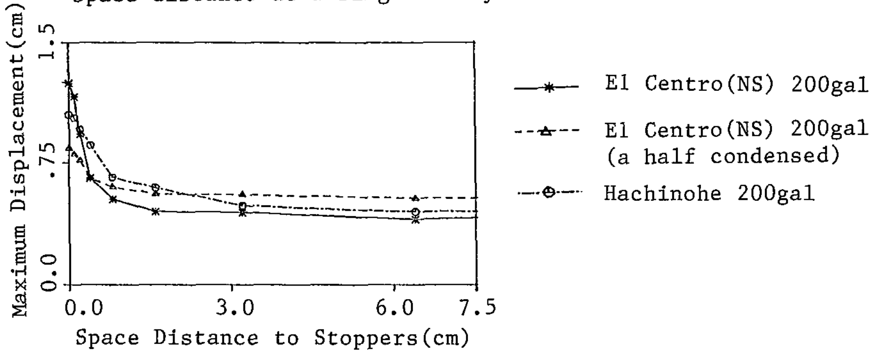


Fig.11 Maximum response story displacement vs. space distance at a 3 story frame

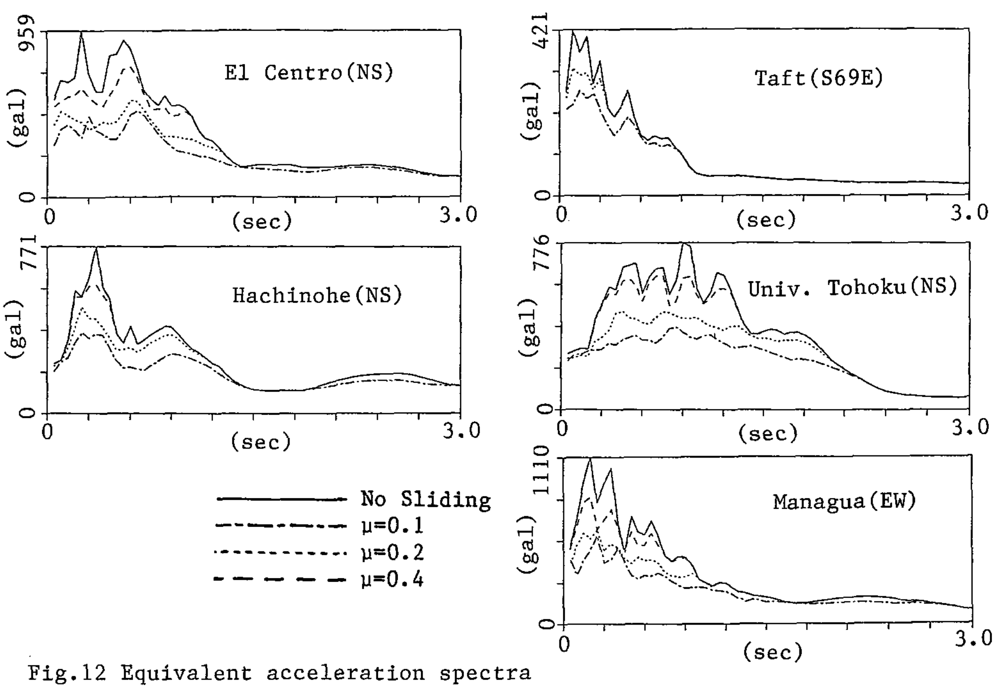


Fig.12 Equivalent acceleration spectra

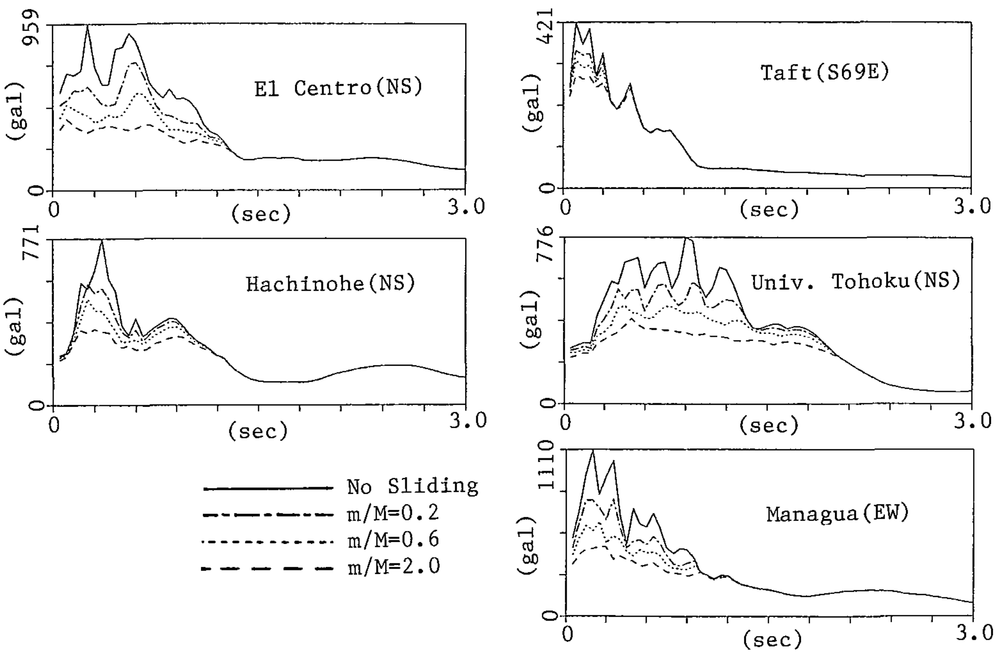


Fig.13 Equivalent acceleration spectra

Table 1 Weights of floor and blocks, and Friction coefficients

		Weights of Floors(kgf)	Weights of Blocks(kgf)	Ratios of Weights	Friction Coeff.
Frame in Fig.6		58.3	31.6	0.542	0.21
Frame in Fig.7	1Fl.	64.8	31.6	0.488	0.11
	2Fl.	64.8	31.6	0.488	0.16
	3Fl.	58.3	31.6	0.542	0.17

Table 2 Natural frequencies and damping ratios

		Without Block		With Block Fixed	
		Frequencies (HZ)	Damping Ratios	Frequencies (HZ)	Damping Ratios
Frame in Fig.6		9.69	0.002	7.69	.003
Frame in Fig.7	1st	3.94	0.007	3.19	.005
	2nd	11.4	0.005	9.14	.003
	3rd	16.4	0.005	13.2	.003