On the Energy Absorption of Laterally Loaded Structural Elements under Axial Forces

by

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INTRODUCTION

In the present report the energy absorption of laterally loaded structural members under constant axial forces will be briefly discussed from a qualitative point of view, based on the energy balance equations for the inelastic column model with single degree of freedom, and its stability and response characteristics will be described. Furthermore the limitation of the rigid-plastic analysis for the present problem will be studied by the comparison of the rigid-plastic solutions with the exact elasto-plastic solutions.

INELASTIC COLUMN MODEL WITH SINGLE DEGREE OF FREEDOM

Fig. 1 shows the column model used in the present analysis for which the following three kinds of analyses were carried out:

- (i) Simplified Elasto-Plastic analysis(SEP)
- (ii) Rigid-Plastic analysis(RP)
- (iii) Elasto-Plastic analysis(EP)

The relation between bending moment(M) and rotational angle(ϕ) in each analysis is illustrated in Fig. 2, where Mp is the yield moment of the rotational spring and Mo is the fully plastic moment of the column section. In the RP and the EP Mp is assumed as

$$M_p = M_0\{1-(P/P_0)^2\} \qquad (1)$$

The following non-dimensional parameters will be used in the later discussions:

$$p=\frac{P}{P_0} \qquad q=\frac{k}{P_0 L} \qquad \theta=\frac{P_0 L}{M_0}\varphi \qquad \bar{E}=\frac{P_0 L}{M_0{}^2}E \qquad (2)$$

where p and q indicate the axial load to yield load ratio and the elastic buckling load (k/L) to yield load ratio respectively.

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ENERGY ABSORPTION OF INELASTIC COLUMN MODEL

[1]Stability

Two types of critical energies can be defined for the present column model. The first one is the 'elastic critical energy' given by the following equations:

$$\tilde{E}_e(\mathrm{SEP})=\frac{1}{2q}\left(1-\frac{p}{q}\right)$$
$$\tilde{E}_e(\mathrm{EP})=\frac{1}{2q}\left(1-\frac{p}{q}\right)(1-p^2)^2 \tag{3}$$

When the external energy $\tilde{E}$ exceeds the elastic critical energy, the plastic deformation occurs. The second one is the 'stable critical energy' expressed as follows:

$$\tilde{E}_p(\mathrm{SEP})=\frac{1}{2p}\left(1-\frac{p}{q}\right)$$
$$\tilde{E}_p(\mathrm{RP})=\frac{1}{2p}(1-p^2)^2 \tag{4}$$
$$\tilde{E}_p(\mathrm{EP})=\frac{1}{2p}\left(1-\frac{p}{q}\right)(1-p^2)^2$$

When the external energy exceeds the stable critical energy, the column loses its stability.

The calculated ratios between these critical energies are shown in Table 1, from which it can be seen that $p<0.2$ in the SEP and $p/q<0.1$ in the RP are required if the tolerance from the EP solution is within 10%. The relation between p and $\tilde{E}$ in the case of $q=1.0$ is illustrated in Fig. 3. The p-$\tilde{E}$ plane is divided into three regions by the elastic critical energy curve($\theta_{max}=\theta_y$) and the stable critical energy curve($\theta_{max}=\theta_c$). The asymptotic stability holds in the first region where the external energy is less than the elastic critical energy. In the second region between the elastic and the stable critical energy curve the permanent deformation occurs after the response due to the external energy. In Fig. 4 the relation between p and $\tilde{E}$ in the EP is shown as a function of the maximum displacement θ_{max}. The stability diagram like Fig. 4 shows the so-called 'practical stability' of the present column model.

[2]Response

The maximum elastic displacement for the given external energy $\tilde{E}$ is expressed as

$$\left.\begin{array}{l}\left.\begin{array}{l}\theta_{max}(\mathrm{SEP})\\ \theta_{max}(\mathrm{EP})\end{array}\right\}=\left(\dfrac{2\tilde{E}}{q-p}\right)^{1/2}\\ \\ \tilde{E}<\tilde{E}_e(\mathrm{SEP}),\ \tilde{E}_e(\mathrm{EP})\end{array}\right\} \tag{5}$$

and the elasto-plastic displacement is given by the following equations:

In case of p=0

$$\left.\begin{aligned} &\theta_{\max}(\mathrm{SEP})=\frac{1}{p}\left\{1-\left(1-\frac{p}{q}-2p\tilde{E}\right)^{1/2}\right. \\ &\tilde{E}_e(\mathrm{SEP})<\tilde{E}<\tilde{E}_p(\mathrm{SEP}) \end{aligned}\right\}$$

$$\left.\begin{aligned} &\theta_{\max}(\mathrm{RP})=\frac{1}{p}\{(1-p^2)-[(1-p^2)^2-2p\tilde{E}]^{1/2}\} \\ &0<\tilde{E}<\tilde{E}_p(\mathrm{RP}) \end{aligned}\right\}$$

$$\left.\begin{aligned} &\theta_{\max}(\mathrm{EP})=\frac{1}{p}\left\{(1-p^2)\right. \\ &\qquad -\left[\left(1-\frac{p}{q}\right)(1-p^2)^2-2p\tilde{E}\right]^{1/2} \\ &\tilde{E}_e(\mathrm{EP})<\tilde{E}<\tilde{E}_p(\mathrm{EP}) \end{aligned}\right\} \qquad (6)$$

In case of p=0

$$\theta_{\max}(\mathrm{SEP})=\tilde{E}+\frac{1}{2q}$$
$$\theta_{\max}(\mathrm{RP})=\tilde{E}$$
$$\theta_{\max}(\mathrm{EP})=\tilde{E}+\frac{1}{2q}$$

The response diagram for the following three cases are shown in Figs. 5-7:

(i) Case[A] (p,q)=(0.1,10.0), (0.0,10.0), (-0.1,10.0)
(ii) Case[B] (p,q)=(0.2, 2.0), (0.0, 2.0), (-0.2, 2.0)
(iii) Case[C] (p,q)=(0.5, 1.0), (0.0, 1.0), (-0.5, 1.0)

From these figures the effect of the parameters p/q, p and q on the accuracy of the simplified elasto-plastic and the rigid-plastic solutions can be clearly understood. For the compressed column(p>0.0) the rigid-plastic solution is considered to be valid in practice when p/q<0.1, however, for the column under axial tension or no axial force the rigid-plastic solution is always valid when the magnitude of the external energy is sufficiently large.

CONCLUSION

The obtained results in the present study (details of which is published in Ref. 1)) can be summarized as follows:

(1) The equations of the elastic critical energy and the stable critical energy were derived and the stability diagrams were presented

(2) The equations of the elastic maximum displacement and the elasto-plastic maximum displacement were derived and the response diagrams were presented.

(3) It was shown that the accuracy of the rigid-plastic solutions largely depends on the ratio of the axial load to the elastic buckling load (p/q).

REFERENCE

1)Yutaka Toi:Qualitative Studies on the Lateral Energy Absorbing Characteristics of Axially Loaded Structural Members, Journal of the Society of Naval Architects of Japan, Vol. 156, (1984), 401-410.

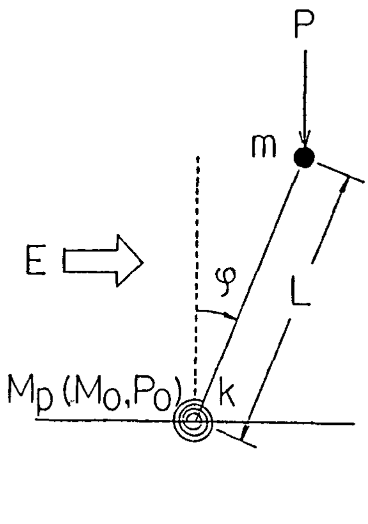

Fig. 1 Nonlinear column model with single degree of freedom

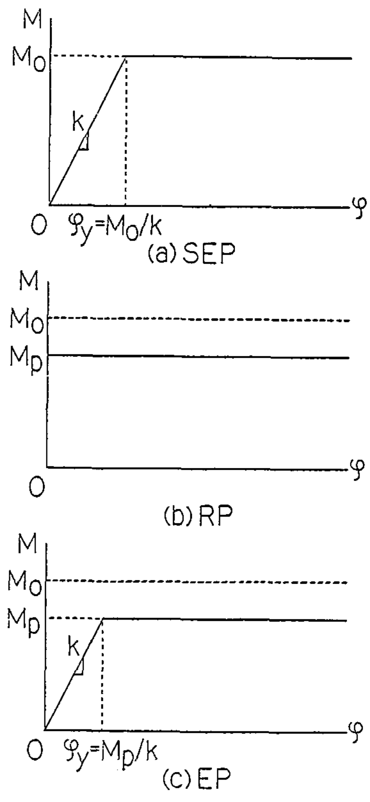

Fig. 2 Nonlinear characteristics of rotational springs

Table 1 Calculated critical energy ratios

p	$\frac{\tilde{E}_e(SEP)}{\tilde{E}_e(EP)}$	$\frac{\tilde{E}_p(SEP)}{\tilde{E}_p(EP)}$	$\frac{\tilde{E}_p(RP)}{\tilde{E}_p(EP)}$			
			q=10.0	q= 2.0	q= 1.0	q= 0.5
0.0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.020	1.020	1.010	1.053	1.111	1.250
0.2	1.085	1.085	1.020	1.111	1.250	1.667
0.3	1.208	1.208	1.031	1.176	1.429	2.500
0.4	1.417	1.417	1.042	1.250	1.667	5.000
0.5	1.778	1.778	1.053	1.333	2.000	∞
0.6	2.441	2.441	1.064	1.429	2.500	
0.7	3.845	3.845	1.075	1.538	3.333	
0.8	7.716	7.716	1.087	1.667	5.000	
0.9	27.701	27.701	1.099	1.818	10.000	
1.0	∞	∞	1.111	2.000	∞	

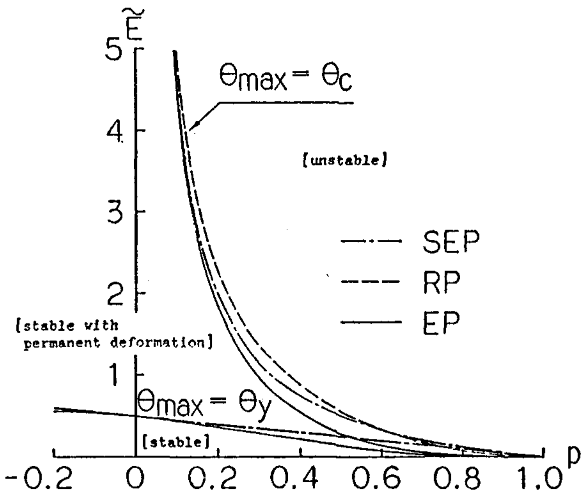

Fig. 3 Stability diagram for q=1.0 (SEP, RP and EP)

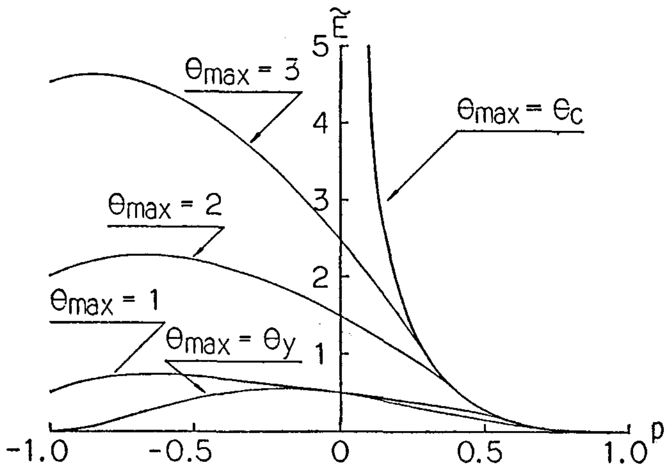

Fig. 4 Stability diagram for q=1.0 (EP)

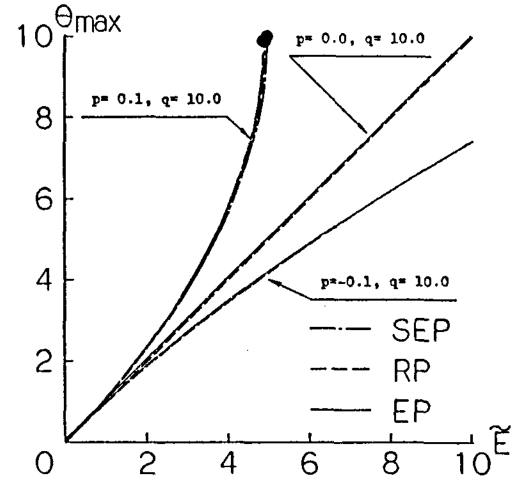

Fig. 5 Response diagram for Case [A]

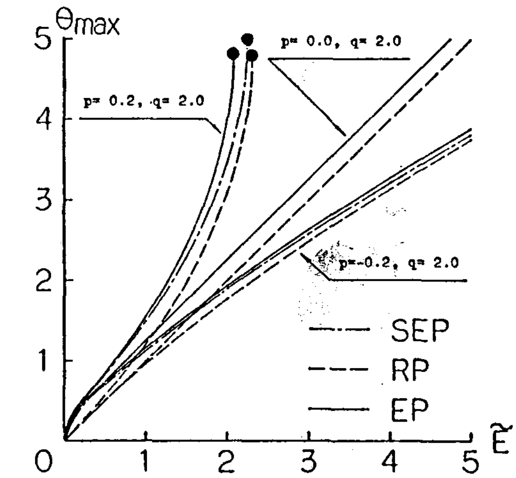

Fig. 6 Response diagram for Case [B]

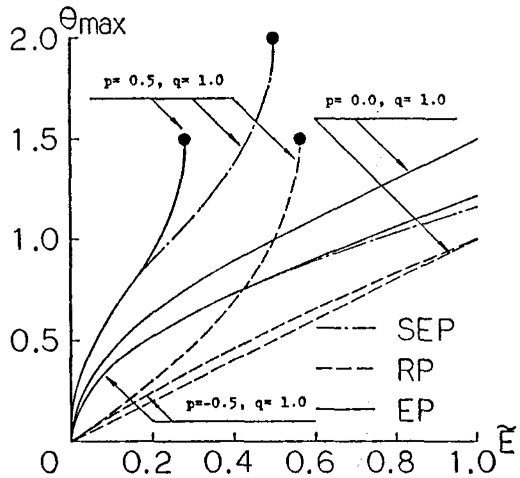

Fig. 7 Response diagram for Case [C]