

Anchored Tank Fundamental Period Approximation
Employing Non-Uniform Shell Thickness

by: George C. Manos *

Introduction

During the past two decades following the Great Alaska and the Nigata earthquakes, the response of ground supported liquid storage tanks subjected to ground motions has drawn considerable attention. Important analytical studies have been published related to the response of tanks anchored at their base and significant experimental work was produced to verify the analysis as well as to investigate areas not yet amenable to analytical treatment. The most refined of the analytical studies use finite elements to discretise both the structure as well as the liquid thus taking into account the important effect of the wall flexibility. Recent work (ref. 4) simulates the coupling between the tank and the foundation employing linear springs at the base of the tank. Based on this analytical research effort simplified design charts have been developed by Housner and Haroun, by Balendra et.al. and simplified approximate formulas have been derived by Sakai et.al. and Shimizu et.al. for predicting impulsive and convective response parameters to be used in design. However, as discussed in reference 7, although the predictions of the convective response parameters seem to agree very well with linear convective response observations, the agreement between predicted and observed impulsive response of anchored tanks is not always of the same satisfactory degree (Fig. 1, Tables I,II); it is believed that the major influence for these differences comes from the base fixity and from the structure-foundation interaction, which will not be discussed further here. This paper deals with the influence of the shell thickness non-uniformity on the fundamental period of vibration for anchored liquid storage tanks. Although this factor may perhaps contribute only a small part to the observed differences between measured and predicted impulsive anchored tank response it is aimed by this study to investigate the degree of this contribution and its relative importance on design.

Basis and Assumptions of the Present Study

A refined analysis based on the work mentioned above can be used to address the problem of wall thickness non-uniformity; however, it would be a cumbersome effort. The simplified design charts as well as the approximate formulas are valid for an equivalent uniform tank-wall thickness. Sakai et.al. assume this equivalent thickness to be equal to the actual thickness at a distance from the bottom equal to 1/3 of the liquid height. Shimizu et.al. suggest averaging the wall thickness along the height. Reference 9 suggests that "in determining the fundamental period of tanks with non-uniform thickness an average equivalent thickness can be used performing this averaging in such a way as to emphasize the section of the tank in which the modal displacements are the largest". This principle, however accurate, will be used here in the form described by eqs. 1. The actual wall thicknesses along the tank height are found by employing the current design practice for primary loads as described in references 10,11. Five different schemes were tried to approximate numerically the fundamental mode radial displacements but because of space limitations here will be discussed in a future publication. Finally, an approximate

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formula has been derived (Eqs. 2, Tapp), based on the charts developed by Balendra et.al., that can be used to predict the fundamental period of anchored tanks with non-uniform wall thickness. These values (Tapp) obtained by the above procedure are next compared with corresponding values obtained using eqs. 4 derived by Sakai (Tjap). This is done because on one hand this formula is relative simple as well as because it is accepted by current design practice in Japan (ref. 5). However, the conclusions from this study are believed to be equally valid when comparison with other simplified formulas or design charts would be employed.

Discussion of Results

Figures 2,3 show the degree of approximation of the fundamental period values derived by eqs. 2 and 3 respectively, when for both the same equivalent thickness is used ($t_{av} = t_1/3$). Figures 4,5 show a comparison between Tapp and Tjap for a variety of tanks with non-uniform thicknesses derived by the design procedures of references 10 and 11. Because of space limitation only representative plots are presented of the full parametric study. From the comparison between Tapp and Tjap the following points can be made:

- Both approximations agree fairly well with each other and with the design charts derived by Haroun and Housner, which for reasons of comparison are considered to represent the "exact" solution when the wall thickness is uniform.
- Differences with maximum values of the order of 20% to 40% can be observed between Tapp and Tjap arising from a realistic representation of the shell thickness non-uniformity, mainly for medium to large storage tanks. Consequently, it can be suggested that this influence should be taken into account, in a way demonstrated by this study, as a more realistic design indicator.

Conclusions

- 1) Simple design formulas and charts are needed to facilitate the earthquake design of liquid storage tanks. Formulas as the one proposed by Sakai and employed in the Japanese practice represents the necessary steps in the right direction. This is also valid for the charts developed by Haroun and Housner, or Balendra et.al as well as the simplified formula and charts derived by Shimizu et.al. The present study has also the same objective.
- 2) The influence of the shell thickness non-uniformity can not explain but a small part of the differences that were observed between predicted and observed fundamental period of vibration values for large model tanks. In this way the present study further demonstrates that the observed differences are mainly due to influences from other sources, which should also be addressed in a realistic way.

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Appendix

$$t_{av}^{mode} = \frac{\sum_{i=1}^n t_i U_{ri}^{mode}}{\sum_{i=1}^n U_{ri}^{mode}} \quad (1)$$

$$T_{app} = 1.7872 R \frac{a^2}{\Omega_1} \sqrt{\frac{\gamma_{sh} (2+0.127G/r_{mode})}{E g}} \quad (2)$$

$$T_{jap} = \frac{2}{\lambda} \sqrt{\frac{W}{\pi g E t_{1/3}}} \quad (3)$$

$$\lambda = 0.067 \left(\frac{H_f}{2R} \right) - 0.30 \left(\frac{H_f}{2R} \right) + 0.46 \quad (4)$$

$$\begin{aligned} a &= H_f / R \\ r_{mode} &= t_{av}^{mode} / R \\ z_i &= \frac{i}{n} H_f \end{aligned}$$

(5)

t_i = Actual shell thickness at z_i distance from the bottom
 U_i^{mode} = Assumed modal radial displacement at distance z_i
 R = Tank Radius, H_f = Liquid height, W = Liquid weight
 γ_{sh} = Shell material specific gravity
 g = acceleration of gravity, $\pi = 3.14159256$
 Ω_1 = Nondimensional frequency parameter approximation (ref. 3)
 $t_{1/3}$ = Actual shell thickness at $H_f/3$ distance from the bottom
 G = Ratio of density of contained liquid over the water density

Table I. Anchored Tank Impulsive Response Fundamental Frequency of Vibration

Tank Description	Type of Excitation	Observed Frequency (Hz)	Predicted Frequency (Hz)			
			ref.1	ref.2	ref.3	ref.4
"Tall" Berkeley Tank (ref.7)	Simulated Earthquake	7.4	11.7	11.3	11.7	12.2
"Broad" Berkeley Tank (ref.7)	Simulated Earthquake	8.7	24.0	24.2	25.5	28.6

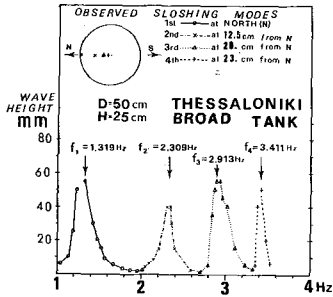


Figure 1

Table II. Convective Response of Storage Tank Fundamental SLOSHING Frequency

Tank Description	Type of Excitation	Observed Frequency (Hz)	Predicted
			(ref.2)
"Tall" Berkeley Tank (ref.7)	Simulated Earthquake	0.60	0.62
"Broad" Berkeley Tank (ref.7)	Simulated Earthquake	0.50	0.48
"Tall" Thessaloniki Tank (ref.8)	Sine Sweep Test	1.30	1.32

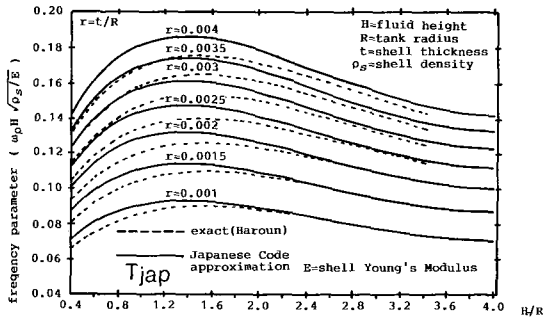


Figure 3

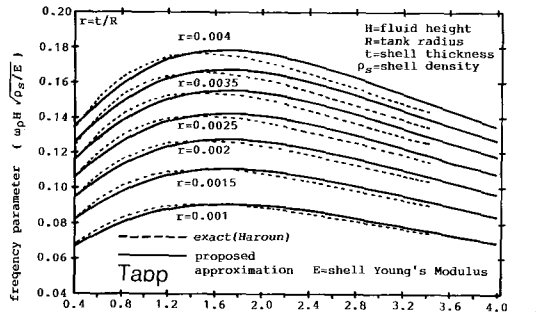


Figure 2

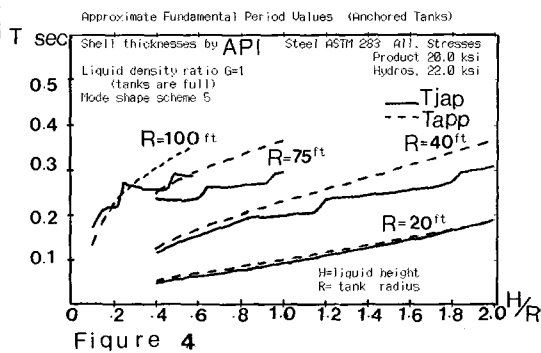


Figure 4

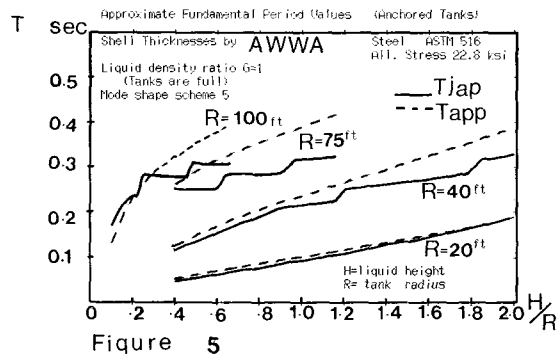


Figure 5