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NUMERICAL AND EXPERIMENTAL STUDIES ON THE CRUSHING OF TUBES

by

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1. INTRODUCTION

Tubes are often used as energy dessipating devices in various situations. The present report is concerned with the numerical and the experimental studies on the crushing behavior of steel tubes under quasi-static loading. The following problems are treated in the present report:

(i) Circular tubes under lateral compression
(ii) Square tubes under lateral compression
(iii) Circular tubes under axial compression
(iv) Square tubes under axial compression

Although the method of theoretical rigid-plastic analysis has extensively been applied to these problems [1-7] to estimate energy absorption capability and load-displacement characteristics, there is a room for improvement of the obtained solutions especially from a quantitative point of view. Furthermore there is a need to establish the method capable of consistently and efficiently dealing with the tubes and the combinations of tubes with various cross-sectional shapes under arbitrary loading and boundary conditions. The nonlinear finite element method using low-order elements based on the reduced integration technique can be considered to have such capabilities. Therefore in the present study the finite element codes for the crush analysis of one-dimensional frame-like structures and arbitrarily shaped thin-walled shell structures are developed and applied to the above-mensioned problems. The crush tests are also carried out in order to obtain the informations on the crushing patterns which are necessary for the finite element analysis and to verify the validity of the developed finite element codes.

2. OUTLINES OF THE DEVELOPED FINITE ELEMENT CODES [15, 16]

The finite element code for the crush analysis of one-dimensional frame-like structures is based on the following algorithm [15]:

(i) Linear Timoshenko beam element with one-point quadrature [8]
(ii) Incremental theory by the updated Lagrangian approach [10]
(iii) Zero normal-stress projection [11]
(iv) Direct treatment of frictional contact

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and the finite element code for the crush analysis of general thin-walled plate and shell structures is based on the following algorithm [16]:

(i) Bilinear quadrilateral shell element with one-point quadrature [9]
(ii) Incremental theory by the updated Lagrangian approach [10]
(iii) Zero normal-stress projection [12]
(iv) Orthogonal hourglass control [13]
(v) Additional stiffness resisting in-plane rotation [14]

3. CIRCULAR TUBES UNDER LATERAL COMPRESSION [15]

The energy dissipating system using circular tubes under lateral compression has the advantage that the effectiveness of such a system does not depend on close control of the loading direction. As a basis of the design of this type of energy absorbers the crushing of circular tubes between rigid plates has been studied by several authors [1-3]. Reid and Reddy [3] gave the highly improved rigid-plastic solution in comparison with the pioneering work conducted by DeRuntz and Hodge [1] by considering the strain hardening and using the theory of elastica, but there still remains a little disagreement between the theoretical and the experimental results.

Fig. 1 shows the results for load-displacement (flattening) curves for the mild steel cylinder of $R/t = 49.5$, in which the finite element solutions under the assumption of zero axial-stress (plane stress) and zero axial-strain (plane strain) are compared with the rigid-plastic solution given by Reid and Reddy and the experimental result. There is a good agreement as a whole between the finite element solution under plane stress condition and the rigid-plastic solution, which shows that the Reid and Reddy's theoretical solution is nearly rigorous. The difference between these solutions and the experimental result is mainly due to the non-uniformity of the deformations in the axial direction which is not taken into account both in the rigid-plastic and the finite element analysis. Based on the experimental observations it can be considered that the restriction on the axial deformations in the final stage of crushing is stronger than that in the initial stage. Consequently in the initial stage of crushing the plane stress solution gives more reasonable result, while in the final stage the plane strain solution agrees better with the experimental result. It should be noted here that P_o in the figure is the plastic collapse load which is given as $P_o = \quad t^2/R$ and D is a cross-sectional diameter which is 112.2 mm.

In Fig. 2 the crushing deformations given by the finite element analysis under plane stress condition are compared with those observed in the experiment. It can be seen that the crushing process accompanied with the movement of contact points between loading plates and the circular cylinder is successfully simulated in the finite element analysis.

4. SQUARE TUBES UNDER LATERAL COMPRESSION [15]

The developed finite element code is applicable to tubes with arbitrary cross-sectional shapes. The flattening problem of square tubes is studied numerically and experimentally as an illustrative example other than circular cylinders.

The calculated load-displacement curves and crushing deformations for the mild steel square tube of H/t = 40.3 (H: side length of a cross section = 87.3 mm) are compared with the experimental results in Fig. 3 and Fig. 4 respectively. In Fig. 3 P_Y is the yield load given by $P_Y = 2tR$. This problem is a little more difficult to simulate rigorously because the result is sensitive to the initial imperfection of the cross-sectional shape, however, as a whole there is a good agreement between the numerical and the experimental results.

5. CIRCULAR TUBES UNDER AXIAL COMPRESSION [17]

The axial crushing of circular cylinders is one of the classical problems in the crush analysis, however, there still remain unsolved problems for the non-axisymmetric crushing, one of which concerns the relation between the circumferential wavenumber (n) in the crushing pattern and the radius-to-thickness ratio [18] and another one is concerned with the mean crushing strength [19]. The first problem is not touched upon here (refer to [18]). In the present study the mean crushing stresses are calculated by the finite element crush analysis for a unit area of the peoriodic crushing pattern. The size of the unit area which is characterized by the axial and the circumferential half wavelength is determined by the experimental observations.

In Fig. 5 the calculated mean crushing stresses are compared with the experimental results. The radius-to-thickness ratios of the calculated and tested circular cylinders range from 16.9 to 108.1. The difference between the numerical and the experimental results especially in the range of small R/t values is due to the thin-walled assumption in the finite element analysis. It can be seen in this figure that the finite element analysis gives highly improved solutions in comparison with the existing rigid-plastic solutions.

Fig. 6 and Fig. 7 show the experimental and the numerical crushing deformations respectively. It can be seen from Fig. 7 that the complicated crushing processes accompanied with the movement of plastic hinges are successfully simulated for the non-axisymmetric crushing modes with various numbers of circumferential waves.

6. SQUARE TUBES UNDER AXIAL COMPRESSION [18]

The crush tests for axially compressed square tubes are carried out and the finite element analysis is also conducted by using the code employed in the preceding section. Square tubes under axial compression are frequently used energy absorbing devices as well as circular cylinders under axial or lateral compression.

Figs. 8 show the comparison with respect to the mean crushing strength for several square tubes whose H/t range from 15.3 to 40.3. In Fig. 8(a) the non-dimensional form expressed in terms of the yield stress is used, while in Fig. 8(b) the non-dimensional form in terms of the tensile strength is employed. It can be seen from these figures that the latter non-dimensional form is preferable for the present problem in which the stress-strain relationship in the range of large strains plays an important role. The calculated mean crushing stresses are about 25% smaller on the average than the experimental values because of the thick-walled effect, however, there is a good agreement between them from a qualitative point of view. The bold line in Fig. 8(b) is the empirical formula given by Meng et al.[7], which agrees well also with the present experimental results.

Fig. 9 and Fig. 10 are the experimetal and the numerical crushing deformations respectively. In fig. 10 the results obtained with different mesh subdivisions are shown, among which the finest mesh (C)is used to calculate the mean crushing strength shown in Fig. 8. The agreement between the calculations and the experiments with respect to the crushing deformations is fairly good as in the case of circular cylinders.

7. CONCLUDING REMARKS

The results of the numerical and experimental studies on the crushing behavior of steel tubes are briefly introduced in the present report, which have been recently conducted in the first author's laboratory. From these results it can be concluded that the finite element method using low-order elements based on the reduced integration technique is a powerful tool for the crush analysis, which has the possibility of solving the remaining problems in the present field especially from the quantitative point of view.

The following items are considered as the future research topics related to the present study:

(i) Other loading conditions and other devices
(ii) 3-dimensional finite element analysis including axisymmetric analysis
(iii) Integrated structures such as automobiles, nuclear-powered ships, bridge piers and offshore platforms
(iv) Dynamic effects and dynamic relaxation method
(v) Mechanical interactions among deformable members and structures

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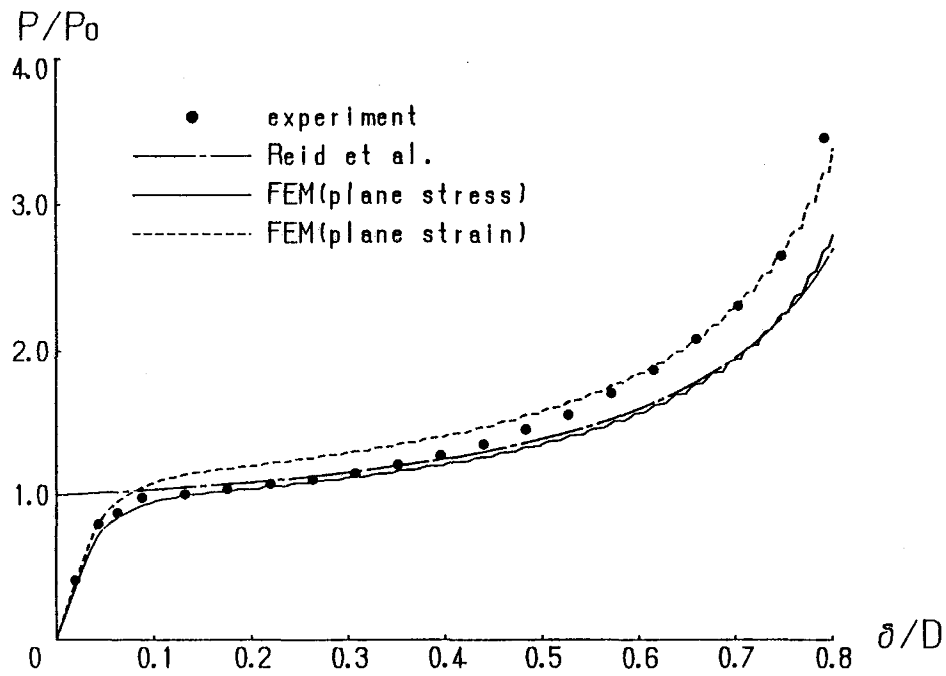


Fig. 1 Load-displacement curves for laterally compressed circular tubes

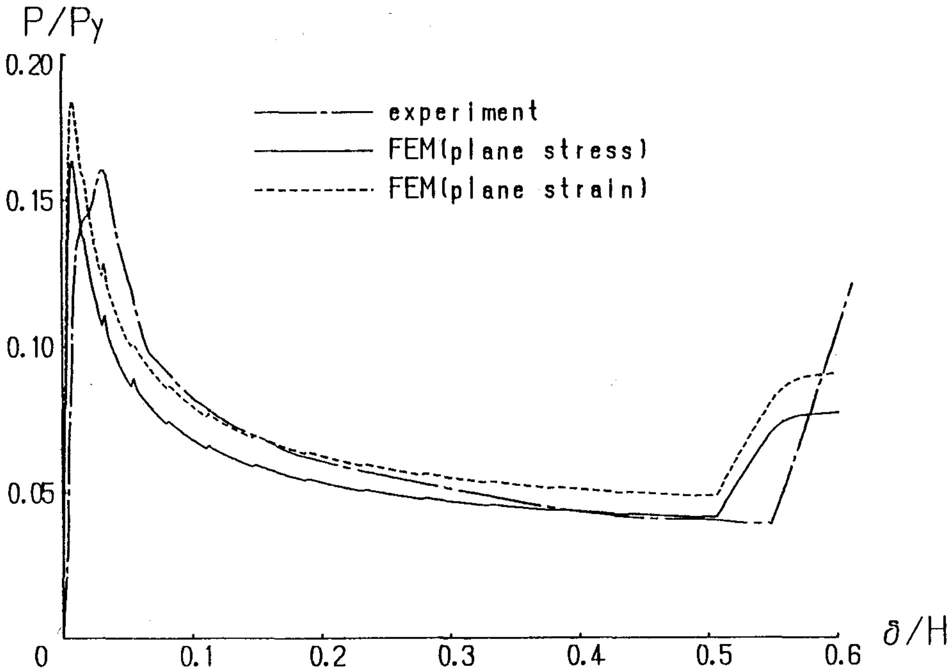


Fig. 3 Load-displacement curves for laterally compressed square tubes

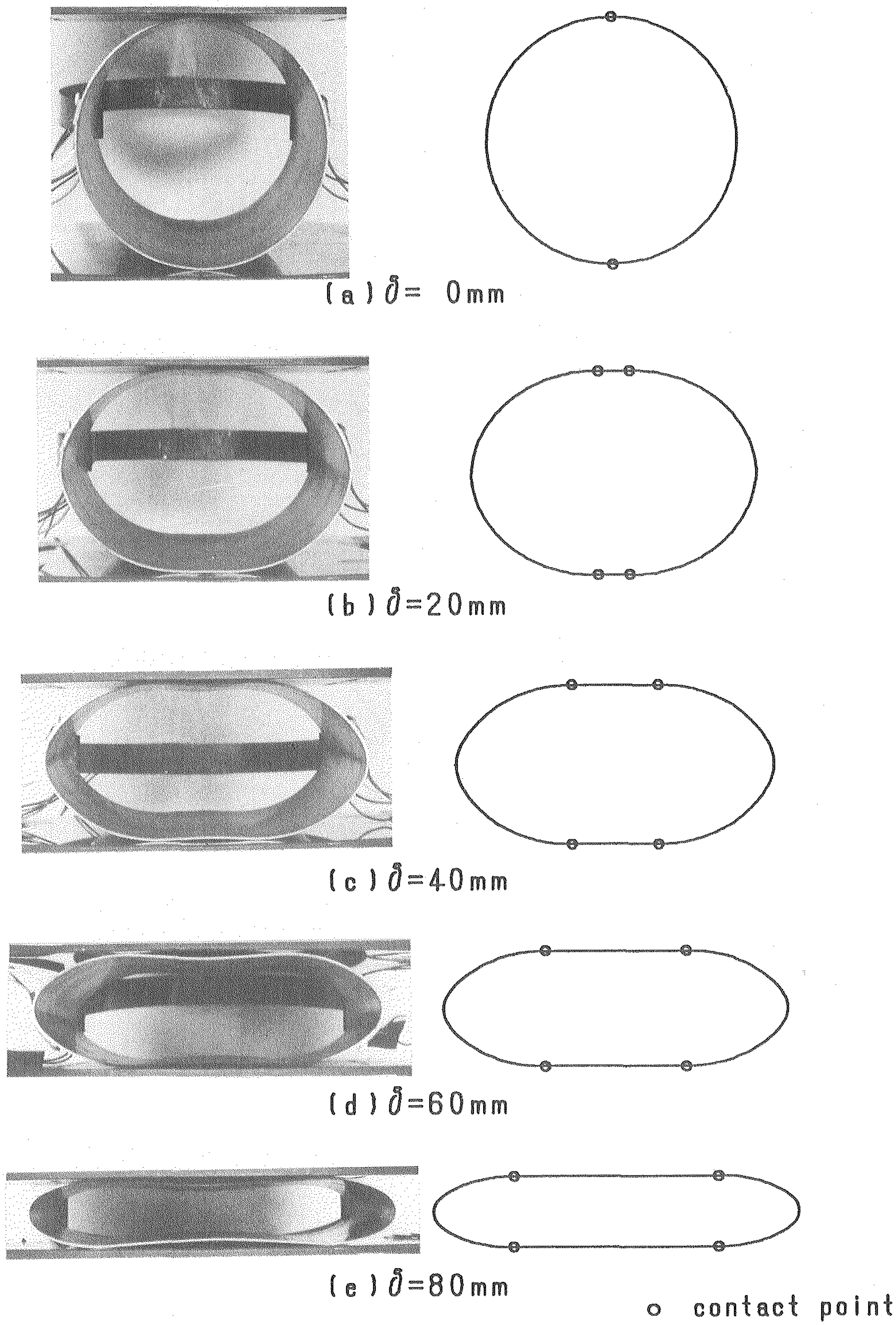


Fig. 2 Crushing deformations of laterally compressed circular tubes (experimental results and finite element solutions)

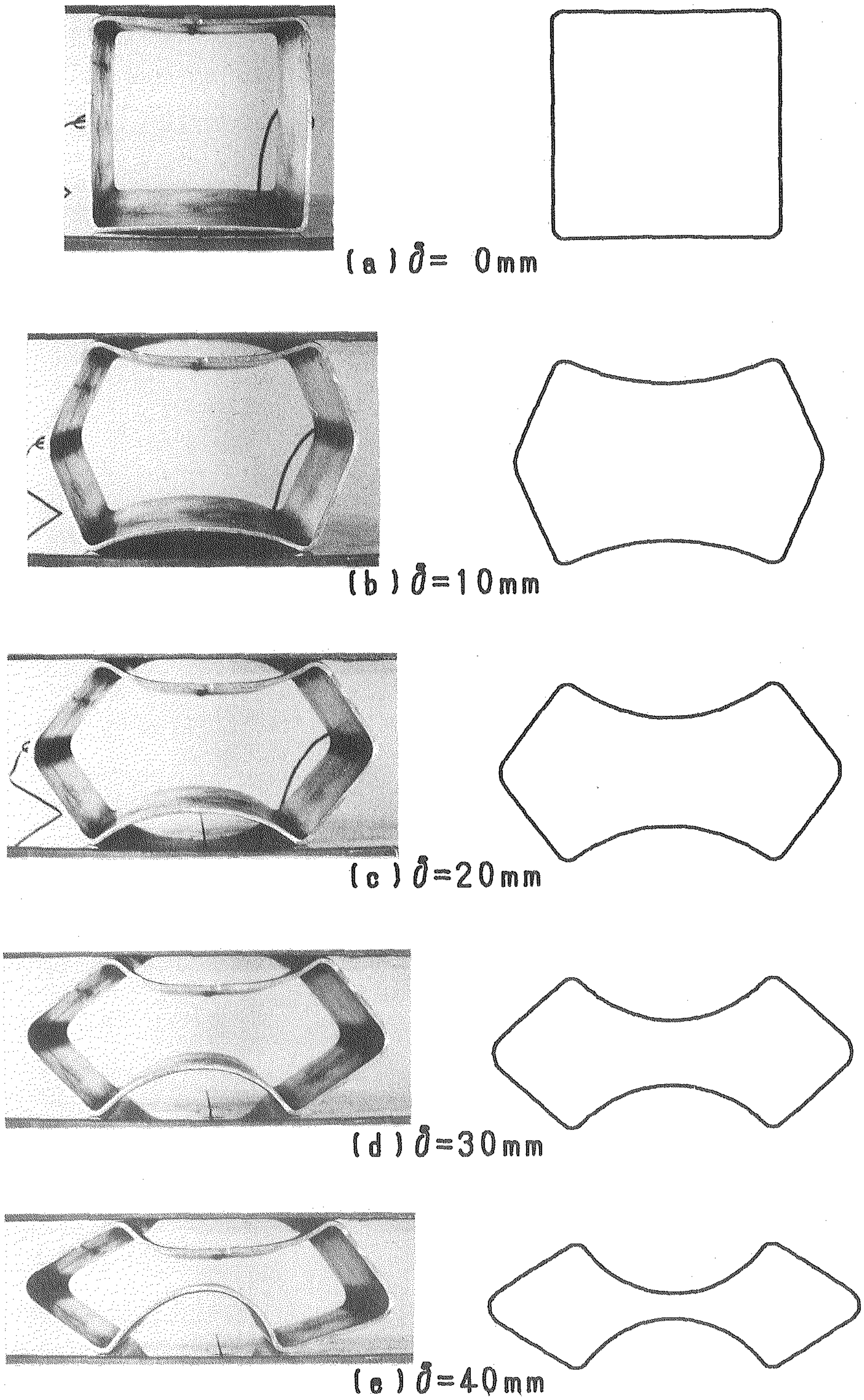


Fig. 4 Crushing deformations of laterally compressed square tubes (experimental results and finite element solutions)

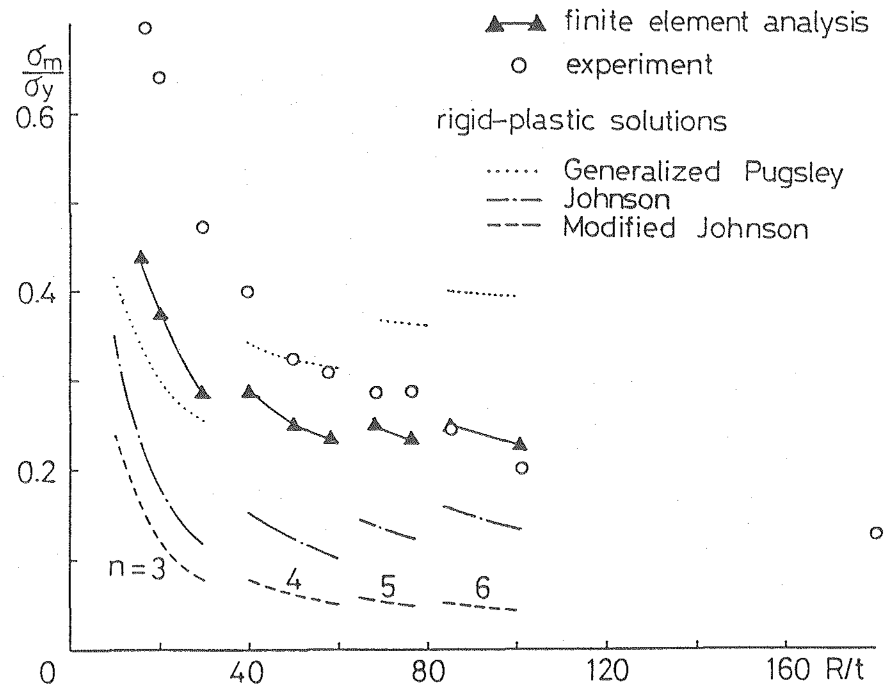


Fig. 5 Mean crushing stresses for axially compressed circular tubes

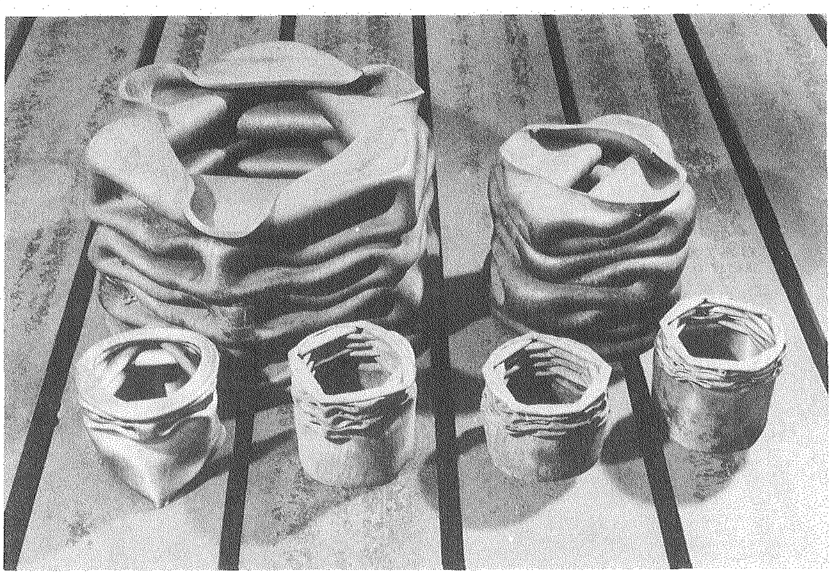


Fig. 6 Axially crushed circular tubes

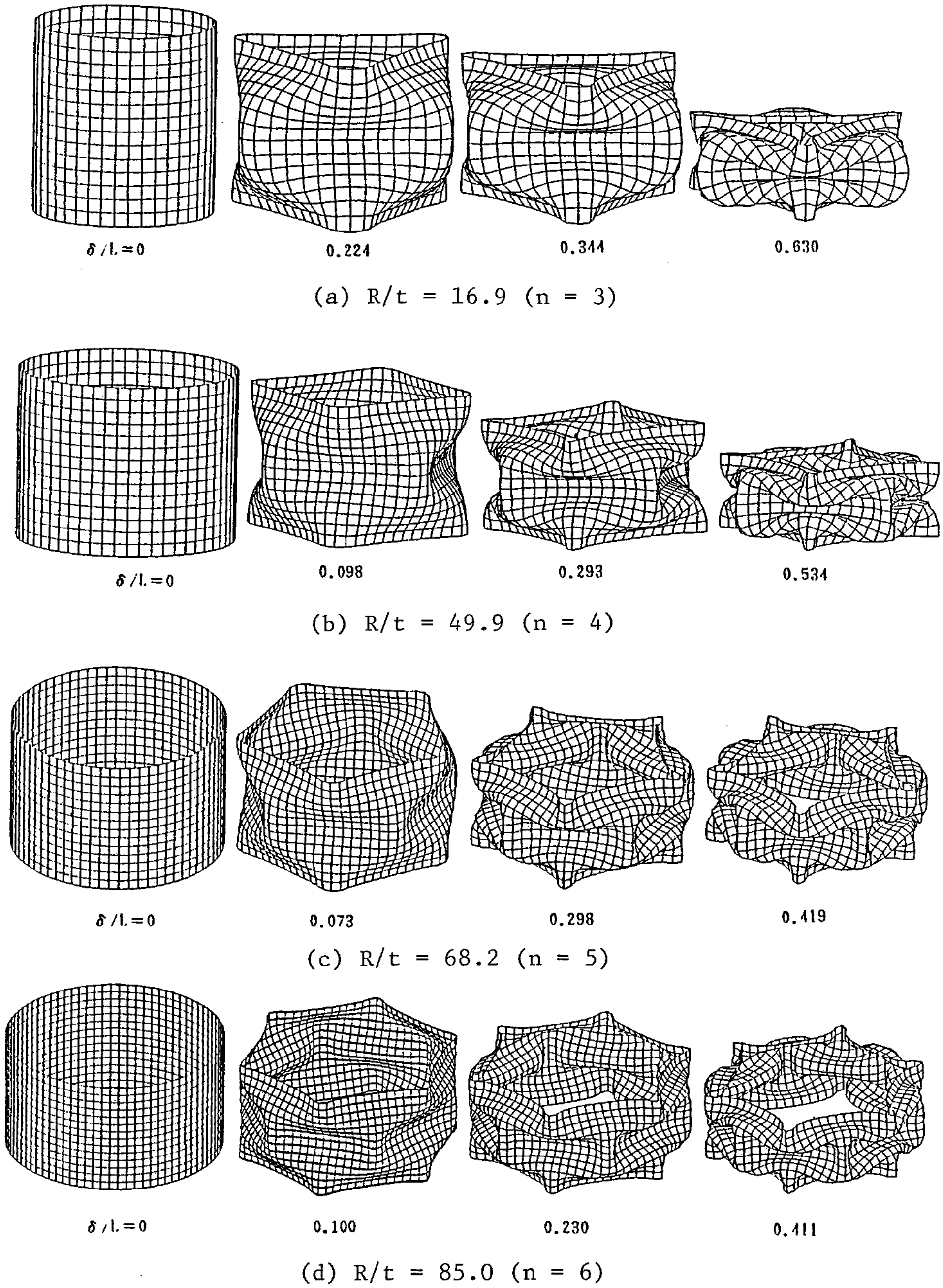
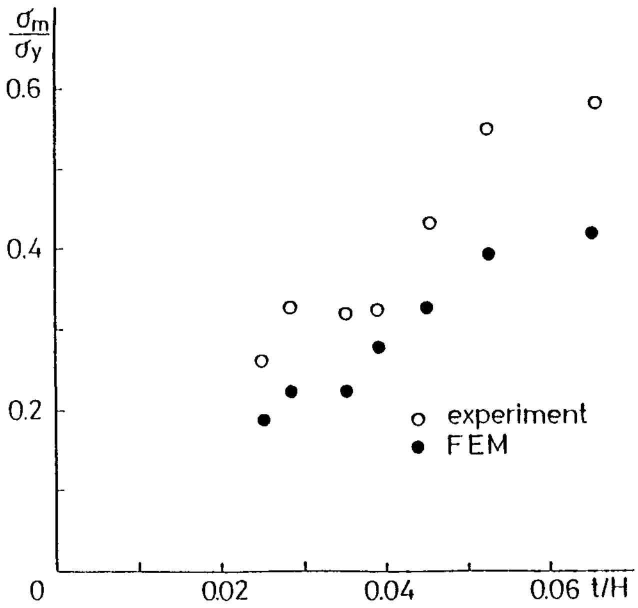
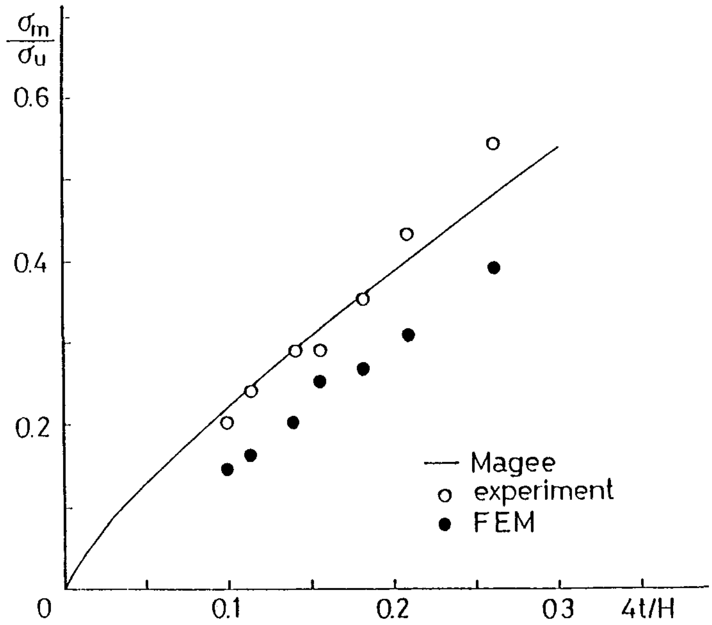


Fig. 7 Crushing deformations of axially compressed circular tubes



(a) non-dimensional form using yield stress



(b) non-dimensional form using tensile strength

Fig. 8 Mean crushing stresses for axially compressed square tubes

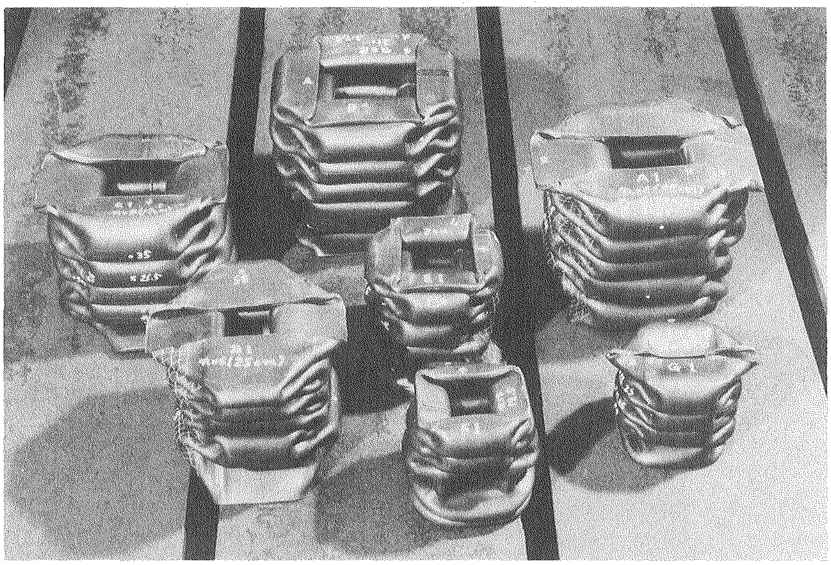


Fig. 9 Axially crushed square tubes

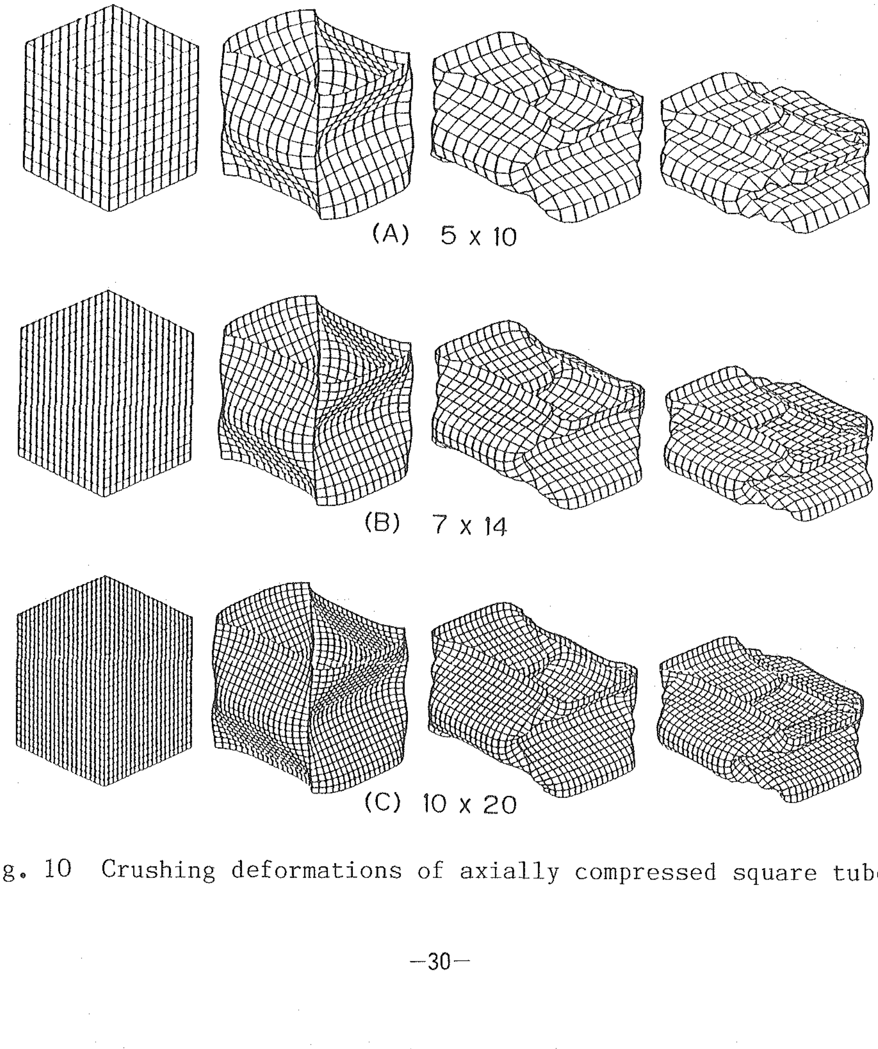


Fig. 10 Crushing deformations of axially compressed square tubes