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PROBABILISTIC MODELING OF SPATIAL VARIABILITY OF SEISMIC GROUND MOVEMENT

by

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SYNOPSIS

A simple new procedure is presented for estimating the spatial variability of seismic ground movement. The seismic ground movement is interpreted as a random function of space variable. The parameter which characterizes the scale of the spatial variation of ground movement is identified from the damage statistics of underground pipelines. The scale of the spatial variation is found to be a function of the predominant period of the site area in question.

INTRODUCTION

The seismic ground movement is known to have considerable variation from point to point in space as well as in time as shown in Fig. 1. Historically, earthquake engineers have focused their attention to the time variability, for example, the time history of acceleration, which is a fundamental factor in dynamic analysis and seismic design of building types of structures. However, recent studies [1,2] have reported that the spatial variation of seismic ground movement is the major source of seismic forces acting on the underground structures such as subways, pipelines and storage tanks below ground etc., which by nature extend for large distance and are widely spaced. The spatial variation has also been understood to be an important parameter affecting the torsional seismic response of tall buildings and of nuclear containment structures having wide spread foundations.

The quantitative information of the spatial variation are very limited so far in earthquake engineering field because of the complexity of the phenomena and of measurement difficulty in practice. The current studies available regarding the spatial variation are in all based on an wave propagation theory. Newmark [2] showed that the strain in ground in the axial direction of wave propagation is directly proportional to the velocity, v, of soil particle at the point in question, and inversely proportional to the wave propagation speed, c, through the ground (v/c). He also showed that the bending strain in ground is similarly proportional to the acceleration, a, of the ground and inversely proportional to the square of the wave speed (a/c^2). These simple relations are obtained by differentiating the solution of the equation of motion in one dimensional wave propagation theory. Hence, these simple relations can not provide the practical values for the strains in ground unless more sophisticated models to be able to assign the specific numerical values to the wave speed, the velocity and the acceleration of soil particle can be established. By taking into account for the horizontally travelling Rayleigh wave in the model, Takada et al. [3] identified the relative displacement between two points from the data recorded during several earthquakes.

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Shinozuka et al. [4] also showed a simulation method for estimating ground strain by modeling the seismic wave as a type of Rayleigh wave horizontally propagating in multiple soil layers. By analysing the total of 123 observed seismic strains in underground structures during 91 earthquakes in Japan, Nakamura et al. [5] reported that the axial strain in buried pipes is more strongly correlated with the two parameters, the peak ground acceleration a_{max} and the predominant period of ground T_g, than with a_{max} alone, suggesting the importance of the ground condition in estimating seismic strain in buried pipes.

The studies mentioned above assume essentially the types of waves and the homogeneity of soils or soil layers. They also use the data observed during low intensity earthquakes causing no damage to underground structures to estimate the parameters involved in their models. It is well known, however, that the seismic wave is composed of many types of waves reflected, refracted by the interface between the soils with different soil properties. Even within nominally homogeneous ground, soil properties may exhibit spatial variability. The presence of the soil with weak strength may cause the failure of soil when a strong earthquake attacks the site. Thus, more realistic model might be required which can incorporate the realistic nature of intense seismic ground movement mentioned above.

This paper presents a simple procedure for estimating the scale of spatial variation of ground displacement during strong earthquake. The principal idea exists in the interpretation that the ground displacement is a random function of space variables. This interpretation together with the recently reported damage statistics of buried pipes with small diameter makes it possible to rationally identifying the scale of spatial variation of seismic ground displacement during strong earthquakes.

PROBABILISTIC MODEL

It is well known that the axial stress in buried small pipe is much higher than the bending stress under the action of seismic forces. Therefore the axial behavior of the buried pipe is considered in this model. The pipe is assumed to be a straight slender beam on flexible foundation as shown in Fig. 2. From these assumptions, the axial equation of motion of the pipe may be given by neglecting the inertial force [1] to the pipe as,

$$- d^2u/dx^2 + n^2u = n^2r(x) \quad , \quad n^2 = K_h/(ES) \tag{1}$$

where E = the modulus of elasticity of pipe, S = the cross sectional area of pipe material, K_h = the stiffness of soil longitudinal to pipe axis per unit length and u = the longitudinal (axial) displacement of pipe. The quantity $r(x)$, which is assumed here to be a homogeneous Gaussian random function with zero mean and variance D^2, represents an ensemble of seismic axial displacement at the time be fixed. In general, the seismic ground displacement r varies with the time and the space. The variation of r with respect to the time produces the particle soil velocity and acceleration which affect the dynamic behaviors of buried pipes. On the other hand, the variation with the space variables forms the relative displacement between two points. Numerous studies [1,5] on the seismic behaviors of buried pipes suggest that the relative displacement between two points is of great importance. Therefore, the assumption is adopted the time be fixed in Eq.(1).

The Fourier transform of Eq.(1) with respect to space variable x gives the well known relationship between "input" and "output" as,

$$S_u(k) = S_r(k)n^4/(k^2 + n^2)^2 \tag{2}$$

where $S_j(k)$ is the one sided spectral density function for $j = u, r$. The quantity k is called the "wave number" with dimension of (1/length). By using the Fourier algorithm, the one sided spectral density function and the variance for the pipe strain, $S_\varepsilon(k)$ and σ_ε^2, can be obtained as,

$$S_\varepsilon(k) = k^2 S_u(k) \tag{3}$$

$$\sigma_\varepsilon^2 = \int_0^\infty S_\varepsilon(k)dk \tag{4}$$

Here it is assumed that a possible analytical expression for the correlation function $R_r(\xi)$ of the homogeneous random function $r(x)$ with zero mean and variance D^2 takes a form as,

$$R_r(\xi) = D^2\exp[-(b\xi)^2] \cos\theta\xi \tag{5}$$

The distance $1/b$, where the amplitude of R_r decays to e^{-1} times its initial amplitude, may be called as the correlation distance. The term $\cos\theta\xi$ represents the phenomena of the wavy form in correlation function due to wave propagation through soil layer. In the face of lack of accurate information about the propagation of wave, it is in this paper decided to choose $\cos\theta\xi = 1$. Thus, the physical fact is taken into account for in this model that the correlation of values of the function $r(x)$ at two points, with the coordinates x and $x + \xi$, may attenuate as the distance ξ increases.

From the Wiener-Khintchine relation together with the expression of the correlation function given by Eq.(5), the one sided spectral density function for the seismic ground displacement $r(x)$ can be obtained as,

$$S_r(k) = [D^2/(b\sqrt{\pi})]\exp[-k^2/(2b)^2] \tag{6}$$

The behavior of $S_r(k)$ given by Eq.(6) is shown in Fig. 3. It can be seen from Fig. 3 that the amplitude is very low in the range of $k/b > 5.0$. In order to estimate the parameter b, the mean break rate v_f^* of the pipe (number of breakes per km) will be considered in the next paragraph.

For the homogeneous Gaussian random process with zero mean, the mean number v_f of crossings of the tensile strain in buried pipe, subjected to the homogeneous Gaussian random displacement, over the limiting admissible strain ε_a in unit length of the pipe can be obtained from the Rice's formula [6] as,

$$\begin{aligned} v_f &= L_e^{-1} \exp[-\varepsilon_a^2/(2\sigma_\varepsilon^2)] \\ L_e &= 2\pi[\sigma_\varepsilon^2/\sigma_{\varepsilon\varepsilon}^2]^{1/2} = 2\pi[\int_0^\infty S_\varepsilon(k)dk/\int_0^\infty k^2 S_\varepsilon(k)dk]^{1/2} \end{aligned} \tag{7}$$

If $\varepsilon_a = 0$, the exponential factor in Eq.(7) goes to one, and $v_f = L_e^{-1}$ becomes the average rate of zero-crossings with positive slope. Hence, the quantity L_e represents the average wave length of the random function of pipe strain. To evaluate L_e, the numerical integrations for σ_ε^2 and $\sigma_{\varepsilon\varepsilon}^2$ in Eq.(7) were performed. Results are shown in Fig. 4. From these results in Fig. 4, the following approximations may be reasonable for σ_ε^2 and $\sigma_{\varepsilon\varepsilon}^2$

$$\begin{aligned} \sigma_\varepsilon^2 &= 2D^2b^2[1 - 3.44\exp(-1.07\sqrt{n/b})] \\ & \qquad\qquad n/b > 3.0, \\ \sigma_{\varepsilon\varepsilon}^2 &= 12D^2b^4[1 - 4.14\exp(-1.00\sqrt{n/b})] \end{aligned} \tag{8}$$

for $n \gg b$, the exponential factor in Eq.(8) goes to zero, and v_f in Eq.(7) becomes

$$
\begin{aligned}
v_f &= [\sqrt{6}b/(2\pi)]\exp[-\varepsilon_a^2/(4D^2b^2)], \quad \text{(No. of Breaks/m)} \\
v_f^* &= v_f \times 10^3, \quad \text{(No. of Breaks/Km)}
\end{aligned}
\tag{9}
$$

where v_f^* is the mean number of breaks per one Km when $1/b$ and D are measured in m. The numerical study performed by using Eqs.(7) and (8) together with a procedure in next section indicates that the condition of $n \gg b$ is valid for small buried pipes like water or gas distribution pipes though these results are not shown in this paper.

Equation (9) represents the relationship among the mean break rate v_f^*, the variance D^2 of seismic ground displacement, the allowable level of pipe axial strain ε_a and the parameter b which characterizes the scale of spatial variation of seismic ground displacement as previously described. Equation (9), thus, makes it possible to estimate the scale of spatial variation b from the data of v_f^*, D^2 and ε_a, without analysing the spatial data recorded by the synchronized transducers.

ESTIMATION OF SPATIAL VARIATION PARAMETER

In order to estimate the spatial variation parameter b in Eq.(9), the statistical damage data of buried small pipes [1] are used. These data are reported in terms of the correlation between the mean number of breaks of buried pipes per one Km (v_f^*) and the peak ground acceleration estimated from the structural damage data. On the other hand, v_f^* in Eq.(9) is a function of the parameter b, the allowable strain of pipe ε_a, and the variance D^2 of seismic ground displacement varying in space. The allowable strain ε_a may be estimated from the mechanical properties of the pipes in the damage data [1] as 10^{-3}. Hence, the question at hand is to determine the relation between the variance D^2 and the peak ground acceleration a_{max}.

The maximum amplitude of seismic ground displacement r_{max} is related to the rms (root mean square) of seismic ground displacement D. The relationship is, of course, a probabilistic in nature. Specifically, the value of r_{max}/D, which has a probability of e^{-1} (0.37) of no exceeding during one Km, is approximately expressed as,

$$
r_{max}/D = [\ 2\ln b + 12.22\ ]^{1/2} \tag{10}
$$

Equation (10) is derived on the basis of the common assumption that the crossings of a specified level occur as a Poisson arrival process [6,7].

By utilizing Eq.(10) and the study of Kanai [8] that shows the relation among r_{max}, a_{max} and the predominant period T_g (sec) of ground, the relationship between the rms displacement D and the actual maximum acceleration a_{max} is obtained as,

$$
D = 2.53[\ 2\ln b + 12.22\ ]^{-1/2} T_g^{\,2} a_{max} \times 10^{-4} \tag{11}
$$

where D is in m and a_{max} is in cm/s^2 (gal).

Substitution of Eq.(11) into Eq.(9) gives the required relationship between v_f^* and a_{max} when T_g and b are known. This relationship is shown in Fig. 5 by the full lines for the four combination of values of T_g and b indicated in Fig. 5. These final combinations of T_g and b indicated in Fig. 5 are obtained by inspecting the results for several combinations in conjunction with the empirical curve by Kubo and Katayama [1] which is also shown in Fig. 5. This empirical curve does not explicitly show the effect of ground conditions on pipe damages. Experiences, however, may show that the pipes buried in soft ground are more vulunerable than those in harder ground. Thus, these effect of ground conditions are approximately taken into account for in the analytical curves of Fig. 5 by choosing the appropriate combinations of values, T_g and b. The upper two curves 1 and 2 in Fig. 5 correspond to the damage rate of the pipes buried in relatively soft ground and the curves 3 and 4 to those in harder ground. In Fig. 5, the variations of v_f^* with the parameter b are also shown by the dashed lines.

From these parameter example studies, the relation between b and T_g can be established as shown in Fig. 6. The line in Fig. 6 may be expressed as

$$\log b = -[1.564\log T_g + 1.971] \tag{12}$$

This simple relation in Eq.(12) may be used to estimate the scale of spatial variability of ground displacement during intense seismic excitation when the predominant period T_g of ground is assigned.

SAMPLE FUNCTION FOR GROUND DISPLACEMENT VARYING IN SPACE

In a usual manner [9], the sample function $\overline{r(x)}$ of ground axial displacement can be effectively simulated from the equation as

$$\overline{r(x)} = \sum_{j=1}^{N}[2S_r(k_j)\Delta k]^{1/2}\cos(k_j x + \phi_j) \tag{13}$$

$$k_j = (j - 0.5)\Delta k$$

$$\Delta k = k_u/N$$

where k_u is the upper limit of the wave number k. From the observation of the behavior of spectral density function shown in Fig. 3, it may be sufficient to take the value $k_u > 5b$. The parameter ϕ_j is the random phase distributed uniformly between 0 and 2π. The term N is the maximum number of summation.

For a numerical example, the following data are used: N = 1024, k_u = 1.0 and D = 1.0m. By using these data, the sample functions of ground axial displacement for four ground conditions (1,2,3, and 4) are shown in Fig. 7. It can be seen from Fig. 7 that the spatial variation in the relatively soft grounds (1 and 2) corresponding to the upper two curves in Fig. 7 is smoother than that in harder grounds (3 and 4) corresponding to the lower two curves, suggesting that the seismic wave with longer wave length (long wave) is predominant in soft ground and the short wave in harder ground.

CONCLUSIONS AND APPLICATIONS

A probabilistic model is developed for the seismic ground displacement varying point to point in one dimensional space. The interpretation that the seismic ground displacement at the time be fixed is a random function of space variable together with the damage statistics of underground pipes makes it possible to identify the scale of the spatial variation of seismic ground displacement during strong earthquakes. From this study, the scale of spatial variation is obtained as a function of the predominant period of ground. The procedure in this paper provides an efficient tool for studying the spatial variation of seismic ground displacement and for the probabilistic seismic response analyses of underground structures. Further extension and applications of this method are currently underway at Miyazaki University to the multidimensional case and to the probabilistic seismic response analysis of underground oil storage tank and subway etc..

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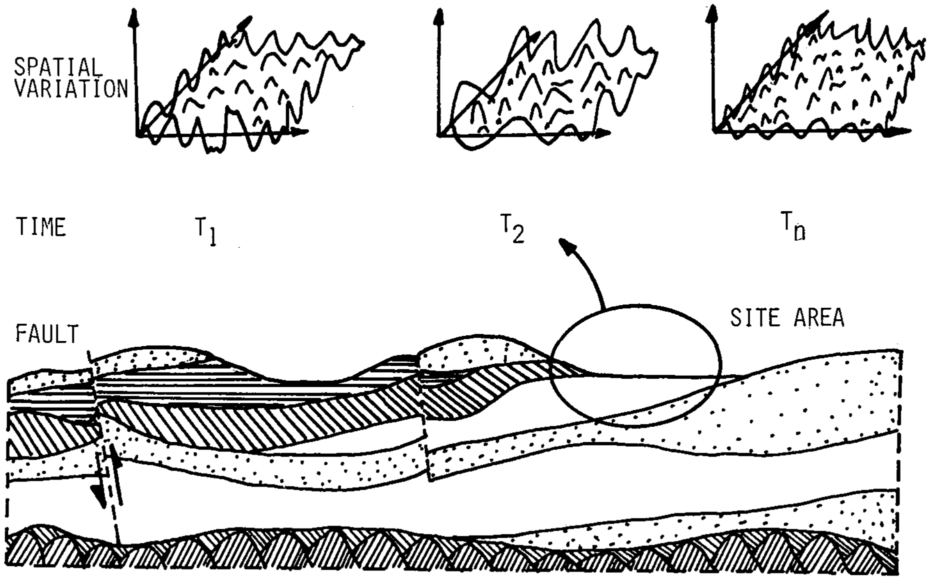

Fig. 1 Schematic Diagram Showing Seismic Source Region, Traveling Path, and Spatial Variation of Seismic Movements in a Region

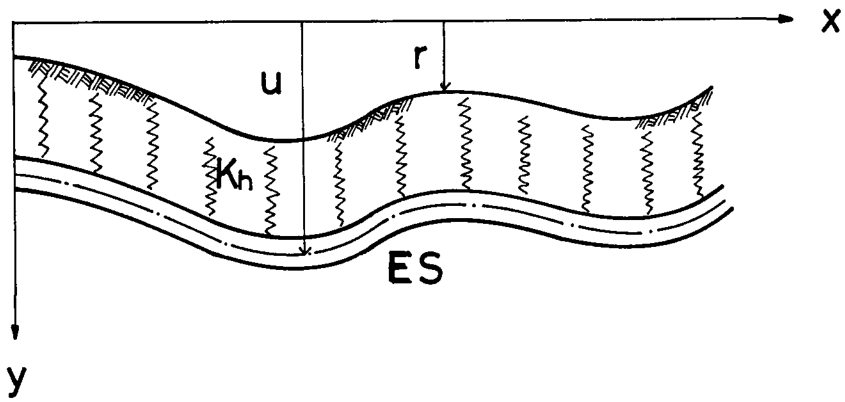

Fig. 2 Buried Pipe Model in Axial Direction and Its Notations

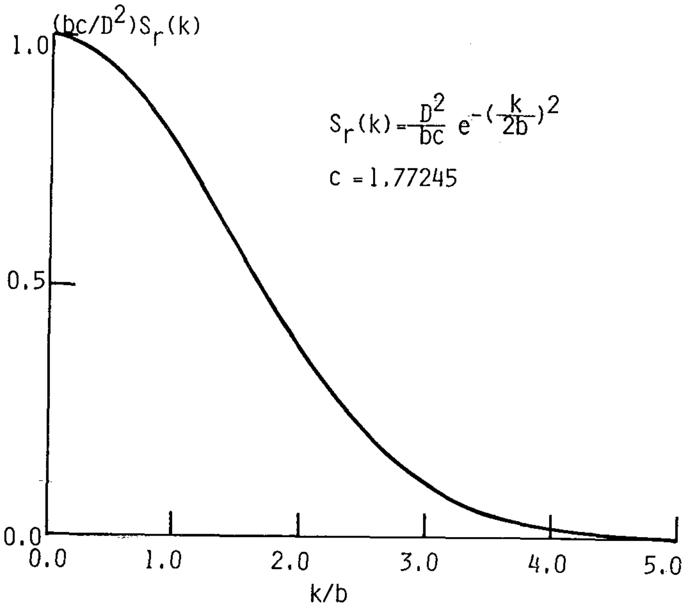

Fig. 3 Graphical Representation of Spectral Density Function for Spatial Variation of Seismic Ground Axial Displacement

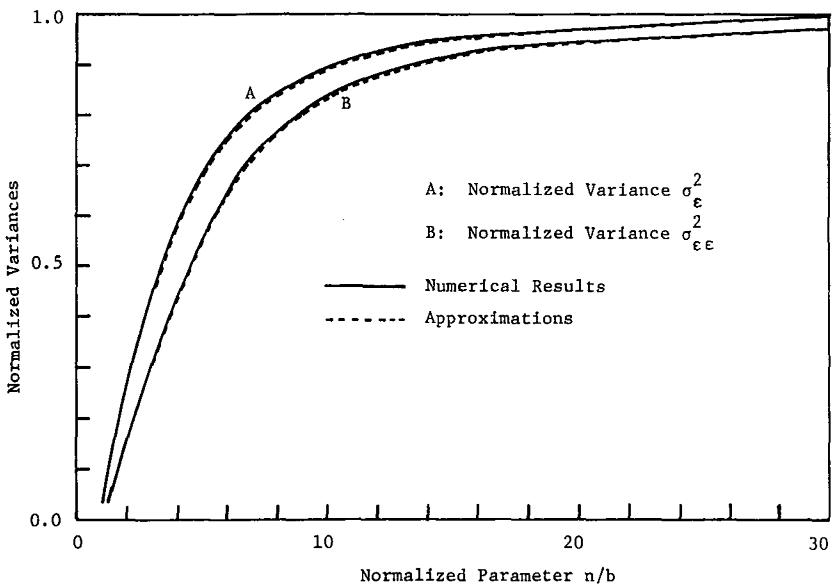

Fig. 4 Numerical Results of Integration in Eq.(7) and These Approximations

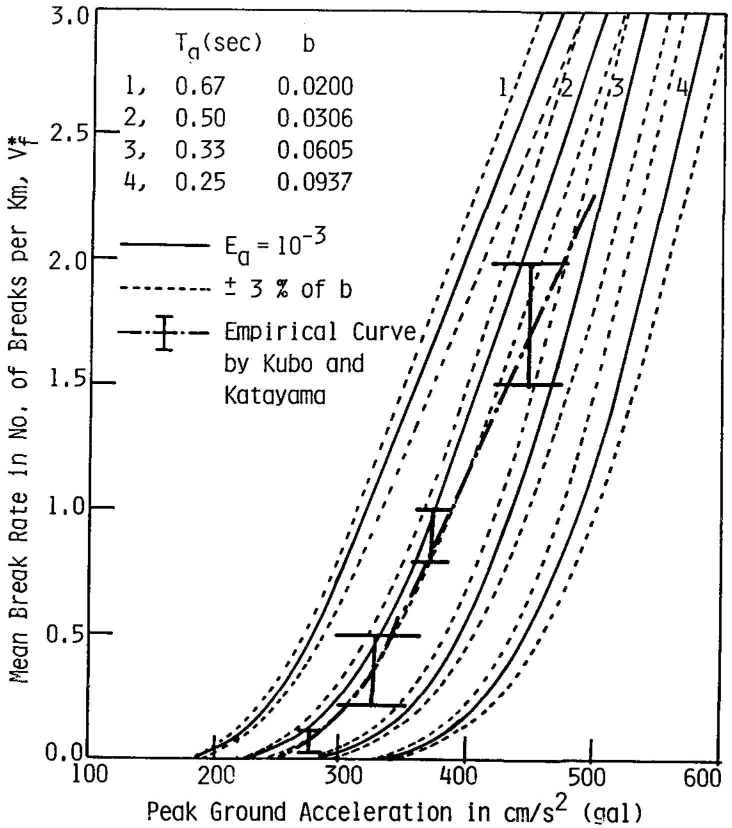

Fig. 5 Correlation Between Mean Break Rate and Peak Ground Acceleration

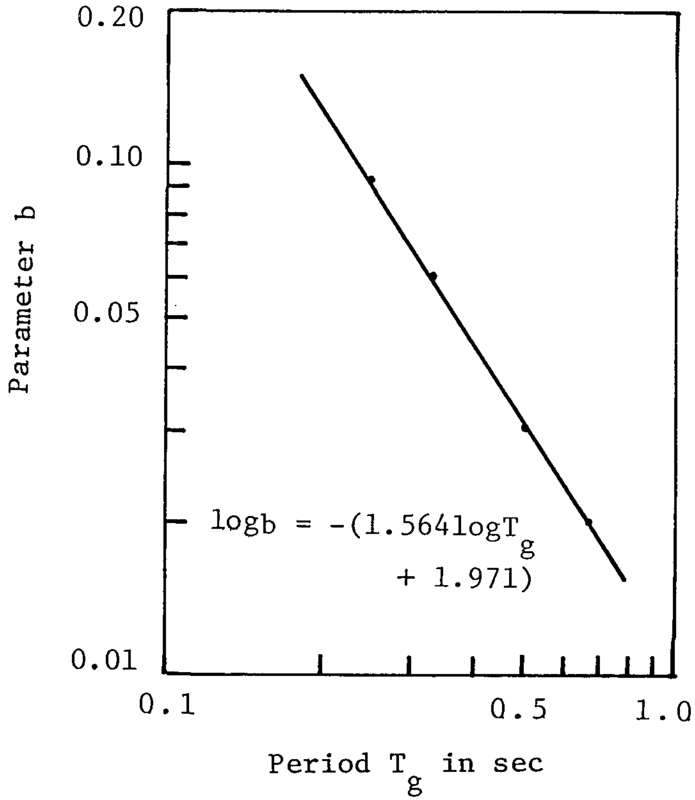

Fig. 6 Relation between the Parameter b and the Predominant Period of Ground

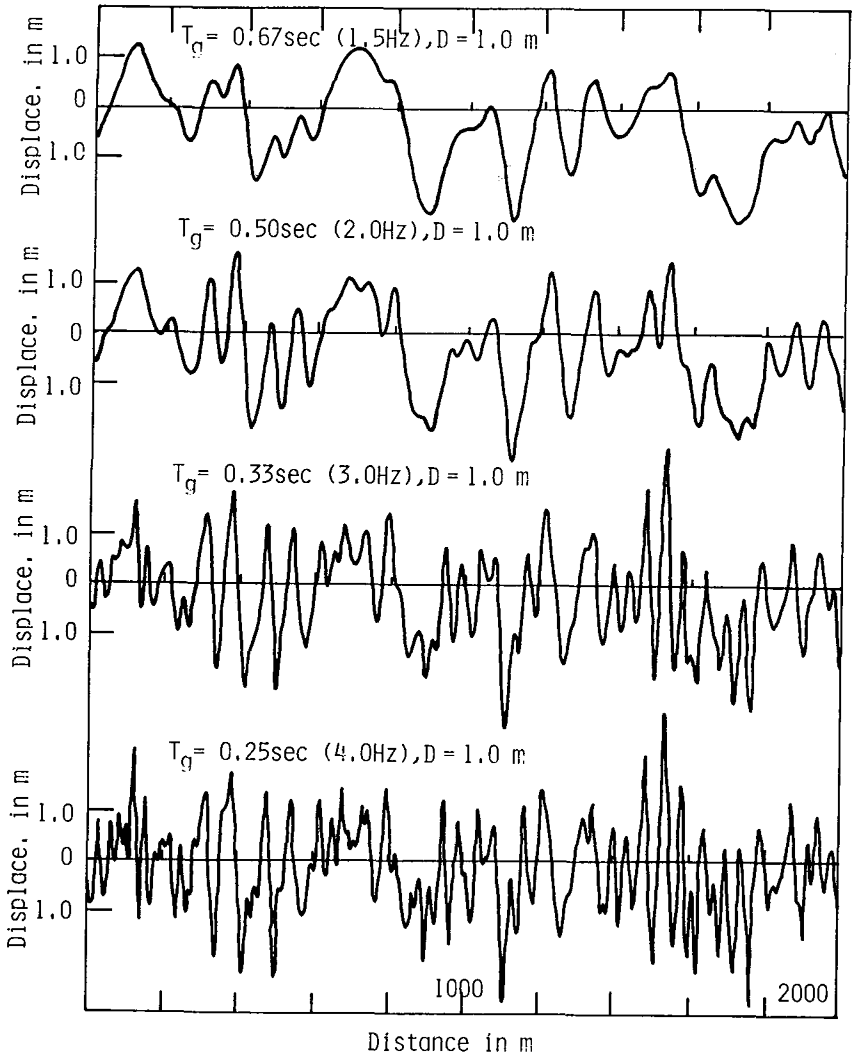

Fig. 7 Sample Functions of Variation of Seismic Ground Axial Displacement along Straight Line for Four Ground Conditions under the Assumption that the rms Displacement D is same for Four Ground Conditions