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NUMERICAL AND EXPERIMENTAL STUDIES ON THE CRUSHING OF TUBES
(FURTHER REPORT)

by

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1. INTRODUCTION

Steel tubes are often used as energy dissipation devices in various situations. The present paper is a further report of the numerical and experimental study on the crushing behavior of steel tubes under quasi-static loading [1]. The following problems are treated in this report:

(i) Toroidal shells under lateral compression
(ii) Circular pipes under concentrated lateral loading
(iii) Thick circular tubes under axial compression

As in Ref. [1], nonlinear finite element method using low-order elements based on the reduced integration technique is employed for numerical calculations, and the obtained solutions are compared with experimental results in order to estimate the validity of the finite element crush analysis.

2. OUTLINES OF THE DEVELOPED FINITE ELEMENT CODES [2, 3]

In the first report [1], the finite element codes for the crush analysis of one-dimensional frame-like structures and general thin shell structures were developed. In the present study, the one-dimensional code has been extended to treat axisymmetric thin shells, and a finite element code for three-dimensional axisymmetric analysis has been newly developed, based on the following algorithm [3]:

(i) Bilinear axisymmetric solid element with one-point quadrature
(ii) Incremental theory by the updated Lagrangian approach with Jaumann stress increments
(iii) Orthogonal hourglass control

3. TOROIDAL SHELLS UNDER LATERAL COMPRESSION [2]

In Ref. [4], the crushing behaviors of two types (a-type and b-type shown in Fig. 1) of toroidal shells under lateral compression are studied in order to estimate the utility of those as shock absorbers

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for radioactive materials' shipping casks, where numerical results by the incremental Rayleigh-Ritz method are compared with the experimental results.

In the present study the axisymmetric finite element method has been applied to the same problems. The upper half of the a-type torus and the whole structure of the b-type torus were respectively subdivided with 120 uniform elements. Both sliding and sticking conditions were assumed at the contact points between the loading slab and the a-type torus. The dimensions and the material constants for each specimen are as follows:

For the a-type torus

$$R_L = 57.3\ (\mathrm{mm}),\quad R_S = 21.6\ (\mathrm{mm})$$
$$T = 3.3\ (\mathrm{mm}),\quad E = 20102\ (\mathrm{kg/mm^2})$$
$$\sigma_Y = 34.69\ (\mathrm{kg/mm^2})$$
$$H' = 88.78\ (\mathrm{kg/mm^2})$$

For the b-type torus

$$R_L = 57.4\ (\mathrm{mm}),\quad R_S = 22.2\ (\mathrm{mm})$$
$$T = 2.5\ (\mathrm{mm}),\quad E = 20102\ (\mathrm{kg/mm^2})$$
$$\sigma_Y = 38.78\ (\mathrm{kg/mm^2})$$
$$H' = 58.16\ (\mathrm{kg/mm^2})$$

Fig. 6 and Fig. 7 show the results of the load-displacement curve and the crushing deformation for each-type specimen respectively. Undoubtedly the present analyses under the assumption of free sliding give considerably improved solutions in comparison with the incremental Rayleigh-Ritz solutions obtained in Ref. [4]. The stiffer solutions given by the·incremental Rayleigh-Ritz method, in comparison with the finite element solutions, is due to the assumed displacement field with substantially smaller number of degrees of freedom and the neglect of the membrane deformation.

4. CIRCULAR PIPES UNDER CONCENTRATED LATERAL LOADING [2]

In this section, the crushing strength of circular tubes under concentrated lateral loading is studied both form numerical and experimental points of view. This problem is related to the estimation of collision damage of tubular members in steel offshore structures [5]. The tested circular pipe has a length of 1000 (mm), a thickness of 1.571 (mm) and a diameter of 101.72 (mm).

Fig. 4 shows the collapsing process of the circular pipe subjected to a lateral load by the loading plate. Both ends of the cylinder are clamped and free to axial displacements. The local crumpling at the central section occurs at first, which is followed by the overall bending in the second stage, and finally the local buckling occurring on the compression sides near both ends causes the loss of load bearing capacity of the specimen.

One quadrant of the cylinder was subdivided with 660 elements as shown in Fig. 5. The contact between the loading plate and the cylinder is taken into account by the penalty function method in this analysis. Free sliding is assumed in the tangential direction. Based on the material test result, bilinearly approximated stress-strain relation has been used, in which the yield stress and the tangent modulus after yielding are assumed as σ_y = 31.0 (kg/mm^2) and E_t = 0.0031E (E = 21000 kg/mm^2), respectively.

The calculated load-deflection curves are compared with the experimental results in Fig. 6, where δ_u and δ_l are the central deflection of the upper surface and the lower surface of the cylinder respectively. Fig. 7 shows the calculated deformations, in which the occurrence of the local buckling on the compression side of both ends can be clearly seen.

5. THICK CIRCULAR TUBES UNDER AXIAL COMPRESSION [3]

The axisymmetric crushing behaviors of thick circular cylinders under axial compression have been studied both numerically and experimentally. Numerical calculations have been conducted by using the three-dimensional, axisymmetric finite element code, in which only a periodic unit area has been analyzed as in the cases of non-axisymmetric crushing treated in the first report.

Fig. 8 shows numerical and experimental crushing deformations of the cylinder which has a radius-to-thickness ratio (R/t) of 4.88. In the numerical calculation, the axial half wavelength is determined by the tangent-modulus plastic buckling theory. Both results are similar, and severe distortion of the cross section due to shearing deformation can be clearly observed in the numerical result.

In Fig. 9 and Fig. 10, the calculated and experimental mean crushing stresses and the experimental crushing strokes (δ) for axial half wavelength (λ) are respectively compared with some theoretical and empirical formulas [6]. In these figures the experimental results for three cylinders which have radius-to-thickness ratios of 4.88, 11.09 and 24.62 are plotted. It can be seen from these figures that the modified version of the Alexander's rigid-plastic solution [7] is the best both for the mean crushing strength and the crushing stroke as was insisted in Ref. [6].

It should be noted here that the numerical solutions for relatively thick cylinders calculated by using solid elements are higher than the experimental values, which is an opposite tendency to the results obtained by using thin shell elements [1]. This fact proves that the error observed in the analysis of axially compressed circular and square tubes which was conducted in Ref. [1] is partly due to the assumption of thin-walled theory.

6. CONCLUDING REMARKS

Further results of the numerical and experimental study on the crushing behavior of steel tubes have been presented in this report. The obtained results can be summarized as follows:

(1) For toroidal shells subjected to lateral loading, improved solutions have been obtained by the finite element analysis, in comparison with the existing incremental Rayleigh-Ritz solutions.

(2) The finite element analysis of a circular pipe subjected to a concentrated lateral loading has given the solution, which corresponds quite well with the experimental results up to details including local

buckling deformations.

(3) From the numerical results obtained for axially compressed circular cylinders by using axisymmetric solid elements, the validity of the modified Alexander's solution has been reconfirmed. And it has also been shown that the errors between calculations and experiments observed for axially compressed tubes in Ref. [1] is partly due to the thin-walled theory assumed in the finite element analysis.

The crush analysis of integrated structures is not necessarily straightforward, because it requires a great amount of computing cost which is often unacceptable from a practical point of view. However, considering rapid progresses both of hardware and software in computational mechanics, it might not be so far that the finite element method will become a truly practical tool in this field, providing that further accumulations of the detailed comparisons between numerical and experimental results are necessary even for simple structural elements in order to estimate and establish reliability of the finite element crush analysis.

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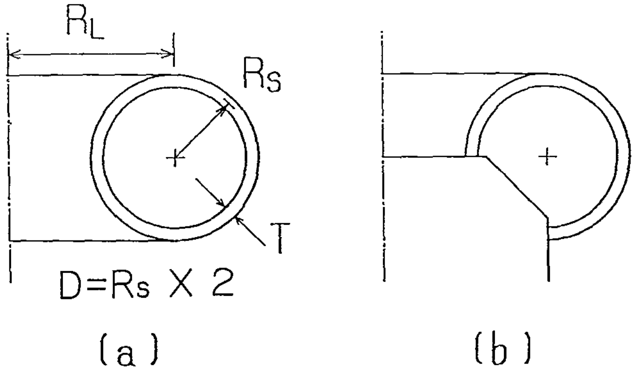
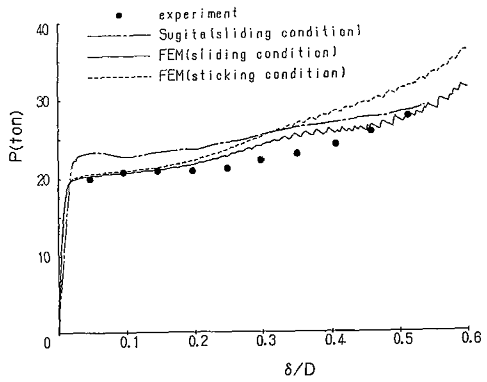
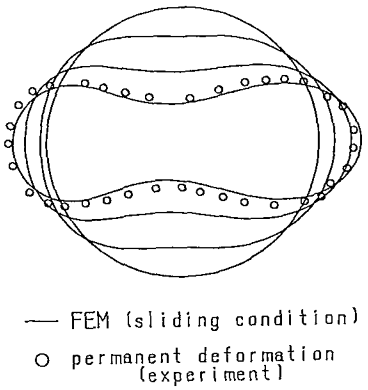


Fig. 1 Two types of toroidal shells under lateral compression

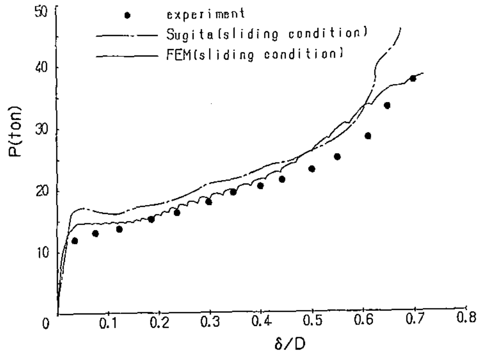


(a) load-displacement curves

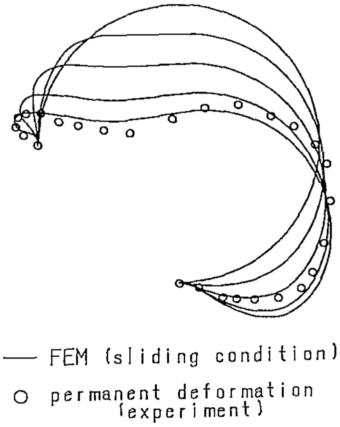


(b) crushing deformations

Fig. 2 Crushing behavior of a-type torus under lateral compression



(a) load-displacement curves



(b) crushing deformations

Fig. 3 Crushing behavior of b-type torus under lateral compression

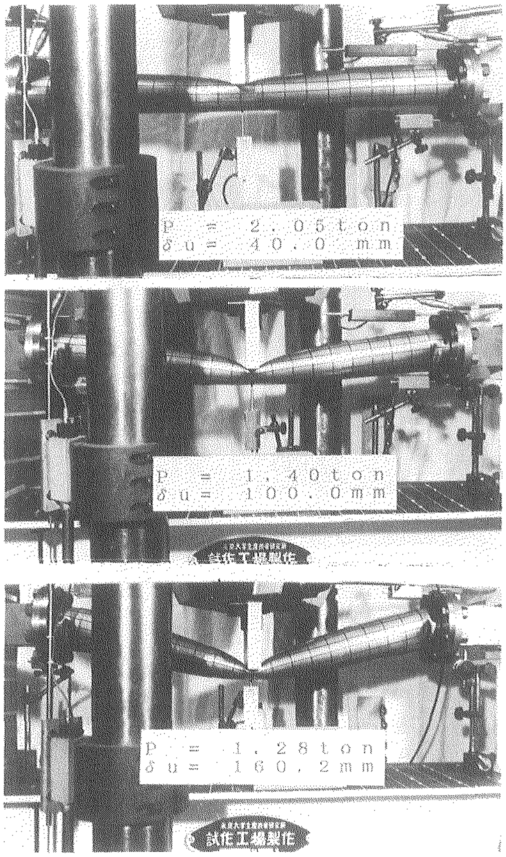
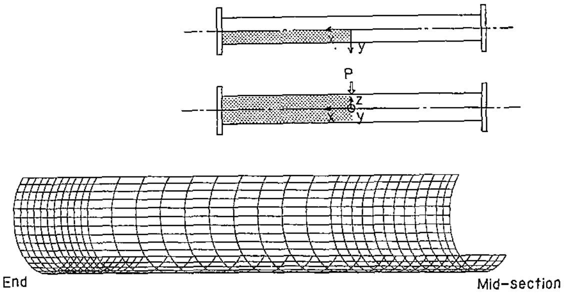


Fig. 4 Experimental collapse behavior of a circular pipe under concentrated lateral loading



1/4 analysis
Number of elements : 660
Number of nodes : 714 + 3(loading plate)
Number of degrees of freedom : 4284 + 3(loading plate)

Fig. 5 Mesh subdivision for a quadrant of the circular pipe

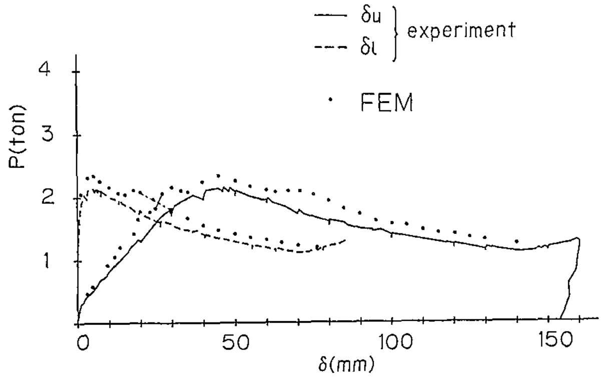


Fig. 6 Numerical and experimental load-displacement curves

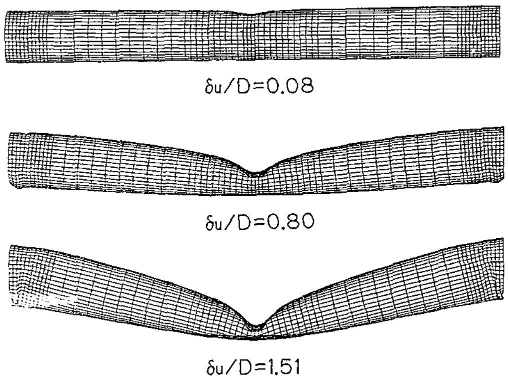
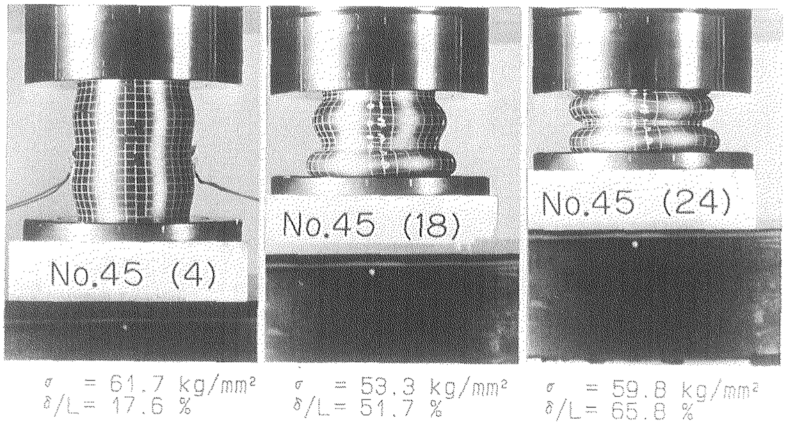
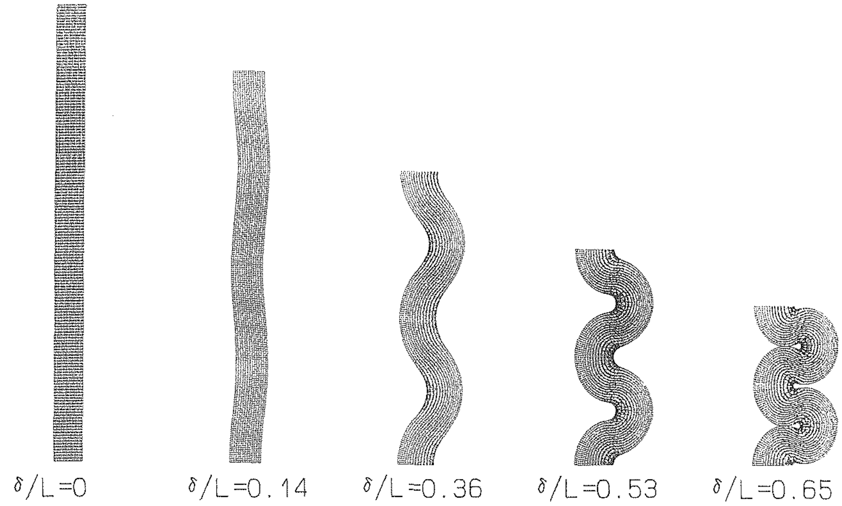


Fig. 7 Calculated collapse behavior of the circular pipe



(a) experiment



(b) finite element analysis

Fig. 8 Numerical and experimental crushing deformations of a circular tube under axial compression

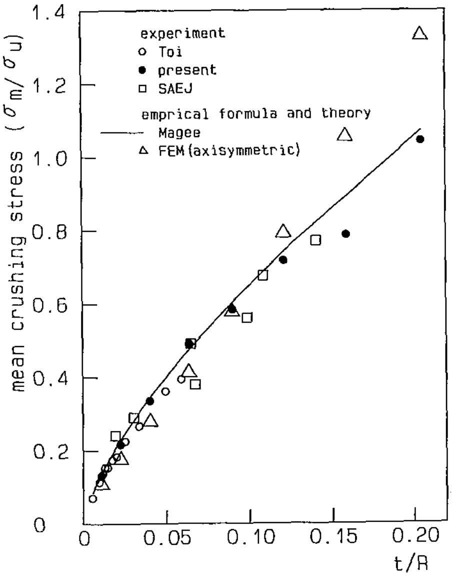


Fig. 9 Mean stresses for axisymmetric crushing of circular tubes under axial compression

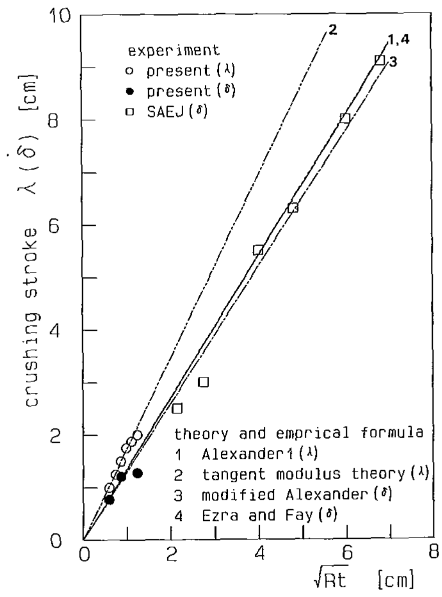


Fig. 10 Axial wavelength for axisymmetric crushing of circular tubes under axial compression