MATRIX CONDENSATION TECHNIQUE IN DISCRETE LIMIT ANALYSIS OF FRAMED STRUCTURES

by

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1 INTRODUCTION

The formulations of Rigid Bodies-Spring Models for the discrete limit analysis of framed structures were previously carried out by Kawai[1)] and Toi[2)], by assuming that each rigid element has 6 degrees of displacement freedom independently in order to take into account the shearing deformations. In this note the matrix condensation technique in neglecting the shearing deformations is briefly discussed, by using the in-plane deformation problem of framed structures as an illustrative example.

2 FORMULATION

Consider two adjacent rigid bars shown in Fig. 1, which are connected by springs resisting the relative movement at the node 2.

2.1 Definition of Coordinate Systems

Some Cartesian coordinate systems used in the formulation are described below in the form of (origin ; coordinate axis 1, coordinate axis 2).

(i) global coordinate system (0 ; x, z)

(ii) element coordinate system (4 ; x_A, z_A), (5 ; x_B, z_B)

(iii) node coordinate system (1 ; x_1), (2 ; x_2), (3 ; x_3)

(iv) joint coordinate system (2; x', z')

The node coordinate axis x_i is defined as the normal direction at the node point i.

2.2 Stiffness Formulation

(i) Displacement Functions

The displacement field in the rigid element A can be expressed by the following equations:

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$$u_A = a_2^A z_A + a_4^A$$
$$w_A = a_6^A \qquad (1)$$

(ii) Relative Displacements

The relative movements can be defined by the following two components:

$$\phi_y = a_2^B - a_2^A$$
$$\delta_z = \{l_z^B (u_B)_2 + n_z^B (w_B)_2\} - \{l_z^A (u_A)_2 + n_z^A (w_A)_2\}$$

where (2)

$$l_z^A = \cos (z', x_A) \qquad n_z^A = \cos (z', z_A)$$
$$l_z^B = \cos (z', x_B) \qquad n_z^B = \cos (z', z_B)$$

or in the matrix form

$$\{d\} = [B] \{a\}$$

where (3)

$$\{d\}^t = \lfloor \phi_y \ \delta_z \rfloor$$
$$\{a\}^t = \lfloor a_2^A \ a_4^A \ a_6^A \ a_2^B \ a_4^B \ a_6^B \rfloor$$

Substituting eq. (1) into eq. (2), the final form of the matrix [B] can be obtained, which is shown in Table 1.

(iii) Degrees of Displacement Freedom

When the shearing deformations are neglected, the degrees of displacement freedom are set as follows:

$$u_1 = l_x^1 (u_A)_1 + n_x^1 (w_A)_1$$
$$u_2 = l_x^2 (u_A)_2 + n_x^2 (w_A)_2$$
$$w_4 = a_6^A$$

where (4)

$$l_x^1 = \cos (x_1, x_A) \qquad n_x^1 = \cos (x_1, z_A)$$
$$l_x^2 = \cos (x_2, x_A) \qquad n_x^2 = \cos (x_2, z_A)$$

Substituting eq. (1) into eq. (4) and solving with respect to a_i^A, the following relation can be obtained:

$$\{a\} = [A] \{u\}$$

where (5)

$$\{u\}^t = \lfloor u_1 \ u_2 \ u_3 \ w_4 \ w_5 \rfloor$$

The matrix [A] is shown in Table 2.

(iv) Resultant Forces

The resultant forces in the springs are calculated by the following relationship:

$$\{s\} = [D]\ \{d\} \tag{6}$$

where

$$\{s\}^t = \lfloor M_y \ N_z \rfloor$$

$$[D] = \ulcorner k_{ry} \ k_{pz} \lrcorner$$

in which M_y and N_z are the bending moment and the axial force respectively. In case of elastic deformation the spring constants can be determined by the following formulas:

$$k_{ry} = EI \ / \ (|(z_A)_2| + |(z_B)_2|)$$

$$k_{pz} = EA \ / \ (|(z_A)_2| + |(z_B)_2|) \tag{7}$$

while in case of plastic deformation the four components in the matrix [D] can be determined by the conventional flow theory.

(v) Stiffness Equation

Applying the Castigliano's theorem to the strain energy stored in the connection springs calculated with the aid of eqs. (3), (5), and (6), the following stiffness equation can be obtained:

$$[k]\ \{u\} = \{f\} \tag{8}$$

where

$$[k] = [A]^t\ [B]^t\ [D]\ [B]\ [A]$$

$$\{f\}^t = \lfloor f_1 \ f_2 \ f_3 \ p_4 \ p_5 \rfloor$$

3 CONCLUSION

The matrix condensation technique in the discrete limit analysis of plane frames was briefly described in this note. The extension to the general space frames is easy and now under way. The same procedure for thin shell structures has already been established and was reported in Ref. 3)

REFERENCES

1) Kawai, T. "Studies on the Ultimate Strength Analysis of a Ship Structure by Means of a New Discrete Thin-Walled Beam Elements (1st Report) ", J. of the Soc. of Nav. Arch. of Japan, Vol. 150, (Dec. 1981) p. 381

2) Toi, Y. "Nonlinear Dynamic Analysis of Frames and Axisymmetric Shell

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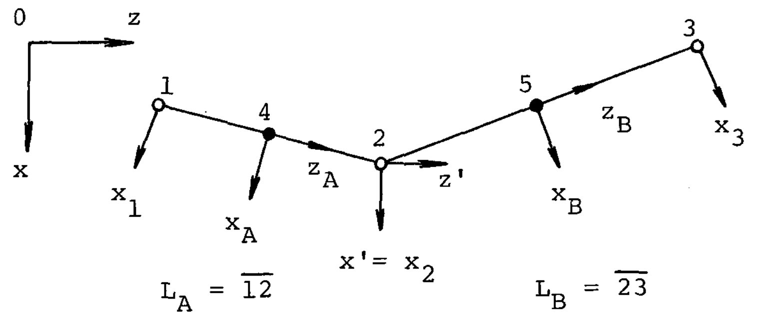

Fig. 1 Two adjacent rigid bars

Table 1 Matrix [B] in eq. (3)

	a_2^A	a_4^A	a_6^A	a_2^B	a_4^B	a_6^B
ϕ_y	-1	0	0	1	0	0
δ_z	$-l_z^A (z_A)_2$	$-l_z^A$	$-n_z^A$	$l_z^B (z_B)_2$	l_z^B	n_z^B

Table 2 Matrix [A] in eq. (5)

	u_1	u_2	u_3	w_4	w_5
a_2^A	$\frac{-l_x^2}{l_x^1 l_x^2 L_A}$	$\frac{l_x^1}{l_x^1 l_x^2 L_A}$	0	$\frac{l_x^2 n_x^1 - l_x^1 n_x^2}{l_x^1 l_x^2 L_A}$	0
a_4^A	$\frac{l_x^2 (z_A)_2}{l_x^1 l_x^2 L_A}$	$\frac{-l_x^1 (z_A)_1}{l_x^1 l_x^2 L_A}$	0	$\frac{l_x^1 n_x^2 (z_A)_1 - l_x^2 n_x^1 (z_A)_2}{l_x^1 l_x^2 L_A}$	0
a_6^A	0	0	0	1	0
a_2^B	0	$\frac{-l_x^3}{l_x^2 l_x^3 L_B}$	$\frac{l_x^2}{l_x^2 l_x^3 L_B}$	0	$\frac{l_x^3 n_x^2 - l_x^2 n_x^3}{l_x^2 l_x^3 L_B}$
a_4^B	0	$\frac{l_x^3 (z_A)_3}{l_x^2 l_x^3 L_B}$	$\frac{-l_x^2 (z_A)_2}{l_x^2 l_x^3 L_B}$	0	$\frac{l_x^2 n_x^3 (z_A)_2 - l_x^3 n_x^2 (z_A)_2}{l_x^2 l_x^3 L_B}$
a_6^B	0	0	0	0	1