

# Discrete Limit Analysis on the Crashworthiness of Structural Components

by

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## INTRODUCTION

In this note the axial crushing behaviors of tubes with square and circular cross-sections are numerically simulated by using the flat rigid plate element which is the discrete element suitable for the crush analysis of thin-walled steel structures. Both members are subdivided with a small number of elements, based on the collapse modes observed in the experiments. The obtained solutions for the mean crushing load are compared with the experimental results.

## FLAT RIGID PLATE ELEMENT

Fig. 1 shows the flat rigid plate element used in the present analysis, which consists of rigid triangular plates and springs distributed over the contact surface between the adjacent elements. The centroid in each element has 6 degrees of freedom independently which represent three-dimensional movement of a rigid body, and therefore the discontinuity of displacements such as plastic hinge lines and slip lines, which will often occur in the crushing mode, can be easily introduced on the interelement boundaries of the present discrete model.

The incremental formulation by the Lagrangean approach for the flat rigid plate element has already been presented in Ref. 1), however, the crushing deformation of thin-walled steel structures is often accompanied with large rotation of rigid elements and in such cases the Lagrangean formulation is not sufficiently accurate. For this reason the Updated Lagrangean formulation with the moving local coordinate systems was adopted in the numerical examples of the present report. All of the calculations were carried out as the static analysis in which the inertial forces and the strain rate effect were neglected.

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## AXIAL CRUSHING OF SQUARE TUBES

Fig. 2 shows the crushing mechanism of a square tube under axial compression. The crushing mode is periodic in the axial direction, so we can deal with only a periodic unit area (shaded area in Fig. 2) as the model to be analyzed. The axial wavelength  $H$  was determined according to the following empirical formula between  $H$  and  $B$ :

$$H/t = 0.99(B/t)^{2/3} \quad (1)$$

The material of specimens was assumed to be mild steel whose tangent modulus-equivalent stress relationship is given in Fig. 3. The calculated load-axial shortening curves are shown in Fig. 4 where the horizontal dotted lines indicate the mean crushing loads computed for the following range of axial shortening:

$$0.0 < \delta/2H < 0.73 \quad (2)$$

The following theoretical rigid-plastic formula for the mean crushing strength is given in Ref. 2):

$$P_m/P_s = 1.3(4t/B)^{2/3} \quad (3)$$

which is compared with the experimental results as well as the numerical solutions by the present analysis in Fig. 5. All of them are comparatively in good agreement. The crushing process can be seen in Fig. 6.

## NON-AXISYMMETRIC CRUSHING OF CIRCULAR CYLINDERS

The non-axisymmetric crushing mechanism as shown in Fig. 7 can be observed in thin-walled circular cylindrical shells subjected to axial compressive force. The static crush simulations for the periodic unit area were carried out, based on the theoretical relation between the circumferential wave number ( $n$ ) in the crushing pattern and the radius to thickness ratio ( $R/t$ ) given in Ref. 3). In the analysis the angle  $\alpha$  in Fig. 7 was assumed as

$$\alpha = \pi/2n \quad (4)$$

In Fig. 8 the obtained mean crushing loads are compared with the experimental values for mild steel specimens given in Refs. 3) and 4). They are considerably in good agreement, however, the assumption expressed by the eq. (4) is not necessarily true in the experiments. Further investigations will be necessary about this inconsistency. Fig. 9 shows the deformed shapes of the unit area at the selected loading steps.

## CONCLUSION

In the present report some numerical studies were conducted for the crushing behaviors of axially compressed tubes with square and circular cross-sections by using the flat rigid plate element. The obtained mean crushing strength agrees well with the experimental results. The present method of discrete analysis can be considered to be a powerful tool for the crush simulations of various kinds of thin-walled steel structural members.

## REFERENCES

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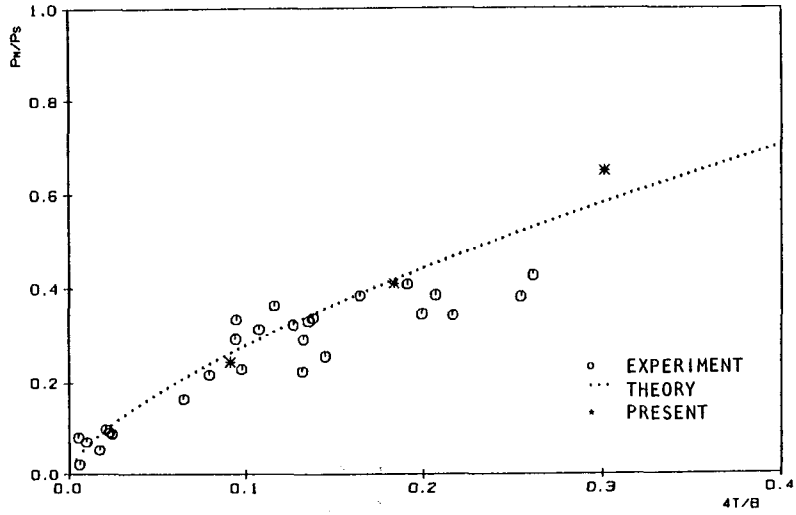


Fig. 5 Mean crushing loads for square tubes

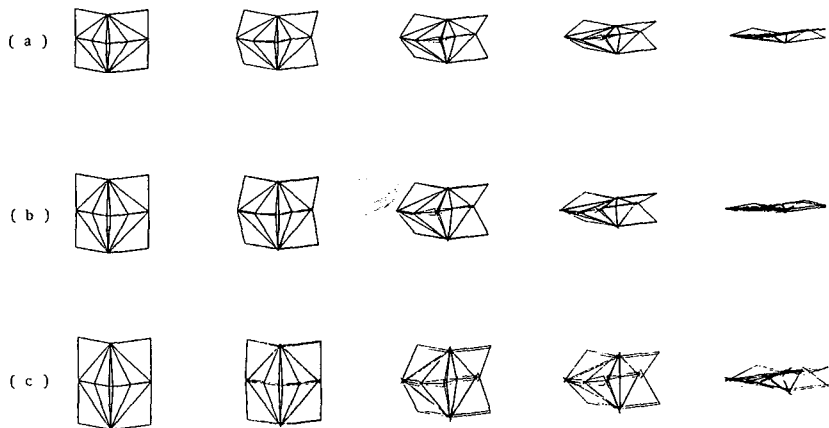


Fig. 6 Crushed profiles of square tubes  
 (a)  $4t/B = 0.0915$  (b)  $4t/B = 0.183$  (c)  $4t/B = 0.301$

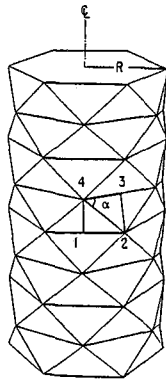


Fig. 7 Crushing mechanism of circular tubes

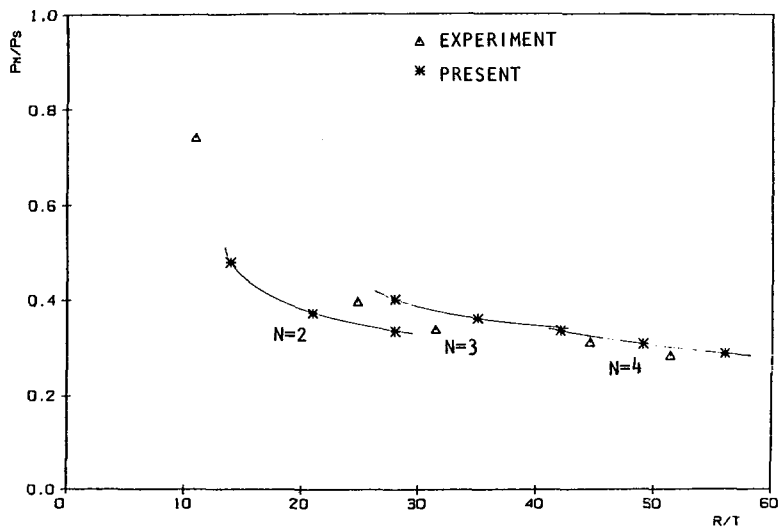


Fig. 8 Mean crushing loads for circular tubes

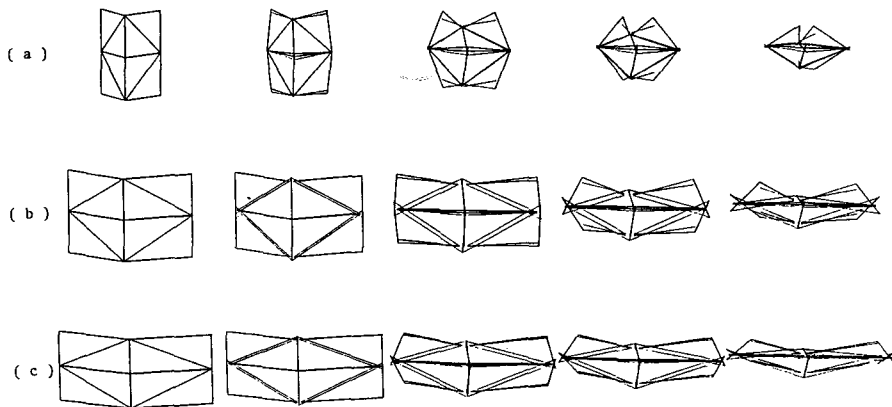


Fig. 9 Crushed profiles of circular tubes  
(a)  $R/t=21$  (b)  $R/t=35$  (c)  $R/t=49$