STATIC AND DYNAMIC BEHAVIOURS OF SILO GROUP UNDER EARTHQUAKE LOADINGS

bу

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SYNOPSIS

Silo group is consisted of many cantilever type cylindrical shells as well as of connection walls between these cylindrical shells. The static and the dynamic behaviours of silo group under earthquake loading are numerically analyzed by using a simple mechanical model, and effect of number of bins, stiffness of connection wall and hight-to-diameter ratio are examined.

INTRODUCTION

The purpose of this paper is to investigate analytically static and dynamic behaviours of silo group subjected to earthquake loadings. Many papers about silos have been published and their main themes are (1) estimation of internal pressure caused by stored materials[1], (2) evaluation of equivalent mass during earthquake [2], (3) buckling behaviours [3,4], and others. However, silo treated in these studies is not a silo group but an isolated silo. Silo group represents a compound shell which is composed of a large number of cantilever type cylindrical shells connected directly or with connection walls (Figs.1 and 2), so that a silo group reveals more complicated mechanical behaviours than an isolated silo. Each cylindrical shell of a silo group is called "bin."

In the paper, static and dynamic analyses are carried out within the elastic range in order to examine the following items:
For the static behaviours of silo group subjected to the unit horizontal load at the silo-top,

 Effect of number of bins, stiffness of connection wall, and hight-todiameter ratio (Lg/R) upon the displacement behaviour and upon the stress distribution.

For the dynamic behaviours,

(2) Effect of number of bins, stiffness of connection wall, and eccentricity of mass between silo group and foundation on natural frequency and on responses of displacements and acceleration to two kinds of earthquakes.

A simple mechanical model, which is called "Beam with Rigid Bar Model," is proposed in order to grasp the overall mechanical behaviour of silo group comparatively easily. In this model, each bin is divided into any beam elements which have rigid bars at both ends, as shown in Fig.3. These rigid bars are used to connect the bin with connection walls, which are

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represented by using the rectangular in-plane and bending finite element. Each beam element admits bending and shear deformations, but cannot represent oval deformation of bin. Hence, this model can be used to grasp global behaviours of silo group by small degrees-of-freedom.

SILO GROUP FOR NUMERICAL ANALYSES

For numerical analysis, let us select a silo group, which is considered to have the standard size, among reinforced concrete silos recently used. Height, radius and thickness of a bin of the silo group are Lg=30m, R=3.75m and H=0.2m, respectively, and length and thickness of a connection wall are L=0.75m and $\rm H_w$ =0.4m, respectively. Young's modulus E, Poisson ratio ν and density ρ of both the bin and the connection wall are E=2.1×10 $^6\rm t/m^2$, ν =0.17 and ρ =2.4t/m 3 , respectively. And also, the density ρ_S of stored material is ρ_S =0.8t/m . Total weight of the stored material estimated by using this value is about four times as much as the whole weight of the silo group itself. For the investigation of the effect of hight-to-diameter ratio, height Lg is changed.

BEAM GROUP MODEL

Let us consider "Beam with Rigid Bar Model," which is called "Beam Group Model" is the following, because a lot of beam models with right bars are used for modelling a silo group. As shown in Fig.3, each bin is represented as an assembly of beam elements which have rigid bars at both ends. Let us consider the degree of freedom of this model. In the case that bins are arranged in the straight line, unknown displacements for each rigid bar are $\mathbf{u_1}$, $\mathbf{u_2}$ and \mathbf{w} , where $\mathbf{u_1}$ and $\mathbf{u_2}$ are vertical displacements at both ends of the rigid bar and \mathbf{w} is horizontal displacement of the center of the bar. By using these displacement components, displacements \mathbf{u} , \mathbf{w} and rotation θ at the end of the beam can be obtained as

A connection wall is represented as an assemble of rectangular plane elements whose node is connected with the end of a rigid bar, so that total degrees of freedom become $3\times(\text{number of rigid bar})$.

In order to examine the validity of the proposed "Beam Group Model," let us analyze a silo group which is composed of three bins, as shown in Fig.4, by using three different models and then compare the results. The first model is a beam model which means that a silo group is represented by one beam only. The second is the beam group model and the third is the cylindrical shell element. Mechanical characteristics being able to obtain by means of these three models are tabulated in Table 1 to compare with. It can be understood from this table that local bending moments caused in the bottom of bin and in the vicinity of connection between bin and connection wall, oval deformation of cylinder, etc. can not be estimated by this beam group model. However, the effect of stiffness of connection wall, which is one of the important items for study, can be investigated.

For the numerical analysis, a silo group which has the same properties

mentioned in the previous section and is composed of three bins in the straight line, was selected. And unit horizontal load was applied to the top of each bin. Fig.6 shows the bending moment of each bin in the case of both five divisions and twenty divisions in the axial direction, calculated by use of the beam group model. The difference of bending moment which appears at the end of each beam element is caused by the additional moment which is made by nodal forces acting at both ends of rigid bar. This indicates the existence of shear force which transmits from the connection wall to the bin. In addition, it can be known in this figure that the average moment diagrams in both cases are almost the same.

Table 2 shows the results by three models. In the table, Wmax is the horizontal displacement at the top of the middle bin, Q and τ max are the transverse shearing resultant and shearing stress, respectively, of the bottom of the middle bin. And σ_X max is the normal stress at the bottem edge of the side bin and τ max is the maximum shearing stress in the connection wall. This table shows that the results by the beam group model are in good agreement with those by other methods.

STATIC ANALYSIS

In order to examine the effects of number of bins, stiffness of connection wall, and hight-to-diameter ratio, let us analyze a silo group, which has the same properties in the previous section and is arranged in a straight line, by using the group beam model. Each bin is represented as an assembly of five beam elements and unit horizontal load is applied to the top of each bin.

(a) Effect of Number of Bin

Main objective of this article is to examine the distribution of shearing stresses in each bin, in other words, the concentration behaviour of shearing stresses which depends on the number of bins as well as to obtain the ratio of the bending deformation to the shear deformation.

Figs.7 shows numerical results with respect to the change of number of bins. Fig.7(1) designates the horizontal displacement at the top of the center bin, which is nondimensionalized by the displacement of a silo group which is consisted of only one bin. Full line gives the total displacement and dotted line shows the bending or flexural displacement. It is seen that as the number of bins increases, the ratio of the shear deformation to the bending one becomes larger. Fig.7(2) shows the shearing of the bottom of the center bin. This figure denotes that the shearing force becomes maximum in the case of three bins, so that the shearing force for the case of three bins can be used for structural design in the safety side. Fig.7(3) gives the normal stresses at the bottom of outer bin in the longitudinal direction. This shows that the normal stresses in the case of considering shear deformation is larger than ones without consideration. The ratio of the shearing stress in the middle connection wall to the maximum shearing stress of isolated silo is shown in Fig.7(4).

(b) Effect of Stiffness of Connection Wall

One of the important problems in the design of silo group is to determine thickness and length of connection wall, which has strong influence in the overall stiffness and strength. To illustrate numerical examples, let us analyze numerically a silo group of three bins in a straight line by changing the length of connection wall. Numerical results are shown graphically in Figs.8 where abscissas denote nondimensionalized length of connection wall to the radius of the bin and ordinates are

nondimensionalized by the corresponding value obtained for the isolated silo. Fig.8(1) gives the decreasing rate of displacement, which can be understood by the increase of the moment of inertia of a cross section. From Figs.8(1) to (3), it can be seen that the longitudinal normal stress changes rapidly though the shearing forces in both bin and connection wall decreases slightly.

(c) Effect of Height of Bin

As the height of bin is larger, the ratis of the flexural deformation to the shear deformation becomes larger, so that the mechanical behaviours such as the distribution of shearing force in each bin are strongly influenced by the height-to-radius ratio as well as by number of bins. Illustrative examples are shown in Fig.9.

Fig. 9(1) shows the relation between horizontal displacement of the top of center bin and number of bin as a parameter of height L_{g} , where ${
m L_g}$ =3.75m is coincident with the radius R=3.75m of a bin. In the case of L_{g}^{-} =R, the displaceemnt is some 90%, and for over L_{g} =15, the displacement is less than 50%. In addition, the difference among the displacements becomes gradually smaller as Lg exceeds 30m. Fog.9(2) denotes the shearing force of the central bin. When height $L_{\rm g}$ is less than 60m, maximum shearing forces are caused in the case of three bins, and the distribution fo shearing forces from bin to bin is given in Fig.9(3). In the case of $L_g=3.75\text{m}$ the shearing force in each bin becomes the average value as the increasing of number of bins, However, if Lg is large, for example, $L_o=240m$, the shearing forces have the parabolic distribution which corresponds to the distribution of shearing stresses in a beam of rectangular cross section. Fig.9(4) indicates the bending moment at the bottom of the central bin. From this figure, it can be known that the magnitude of the bending moment is almost constant not with standing the number of bins. In Fig.9(5), the distribution of the normal stresses caused by the bending moment is shown for the two cases of 5 bins and 9. The ordinate is normalized by the maximum normal stress which appears at the bottom of the outer bin. Normal stresses are distibuted linearly when the height is large, and this corresponds with the results given in Fig. 9(3). Fig. 9(6) shows the influence of both the length and thickness of the connection wall upon the horizontal displacement with a parameter of height in the case of three bins. When the thickness of the wall is less than 0.1m, the connection wall cannot reduce the displacement though the length of the wall is made larger. This means that the stiffness of the connection wall is too small to make a silo group monolithic.

DYNAMIC ANALYSIS

In order to obtain the dynamic behaviour of silo groups under the earthquake loading, the eigenvalue analysis and the response analysis are carried out. In the eigenvalue analysis, the effects of number of bins, the stiffness of connection wall and the eccentricity of mass on both natural period and the vibration mode. And in the response analysis, maximum responses of displacement and shearing force are calculated in order to examine the effect of number of bins.

The stiffness matrix used in the dynamic analysis is the same as in the static analysis, and the mass matrix is derived as the lumped mass by adding the weight of a silo group it self to the weight of stored materials. For the beam group model, number of division in the axial direction of each bin is made three. Notations used in this section is given in Appendix.

(a) Natural Vibration Analysis

Let us first consider the silo groups of square plan. Natural periods are shown in Table 3 and in Fig.10, and vibration modes are depicted in Figs.11(1) and (2) concerning silo groups of one bin and three bins, respectively. From these tables and figures, it can be known that the first and the second eigenvalue is the same, so that the first and the second are translational modes in the Y and Z direction, respectively. Next the third mode is global torsional mode and the fourth mode is a local torsional mode in which each silo is rotated in the opposite direction with respect to the neighboring bin. In Fig.10, it can be known that the decreasing ratio of the natural period corresponding to the translational mode for more than three bins is little and this tendency is similar to the decreasing ratio of displacement in the static analysis.

For silo groups of rectangular, plan natural periods are shown in Table 4 and Fig.12 and modes of vibration are depicted in Figs.11(3) and (4). And for one row plan, natural periods and vibration modes are shown in Table 5 and in Fig.11(4), respectively.

Let us consider the case where there are empty bins, that is, the distribution of mass does not uniform. Numerical silo group has the arrangement of 3×5 bins as shown in Fig.12 and shaded bins has stored material, which corresponds to numbers of abscissa in Fig.14 and to numerators in Table 6. Natural periods in the first column are the same as in the third column in Table 4. Eccentricity of mass causes the change of vibration, for example, from the translational mode in the Y direction to the combination mode of translation in the Y direction and global torsion.

In Table 7 and Fig.15, the effect of the change of both thickness and length of connection wall on the natural period and the mode of vibration is shown for the case of 3×5 arrangement of bins. Fig.15 shows that the period decreases according to the increase of length when the thickness of the wall is 0.4m, and also that the period increases when the thickness is 0.025m.

(b) Response Analysis

The response analysis to two kinds of earthquakes is carried out within the elastic range for silo group of square plan given in Table 3, where $\rm L_g = 30m$ and bins are represented by three beam elements. The earthquake waves used in the paper are the N-S component of El Centro in 1940 and the E-W component of Hachinohe in 1968. Max acceleration is standardized to 100 gal, and the direction in which silo groups are skaked is Y-direction and the damping constant of two percent is used. The modal analysis adopting the first to the sixth modes is used in combination with the Runge-Kutta method of the time interval of 0.01 second for numerical integration.

Fig. 16 shows the maximum responses of horizontal displacement, maximum absolute accelerations, and shear coefficients in the middle point of the beam element, respectively. The maximum displacement, absolute acceleration and coefficient of shear force are 3.86cm, 542gal and 0.31, respectively, in the case of an isolated silo of 1×1 under the El Centro N-S wave. In addition, it can be known that the maximum displacements take the values from 0.43 to 0.78cm for 3×3 and 5×5 and that the maximum coefficients of shear force cover 0.19 to 0.31.

In order to investigate whether six modes adopted in the modal analysis

are satisfactory or not, the previous results are compared with values obtained by selecting up to the second mode, and it was proved that both displacement responses were almost the same and that the difference as to the coefficient of shear force was about 10 percent. Fig. 17 and 18 shows the examples of time histories.

In Fig.19, response curves of horizontal displacement to the harmonic force with amplitude of 100 gal, which is applied at the base of the silo group, are depicted.

CONCLUSION

The static and the dynamic behaviours of silo group were investigated by using "Beam Group Model." Major points of interest found from the study reported in this paper are summarized below.

- (1) "Beam with Rigid Bar Model" can be used to grasp the overall mechanical behaviours of silo groups by small degrees-of-freedom.
- (2) The shearing force of the central bin is maximum for the silo group consisted of three bins when horizontal forces are loaded at the top of each bin
- (3) In order to make a silo group monolithic, thickness of the connection wall should be more than 0.1m for H=0.2m.
- (4) In the case that the height is comparatively small, the shearing force in each bin becomes the avarage value as the increasing of number of bins. However, if the height is large, the shearing forces have the parabolic distribution.
- (5) The shape and the appearing order of modes of vibration is much influenced by the difference of number of bins in both Y- and Zdirections.
- (6) Maximum responses due to two kinds of earthquakes and response curves to the harmonic force are depicted to get magnification factors and coefficient of shearing force.

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APPENDIX: NOTATION

Lg : Height of bin (m)
R : Radius of bin (m)
H : Thickness of bin (m)

L : Length of connection wall (m)
Hw : Thickness of connection wall (m)

E : Young's modulus of bin and connection wall (t/m^2)

ν : Poisson ratio of bin and connection wall ρ : Density of bin and connection wall (t/m^3)

 ho_{S} : Density of stored material (t/m³) u : Vertical displacement of bin w : Horizontal displacement of bin

Notations used in Dynamic Analysis indicate the following modes.

Xn : Vertical mode in X direction
Yn : Horizontal mode in Y direction

Gn : Global torsional mode
Ln : Local torsional mode

Bn : Bow shape mode

 ${\tt YG}_n$: Combination mode of translation in Y direction and global

torsion

 $\mathbf{Z}_{n}\,\mathbf{Xm}$: Combination mode of translation in Z direction and vertical

mode in X direction

where n and m represent the number of mode.

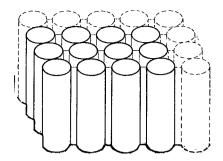


Fig.1: Silo Group Structure

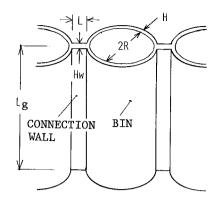


Fig.2: Symbols

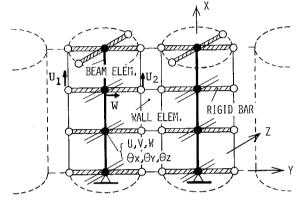


Fig.3: BEAM GROUP MODEL

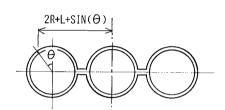


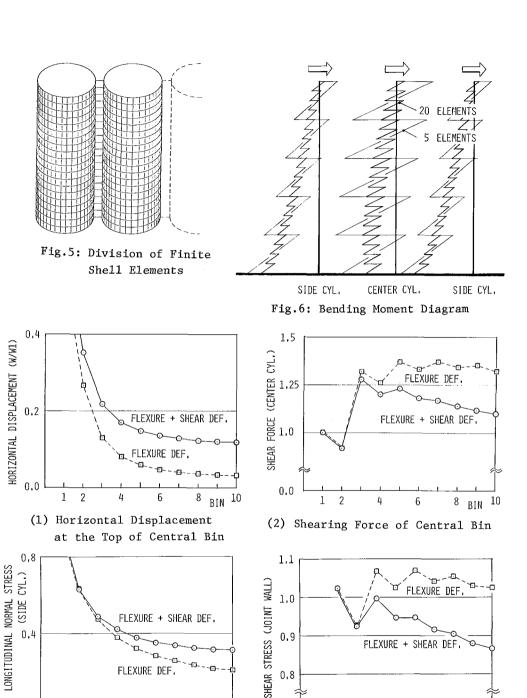
Fig.4: Silo Group with Three Bins

Table 1: Characteristics of Three Models

MODEL	Computation of Displacement	Computation of Shear Force	Estimation of Stiffness of Cross Wall	Computation of Local Stress
BEAM	able	able	unable	unable
BEAM GROUP	able	able	able	unable
SHELL (FEM)	ab1e	able	able	ab1e

Table 2: Comparison of Results by Three Models

	CE	CENTER CYLINDER			CENTER WALL
MODEL	W max	Q	てmax	Ox max	7 max
BEAM	0.236	1.32	1.06	0.413	0.915
BEAM GROUP	0.217	1.28	1.03	0.488	0.920
SHELL(FEM)	0.233	1.26	1.01	0.456	0,879



0.0 0.0 BIN 10 1 2 6 8 1 2 (3) Longitudinal Normal Stresses (4) Shearing Stresses of Central at the Bottom of Outer Bin

FLEXURE + SHEAR DEF.

FLEXURE DEF.

0.4

Fig.7: Effect of Number of Bins (Flexural Deformation and Shear Deformation)

0.9

0.8

Connection Wall

FLEXURE + SHEAR DEF.

BIN 10

8

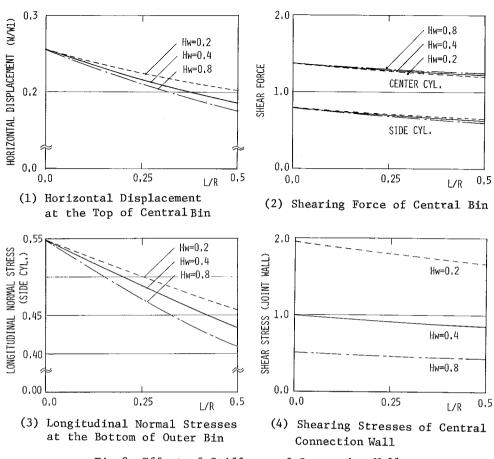


Fig.8: Effect of Stiffness of Connection Wall

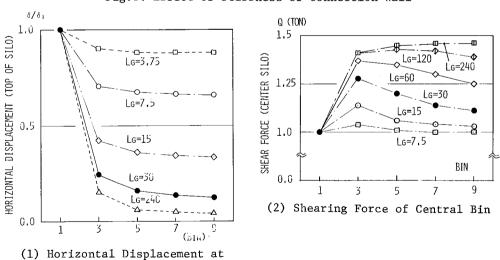


Fig.9: Effect of Height of Bin

the Top of Central Bin

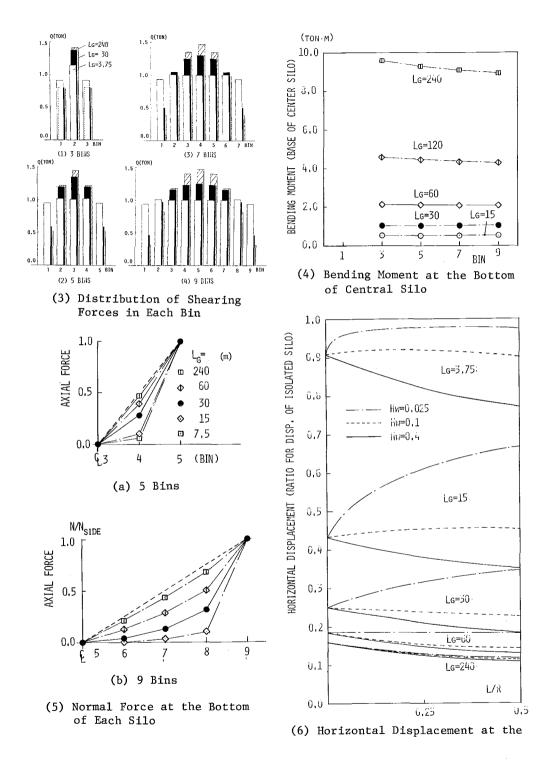


Fig.9: Effect of Height of Bin

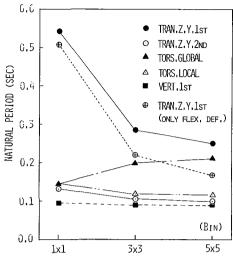


Fig.10: Natural Periods in the Case of Square Plan

Table 3: Natural Periods in the Case of Square Plan

	NO.	BINS		
	MODE	1×1	3×3	5×5
	1st	0.542 Y1	0, 286 Y1	0.249 Y1
	2nd	0.542 Z1	0.286 Zl	0.249 Z1
	3rd	0.145 L1	0.200 Gl	0.214 G1
FLEXUAL	4th	0.133 Y2	0.118 L1	0.115 L1
DEF.	5th	0.133 Z2	0.106 Y2	0.100 Y2
SHEAR	6th	0.095 X1	0.106 Z2	0.100 Z2
DEF.	7th	0.070 L2	0.091 X1	0.090 X1
	8th	0.070 Y3	0.073 G2	0.079 G2
	9th	0.053 23	0.064 —	0.076 -
	10th	0.039 L3	0.064 —	0.070 —
	lst	0.507 Y1	0.220 Y1	0.166 Y1
	2nd	0.507 Z1	0.220 Z1	0.166 Z1
	3rd	0.145 L1	0.120 G1	0.116 G1
	4th	0.097 Y2	0,118 L1	0.115 L1
ONLY FLEXUAL DEF.	5th	0.097 Z2	0.091 X1	0.090 X1
	6th	0.095 X1	0.069 Y2	0.071 Y2
	7th	0.053 L2	0.069 Z2	0.071 Z2
	8th	0.039 Y3	0.052 -	0.065 —
	9th	0.039 Z3	0.044 -	0.053 —
	10th	0.039 L3	0.044 -	0.053 -

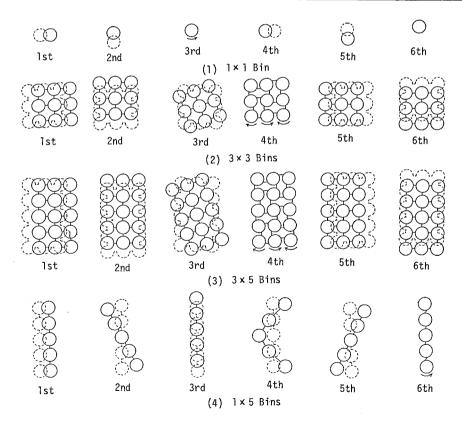


Fig.11: Vibration Modes

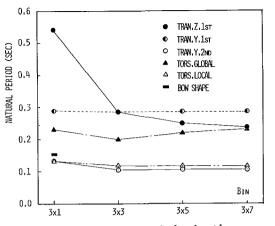
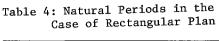


Fig.12: Natural Periods in the Case of Rectangular Plan



NO. MODE	BINS				
	1×1	1×3	1×5	1×7	
lst	0.542 Yl	0.542 Y1	0.542 Y1	0.542 Y1	
2nd	0.542 Zl	0,288 Zl	0.315 G1	0.376 G1	
3rd	0.145 L1	0,232 G1	0.250 Zl	0.253 B1	
4th	0.133 Y2	0.153 B1	0.204 Bl	0.237 Zl	
5th	0.133 Z2	0.138 Y2	0.158 G2	0.194 B2	
6th	0.095 X1	0.130 L1	0.137 L1	0.161 B3	
7th	0.070 L2	0.107 Z2	0.133 Y2	0.142 B4	
8th	0.070 Y3	0.093 X1	0.127 G3	0.133 Y2	
9th	0.053 Z3	0.086 G3	0.108 B2	0.131 L1	
10th	0.039 L3	0.070 Y3	0.101 Z2	0.125 B5	

Table 5: Natural Periods in the Case of One Row Plan

NO. MODE	BINS			
	3×1	3×3	3×5	3×7
1st	0.542 Z1	0.286 Yl	0.286 Y1	0.286 YI
2nd	0.288 Y1	0.286 Z1	0.249 Z1	0.237 Z1
3rd	0.232 G1	0.200 G1	0.218 G1	0.235 G1
4th	0.153 Bl	0.118 L1	0.117 L1	0.116 L1
5th	0.133 Z2	0.106 Y2	0.106 Y2	0.106 Y2
6th	0.130 L1	0.106 Z2	0.100 Z2	0.097 Z2
7th	0.107 Y2	0.091 X1	0.090 X1	0.092 G2
8th	0.093 X1	0.073 G2	0.084 G2	0.090 X1
9th	0.086 G2	0.064 Y3	0.070 Y3	0.075 —
10th	0.070 Z3	0.064 Z3	0.064 Z3	0.072 —

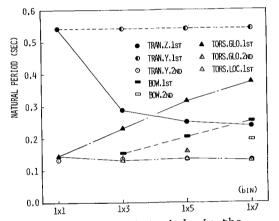


Fig.13: Natural Periods in the Case of One Row Plan

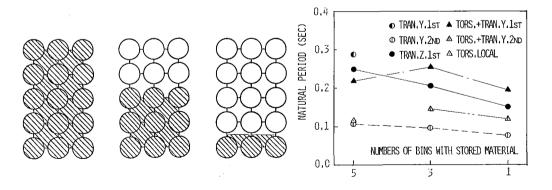


Fig.14: Natural Periods in the Case of Eccentricity of Mass

Table 6: Natural Periods in the Case of Eccentricity of Mass

NO.	NUMBERS OF BIN LOADED				
MODE	5/5	3/5	1/5		
lst	0.286 Y1	0.254 YG1	0.195 YG1		
2nd	0.249 Zl	0.205 Z1	0.151 Z1		
3rd	0.218 G1	0.147 YG2	0.118 YG2		
4th	0.117 L1	0.100 G2	0.076 Z2X1		
5th	0.106 Y2	0.095 Y2	0.076 Y2		
6th	0.100 22	0.090 Z2X1	0.074 G2		

Table 7: Natural Periods as a Parameter Stiffness of Connection Wall

OI	Stir	iness o	I Comile	CLIOII W
WALL THICK. (Hw)	NO.	L/R		
	MODE	0.05	0.25	0.50
0.40	lst	0.305 Yl	0.286 Yl	0.259 Y1
	2nd	0.264 Z1	0.249 Zl	0.227 Zl
	3rd	0.227 G1	0.218 G1	0.207 G1
	4th	0.110 Y2	0.117 L1	0.131 L1
	5th	0.104 22	0.106 Y2	0.127 Y2
	6th	0.094 X1	0.100 Z2	0.123 Z2
0.025	1st	0.317 Y1	0.339 L1	0.351 L1
	2nd	0.280 Z1	0.332 Y1	0.346 Y1
	3rd	0.263 G1	0.311 L2	0.339 L2
	4th	0.199	0.302 L3	0.335 L3
	5th	0.188 L1	0.299 L4	0.330 L4
	6th	0.146 G1	0.288 L5	0.321 L5

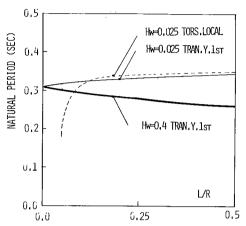


Fig.15: Natural Periods as a Parameter of Stiffness of Connection Wall

0.6

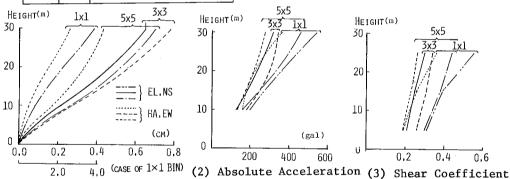
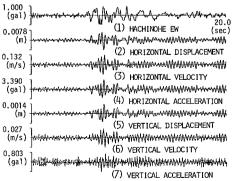


Fig.16: Maximum Response and Shear Coefficient in the Case of Square Plan



(1) Horizontal Displacement

Fig.17: Time histories of Response Resulted by El Centro N-S

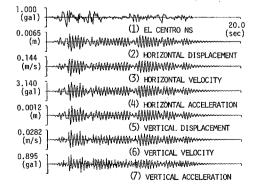


Fig.18: Time histories of Response Resulted by Hachinohe E-W

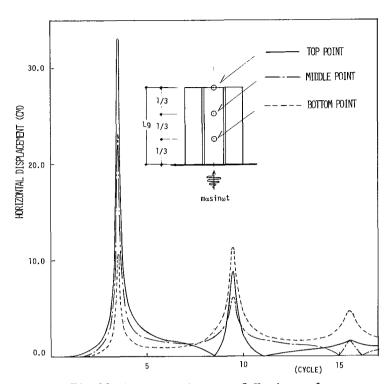


Fig.19: Response Curves of Horizontal
Displacement to the Harmonic Force