Bull. ERS, No. 14 (1981)

# ULTIMATE STRENGTH OF STEEL COLUMN BASES

by

Kenichi OHI,[I)] Hisashi TANAKA,[II)] and Koichi TAKANASHI[II)]

## SYNOPSIS

This paper deals with theoretical estimation of the ultimate strength for ordinary types of column bases in small steel buildings.

In this paper, the tensile strength of anchor bolts and the flexural strength of a base plate are chosen for the parameters governing the column base strength. Then, the column base subjected to a bending moment and an axial force is substituted by a simple mechanical model.

A rational solution for the interaction surface of flexural and axial resistances in the column base is presented through the upper and the lower bound theorems in the limit analysis.

The solution is compared with the maximum strength observed in past experimental results, and proved to provide satisfactory predictions.

## INTRODUCTION

Ordinary types of column bases, which are usually encountered in small steel buildings, consist of anchor bolts embeded in a concrete footing and a base plate not covered by reinforced concrete.

In conventional structural analyses, most of these types of column bases are easily substituted by the idealistic supports, such as simple or completely fixed ones. It has been reported, however, some of the structural damages due to recent severe earthquakes in Japan are caused by the actual inelastic behavior, not expected in the design procedure, of these types of column bases [1,2]. In the advanced aseismic design, it is indispensable to obtain an appropriate estimation of not only elastic stiffness

I) Research Associate, II) Professor, Institute of Industrial Science, University of Tokyo

but also ultimate strength and deformability over inelastic range.

Most of the preceding research works in Japan on these types of column bases have been devoted to the estimation of elastic stiffness [3], and recently, several experimental studies have been made in order to investigate the inelastic behavior [4,5,6,7]. Theoretical studies, however, are not sufficiently enough to provide a criterion for predicting inelastic behavior of these types of column bases.

This paper presents the theoretical solution for the ultimate strength of the column base subjected to a bending moment and an axial force. For the similar purpose, Salmon, C.G. studied the upper bound for maximum resisting moments of these column bases as early as in 1955 [8], and a few proposals have been made also in Japan [9,6].

Most of these works, however, are restricted to a certain mechanism or a certain stress field of the column base, and especially, neglect the inelastic deformation in the tension-side portion of the base plate, which affects considerably the column base strength. In this paper, various mechanisms and stress fields are considered, and the more rational solution is presented for the ultimate strength.

## ASSUMPTIONS OF THE ANALYSIS

The most simplified types of column bases, shown in Fig.1, are studied in this paper; a rectangular base plate is welded to an end of a steel column, and firmly attached to the foundation through several anchor bolts. The same number of anchor bolts are located in a single row at the symetrical position outside of the column. In this analysis, the effects of stiffners, such as rib plates or wing plates, are not considered.

The assumptions of the analysis are summarized as follows:

1) A bending moment M and an axial force N transferred through the column are considered as external loads.
2) The base plate is substituted by a beam uni-directionally deformable in a single plane.
3) The anchor bolts resist only tensile forces.
4) No failure occurs in the foundation or the concrete footing.
5) The flexural failure of the base plate does not occur in the portions beneath the column section and the nuts of anchor bolts. These portions in the base plate are called as 'rigid zone' in this paper.

Parameters of the location and the size of these rigid zones, which are non-dimensionally expressed to the total length D of the base plate, are shown in Fig.1.

Evidently, these parameters satisfy the following equation and inequalities.

$$h = 1 - 2a - 2b \tag{1}$$

$$a > 0, \quad b > 0, \quad c > 0, \quad a - c > 0, \quad b - c > 0 \tag{2}$$

When the full-plastic tension of an anchor bolt is denoted by $p_P$, the summation of $p_P$ for the either side of anchor bolts is denoted by $B_P$:

$$B_P = n\, p_P \tag{3}$$

where $n$ : number of the either-side anchor bolts

The full-plastic moment of the base plate is denoted by $M_P$.

In an attempt to indicate the state of the column base, we define three normalized parameters, x,y, and z, as follows:

$x = M_P / B_P D$ : normalized full-plastic moment of the base plate

$y = N / B_P$ : normalized axial force

$z = M / B_P D$ : normalized bending moment

The parameter x is positive, according to its definition. The symmetric modelling allows us to consider only positive sign for the parameter z. As for the parameter y, both the positive sign for compression and the negative sign for tension are considered in this analysis.

This paper aims to describe the interaction surface of these parameters in steel column bases on x-y-z three-dimensional space.

## KINEMATICAL ANALYSIS

Thirteen mechanisms are shown in Table 1, where a virtual rotation occurs in the central rigid zone. As for the codes of each mechanisms, also shown in Table 1, the alphabetical and the numeral codes correspond to the tension-side shapes of mechanisms and the compression-side ones, respectively. The codes of shapes of the mechanisms in each side are tabulated in Table 2. Additionally, the numbers in the first column of Table 1, i (=1,2,···, 13), are assigned to each mechanism.

By the application of the virtual work principle to the i-th mechanism, the normalized bending moment z is obtained as a function of x and y, $f_i(x,y)$. These functions $f_i(x,y),(i=1,2,\cdots,13)$, are also tabulated in Table 1. Every $f_i(x,y)$ is a linear function of x and y, and it expresses a certain plane in the x-y-z three-dimensional space, as shown in Fig.2.

According to the limit analysis theorem, every $f_i(x,y)$ gives an upper bound of the normalized collapse moment for the combination of x and y.

## STATICAL ANALYSIS

The combinations of x and y, under which an upper bound $f_i(x, y)$ satisfies a statically admissible stress field, are obtained in the following paragraphs.

Stress fields in this chapter are expressed in the following conventions.

1) The bending stresses in the base plate are normalized by $B_pD$.
2) The concentrated forces acting on the base plate are normalized by $B_p$.
3) The distances along the base plate are normalized by D.

The sign conventions of each parameters are shown in Fig.3. The normalized tension in the anchor bolts of either side is denoted by $\beta_t$ or $\beta_c$, respectively. And the normalized reaction force between the base plate and the foundation is denoted by $r_t$ or $r_c$. The subscripts, t or c, are assined according to the location in the tension side or compression side, respectively.

According to the assumption 5), the bending stresses in the base plate can reach to full-plastic moment at the points, E,F,G, O,P, and Q, shown in Fig.3; the normalized bending stresses at these points are denoted by $m_{c3}$, $m_{c2}$, $m_{c1}$, $m_{t1}$, $m_{t2}$, and $m_{t3}$, respectively.

The normalized distance between the location of $r_c$ and the point G is denoted by $d_c$; the distance between the location of $r_t$ and the point Q is denoted by $d_t$.

The stress fields for each mechanism and their codes are shown in Fig.4. These stress fields are composed of 5 tension-side stress fields and 21 compression-side stress fields. Every tension-side stress field satisfies the following conditions in relation to the mechanism which has the same alphabetical code as the subscript of the stress field code.

1) The normalized bending stresses at the points where the mechanism has plastic hinges are equal to x $(=M_p/B_pD)$.
2) The normalized tensions in the anchor bolts which are in plastic flow are equal to 1 $(=B_p/B_p)$.

Similarly, every compression-side stress field satisfies the above two conditions with the mechanism which has the same numeral code as the subscript of the stress field code.

First, we describe the domains of (x,y) in which the either-side stress fields are possible and statically admissible. In the following paragraphs we use the description of the set theory, and we express the set T, defined by all the element $\omega$ which belongs to the set V and posseses a property F, as:

$$T = \{ \omega \in V : F(\omega) \} \tag{4}$$

Now we define the universal set U by:

$$U = \{ (x,y) \in E^2 : x > 0 \} \tag{5}$$

where $E^2$ : two-dimensional Euclidean space

The subsets of U, ${}_tT_K$ and ${}_cT_{Kj}$ , (K =A,A',B,C,D,j=1,1',2,2',3, 4) are defined by:

$$\begin{aligned} {}_tT_K = \{ & (x,y) \in U : r_t \geq 0 \text{ and } 0 \leq \beta_t \leq 1 \text{ and} \\ & d_t \leq a - c \text{ and } -x \leq m_{tk} \leq x, \ (k=1,2,3), \\ & \text{in the tension-side stress field } {}_tS_K \} \end{aligned} \tag{6}$$

$$\begin{aligned} {}_cT_{Kj} = \{ & (x,y) \in U : r_c \geq 0 \text{ and } 0 \leq \beta_c \leq 1 \text{ and} \\ & d_c \leq a + b \text{ and } -x \leq m_{ck} \leq x, \ (k=1,2,3), \\ & \text{in the compression-side stress field } {}_cS_{Kj} \} \end{aligned} \tag{7}$$

The normalized stresses and distances are expressed as functions of x and y in Fig.4. Therefore, we can define the sets, ${}_tT_K$ and ${}_cT_{Kj}$ , also by a series of inequalities including x and y. We can simplify these expressions of the sets, considering eq.(1) and inequalities (2), and we obtain the expressions shown in Table 3.

Now we denote the total stress field, composed of ${}_tS_K$ and ${}_cS_{Kj}$, by ${}_tS_K \oplus {}_cS_{Kj}$ . The stress fields, ${}_tS_A \oplus {}_cS_{A1}$ , ${}_tS_{A'} \oplus {}_cS_{A1}$ , ${}_tS_A \oplus {}_cS_{A1'}$ , and ${}_tS_{A'} \oplus {}_cS_A$ are all in equilibrium with the external loads, y and $f_1(x,y)$, and we define the set $W_1$ by:

$$W_1 = ({}_tT_A \cap {}_cT_{A1}) \cup ({}_tT_{A'} \cap {}_cT_{A1}) \cup ({}_tT_A \cap {}_cT_{A1'}) \cup ({}_tT_{A'} \cap {}_cT_{A1'}) \tag{8}$$

where $\cap$ : the intersection of sets
$\cup$ : the union of sets

If (x,y) is an element of the set $W_1$, one of the four stress fields is possible and statically admissible for this combination of x and y, and $f_1(x,y)$ gives a lower bound of the normalized collapse moment. At the same time, it is already shown that $f_1(x,y)$ also gives an upper bound, and we arrive at:

$$(x,y) \in W_1 \rightarrow f_1(x,y) \text{ : the normalized collapse moment} \tag{9}$$

Similarly, we can define the set $W_i$ for $f_i(x,y)$, (i=2,3, 13), as follows:

$$W_2 = ({}_tT_A \cap {}_cT_{A2}) \cup ({}_tT_{A'} \cap {}_cT_{A2}) \cup ({}_tT_A \cap {}_cT_{A2'}) \cup ({}_tT_{A'} \cap {}_cT_{A2'}) \tag{10}$$

$$W_3 = ({}_tT_B \cap {}_cT_{B1}) \cup ({}_tT_B \cap {}_cT_{B1'}) \tag{11}$$

$$W_4 = ({}_tT_B \cap {}_cT_{B2}) \cup ({}_tT_B \cap {}_cT_{B2'}) \tag{12}$$

$$W_5 = {}_tT_B \cap {}_cT_{B3} \tag{13}$$

$$W_6 = ({}_tT_C \cap {}_cT_{C1}) \cup ({}_tT_C \cap {}_cT_{C1'}) \tag{14}$$

$$W_7 = ({}_tT_C \cap {}_cT_{C2}) \cup ({}_tT_C \cap {}_cT_{C2'}) \tag{15}$$

$$W_8 = {}_tT_C \cap {}_cT_{C3} \tag{16}$$

$$W_9 = {}_tT_C \cap {}_cT_{C4} \tag{17}$$

$$W_{10} = ({}_tT_D \cap {}_cT_{D1}) \cup ({}_tT_D \cap {}_cT_{D1'}) \tag{18}$$

$$W_{11} = ({}_tT_D \cap {}_cT_{D2}) \cup ({}_tT_D \cap {}_cT_{D2'}) \tag{19}$$

$$W_{12} = {}_tT_D \cap {}_cT_{D3} \tag{20}$$

$$W_{13} = {}_tT_D \cap {}_cT_{D4} \tag{21}$$

Finally

$$(x,y) \in W_i \rightarrow f_i(x,y) \text{ : the normalized collapse moment } (i=1,2,\cdots,13) \tag{22}$$

The sets ${}_tT_K$ and ${}_cT_{Kj}$ are already given in Table 3, and we can express the sets $W_i$ in the term of x and y, as shown in Table 4.

## INTERACTION SURFACE

Several domains are formed on x-y plane by the sets $W_i$ . These domains are illustrated in Fig.5 (I) to (IV), according as the four restrictions on a,b, and c. These figures and Table 4 lead us to:

1) If $U' = W_1 \cup W_2 \cup \cdots \cup W_{13}$ , ( 23 )

$$U' = \{ (x,y) \in U : y \geq -g(x) \}$$

where $g(x)$ : normalized tensile force shown in Table 5

2) $(x,y) \in W_i$ and $y = -g(x) \rightarrow f_i(x,y) = 0$ ( 24 )

The normalized tensile force g(x) coincides with the tensile strength of T-stub connection already reported by one of the authors [10]. The tensile strength of the column base is derived from four mechanisms, also shown in Table 5.

Now we consider a normalized axial force $y < -g(x)$ and any normalized bending moment z. The work done by these two loads in the mechanism, shown in Table 5, are greater than the work dissipated by the internal forces. According to the limit analysis theorem, this involves:

3) If $U'' = \{ (x,y) \in U : y < -g(x) \}$ , ( 25 )

$(x,y) \in U'' \rightarrow$ The collapse of the column base occurs.

The union of U' and U" makes the universal set U, and we have described the collapse of the column base for all the possible combination of x and y.

According as the domains of (x,y), shown in Fig.5, the function $f_i(x,y)$ can be chosen to calculate the normalized collapse moment, and the interaction surface can be illustrated on x-y-z three-dimensional space, as shown in Fig.6 (I) to (IV).

## COMPARISON WITH PAST EXPERIMENTAL RESULTS

In this chapter 22 cases of experimental works [4,5,6,7] studied by several researchers in Japan are chosen, in which inelastic behavior of base plates and anchor bolts are observed, and the observed maximum strength of column bases in these experiments are compared with the strength predicted by the presented solution.

The yield strength is also an important factor in the design of column bases, but sometimes the yielding can be hardly determined by the experimental results, because the yielding in actual column bases occurs gradually with the extension of inelastic zone.

Therefore, the comparison is made in the term of the maximum strength due to the fracture at the threaded portion of anchor bolts.

Parameters in the prediction are estimated as follows:

1) The size of rigid zone beneath the nut, 2cD, is estimated by the distance between two pararell sides of the nut.
2) The size of the rigid zone connected with the column, hD, is estimated by the total length of the column depth and the half size of both flange fillet welds outside the column.
3) $B_P$ is calculated by:

$$B_P = 0.75\ n\ {}_a\sigma_B\ A_d \tag{26}$$

where ${}_a\sigma_B$ : the tensile strength of anchor bolt material
$A_d$ : the unthreaded-body area of an anchor bolt

4) $M_P$ is estimated by the following formula, taking into acount the stress increment due to strain hardening in the inelastic zones of the base plate.

$$M_P = B\ t^2\ ({}_b\sigma_Y + {}_b\sigma_B)\ /\ 8 \tag{27}$$

where B : the width of base plate
t : the thickness of base plate
${}_b\sigma_Y$ : the yield stress of base plate material
${}_b\sigma_B$ : the tensile strength of base plate material

The informations about each experimental results are summarized in Table 6.

Here, we denote, by $\kappa$, the ratio of the predicted maximum strength to the experimental one. The values of $\kappa$ are calculated for each case and they are plotted in Fig.7, and the statistic informations about $\kappa$ are tabulated in Table 7; the mean of $\kappa$ is 1.01 and the coefficient of variation is 13% of the mean value.

The comparison shows that the predicted values of the maximum strength coincide with the experimental ones with sufficient accuracy for the practical purpose.

## APPLICATIONS AND MODIFICATIONS OF THE SOLUTION

1) The presented solution is derived by means of the limit analysis, and it is required that the components of the column bases, such as base plates and anchor bolts, are deformable to some extent over inelastic range. If these components show brittle failure, the solution may overestimate the actual strength.
2) The base plate is substituted by a beam in the presented analysis. If the width of the base plate is very large in

comparison with the size of column section, the solution may overestimate the actual strength, which is affected by the two-dimensional inelastic behavior as a plate. Empirically, the solution is applicable to the base plate width smaller than two times of the column size.

3) The failure of concrete footing is not considered in the analysis. It has been widely reported, when the concrete footing is loaded over a small area on its surface, the critical stress for the loaded area is considerably greater than the compressive strength observed in standard cylinder tests.

This passage deals only with the modification of the solution for an upper limit of the reaction force between the base plate and the footing, since the actual failure of the concrete footing is affected by various parameters.

Now we denote the normalized critical compression by $r_{cr}$, which is loaded, accompanied with no bending, over the total area of the base plate. And also another normalized critical compression is denoted by $r'_{cr}$ , which is loaded over the compression-side portion of the base plate outside the column.

When the failure of the base plate and the anchor bolts occurs in the column base subjected to both a bending moment and an axial force, the normalized reaction between the base plate and the footing, $r_c$ , can be substituted by the values shown in Fig.4. The condition that the footing failure and the column base collapse occur at the same time is expressed by:

$$r_c = r'_{cr} \qquad (28)$$

For example, if $a<b<x$, the normalized reaction force $r_c$ is plotted against y, as shown in Fig.8(I). If $r'_{cr}$ is given, the value of y that satisfies eq.(28) is determined as $y'_{cr}$ , and the point G can be also determined on the y-z interaction curve, as shown in Fig.8(II). When only an axial force is applied, the point H is determined on y-axis by the value of $r_{cr}$. Taking into account the footing failure, the z-y interaction curve is modified by a straight line between the two points approximately.

## CONCLUSIONS

Theoretical interaction surfaces of the flexural and the axial resistances in steel column bases have been presented by the limit analysis of the simplified mechanical model. It has been confirmed through the comparison with past experimental results that the solution provides an effective method for predicting the ultimate strength of column bases. One of the formulas in Table 1 is chosen to calculate the collapse moment, according as the domains of the normalized parameters for the axial load and the base plate strength, y and x, shown in Fig.5. This solution is applicable to the advanced design of the steel column base.

ACKNOWLEDGEMENT

The authors would like to express their appreciation to Kajima Foundation for supports during this study, and also to Mr. H. Kondo and Mrs. Ogawa for assistance in preparing the manuscript.

REFERENCES

1. The Architectural Institute of Japan, "Report on Structural Damages caused by the Miyagiken-Oki Earthquake in 1978", May 1980.
2. The Architectural Institute of Japan, "Report on Structural Damages caused by the Izuhanto-Oki Earthquake in 1974 and the Izuohshima-Kinkai Earthquake in 1978", June 1980.
3. Taguchi,T., "Experiments on the Fixity of Steel Column Bases," (Report I), (Report II), (Report III), (Report IV), Transactions of the Architectural Institute of Japan, No.51, September 1955, No.52, March 1956, No.53, June 1956, No.55, February 1957.
4. Masuda,K., "Experimental Studies on Mechanical Characteristics of the Steel Column Bases", Transactions of the Architectural Institute of Japan, No.297, November 1980.
5. Akiyama,H., Kurosawa,M., and Wakuni,N., "Strength and Deformability of Steel Column Bases," (No.1), Proc. of the Annual Convention of the Architectural Institute of Japan, September 1980.
6. Wakabayashi,M., Nakamura,T., and Isobe,T., "Experimental Study on the Ultimate Load Carrying Capacity of the Base of Columns of Steel Structures," (Part I), ibid.
7. Tanaka,H., Takanashi,K., Ohi,K., and Maeda,Y., "Studies on Elasto-Plastic Behavior of Simple Column Bases in Steel Structures," (Part I), ibid.
8. Salmon,C.G., Schenker,L., and Johnston,B.G., "Moment-Rotation Characteristics of Column Anchorages", Proc. of the ASCE, Jour. of the Structural Division, Vol.81, April 1955.
9. Akiyama,H., "Required Performance of Column Anchorages in Steel Frames to Resist Severe Earthquakes", Proc. of the Annual Convention of the Architectural Institute of Japan, September 1980.
10. Tanaka,H. and Tanaka,A., "Design Formulas for the Strength of T-stub Connection", Jour. of the Society of Steel Construction of Japan, Vol.11, No.120, December 1975.

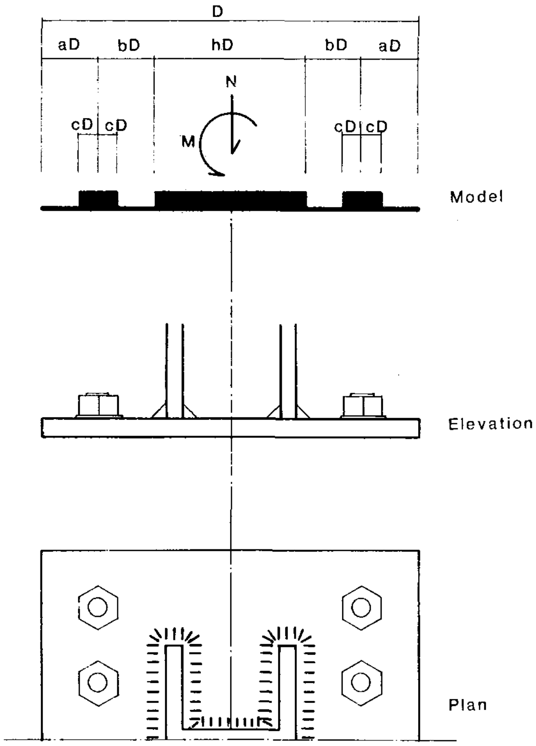
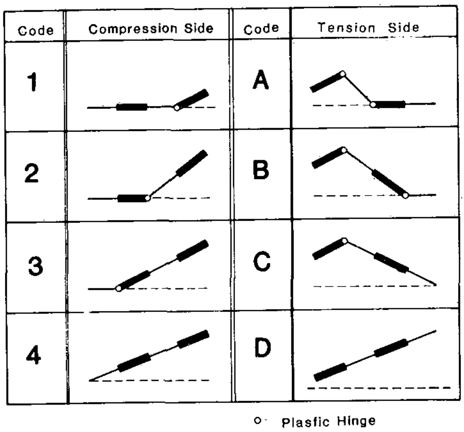



FIG. 1 MECHANICAL MODEL
FOR STEEL COLUMN BASES

TABLE 2 SHAPES OF MECHANISM IN EACH SIDE



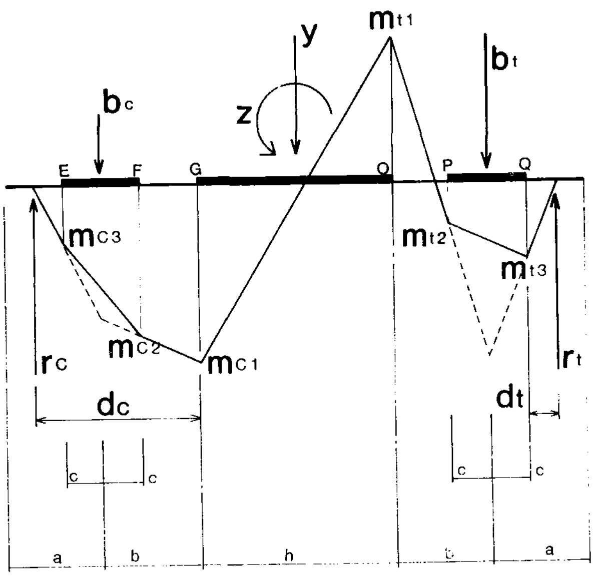




FIG. 3 NORMALIZED STRESS FIELD FOR COLUMN BASES

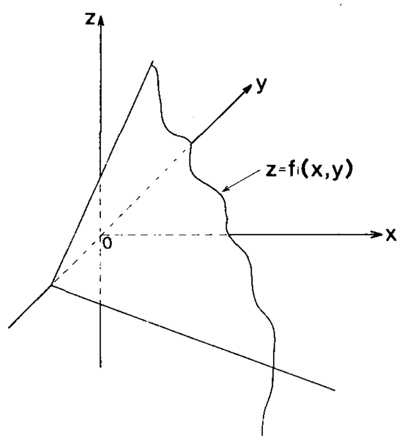
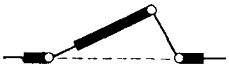
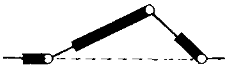
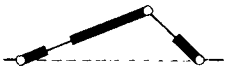
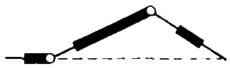
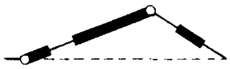
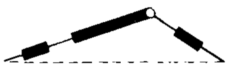



FIG. 2 PLANE ON x-y-z SPACE CORRESPONDING TO A MECHANISM

TABLE 1 MECHANISMS AND UPPER BOUNDS FOR NORMALIZED COLLAPSE MOMENT

| i | Code | Mechanism | $f_i(x,y)$ |
|---|---|---|---|
| 1 | A1 |  | $2(1+\frac{h}{b-c})x+\frac{hy}{2}$ |
| 2 | A2 |  | $2(2+\frac{h}{b-c})x+(b-c+\frac{h}{2})y$ |
| 3 | B1 |  | $\frac{ch}{b+c}+2(1+\frac{h}{b+c})x+\frac{hy}{2}$ |
| 4 | B2 |  | $\frac{c(h+b-c)}{b+c}+2(1+\frac{h+b-c}{b+c})x+(b-c+\frac{h}{2})y$ |
| 5 | B3 |  | $\frac{c(h+2b+2c)}{b+c}+2(1+\frac{h+b+c}{b+c})x+(b+c+\frac{h}{2})y$ |
| 6 | C1 |  | $\frac{ah}{a+b}+\frac{x}{a+b}+\frac{hy}{2}$ |
| 7 | C2 |  | $\frac{a(h+b-c)}{a+b}+\frac{(1+b-c)x}{a+b}+(b-c+\frac{h}{2})y$ |
| 8 | C3 |  | $c+\frac{a(h+b+c)}{a+b}+\frac{(1+b+c)x}{a+b}+(b+c+\frac{h}{2})y$ |
| 9 | C4 |  | $\frac{a}{a+b}+\frac{x}{a+b}+\frac{y}{2}$ |
| 10 | D1 |  | $h+b+x+\frac{hy}{2}$ |
| 11 | D2 |  | $h+2b-c+x+(b-c+\frac{h}{2})y$ |
| 12 | D3 |  | $h+2b+2c+x+(b+c+\frac{h}{2})y$ |
| 13 | D4 |  | $1+\frac{y}{2}$ |

o : Plastic Hinge

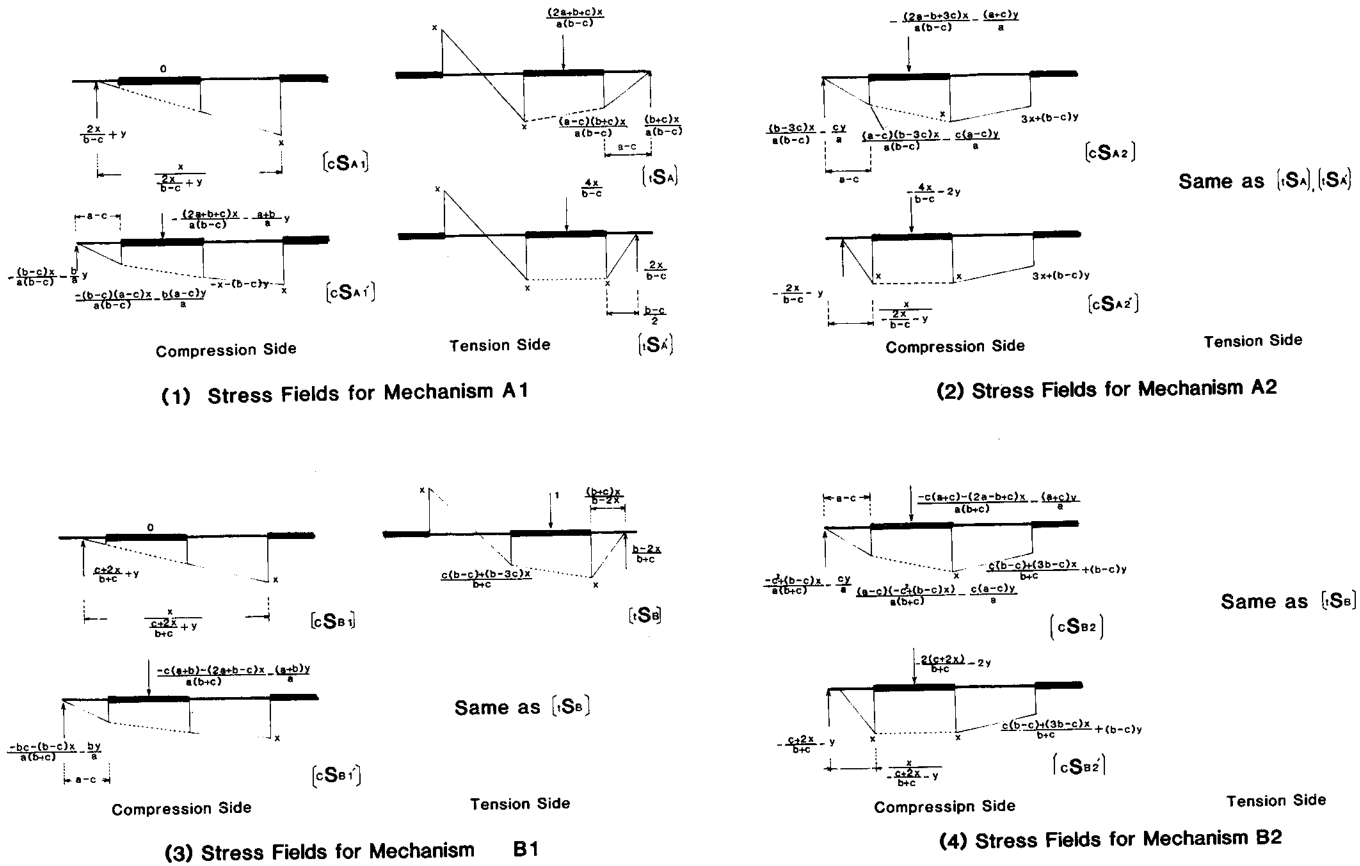
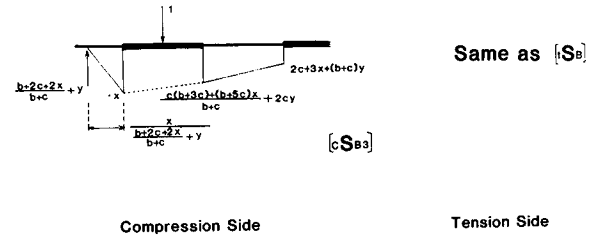
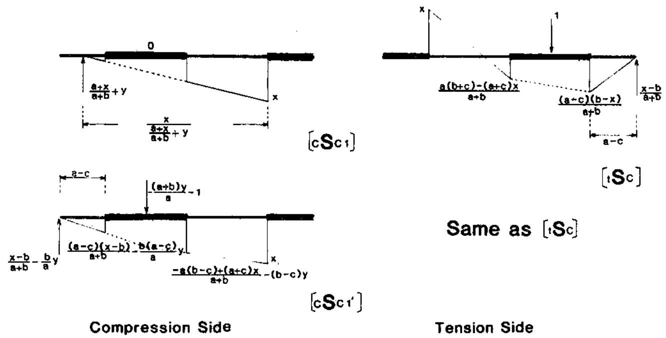


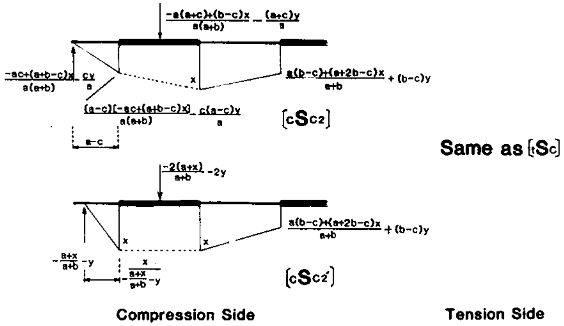

FIG. 4 STRESS FIELDS FOR EACH MECHANISM



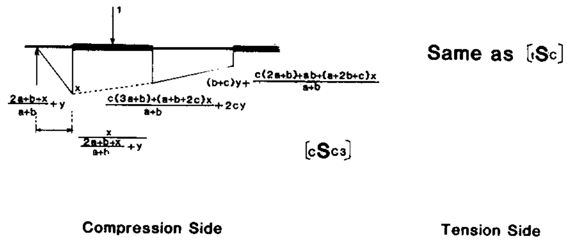

(5) Stress Fields for Mechanism B3



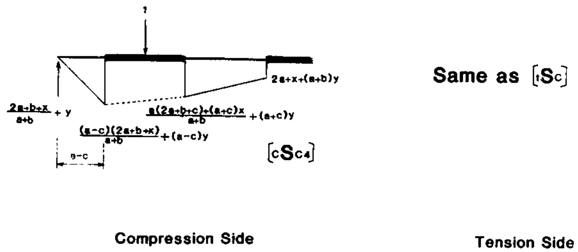

(6) Stress Fields for Mechanism C1



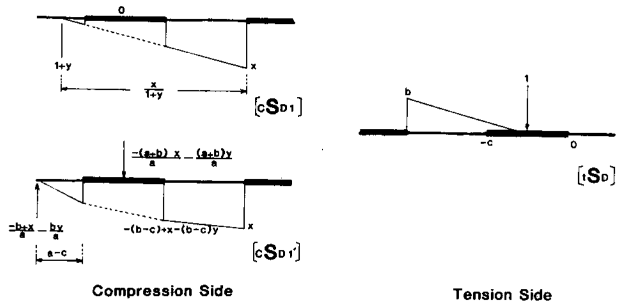

(7) Stress Fields for Mechanism C2




(8) Stress Fields for Mechanism C3

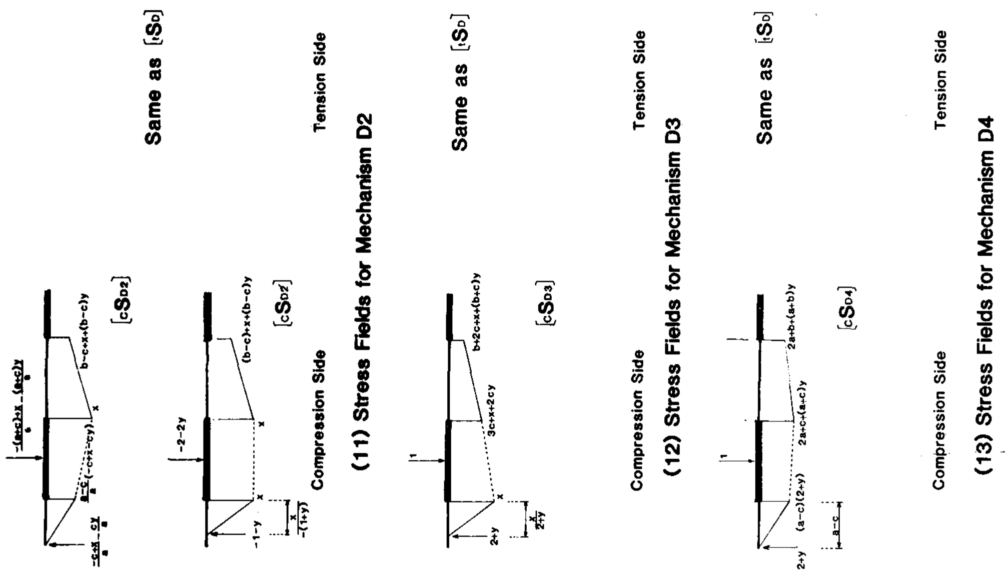



(9) Stress Fields for Mechanism C4




(10) Stress Fields for Mechanism D1

FIG. 4 CONTINUED




(11) Stress Fields for Mechanism D2

(12) Stress Fields for Mechanism D3

(13) Stress Fields for Mechanism D4

FIG. 4 CONTINUED

## TABLE 3 EXPRESSIONS FOR THE SETS $_tT_K$ AND $_cT_{Kj}$

( $\phi$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $_tT_A$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : x \leq \frac{a(b-c)}{2a+b+c} \right\}$ |
| | $b+c < 2a$ | $\phi$ |
| $_tT_{A'}$ | $2a < b+c$ | $\phi$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : x \leq \frac{b-c}{4} \right\}$ |
| $_tT_B$ | $2a < b+c$ | $\phi$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b-c}{4} \leq x \leq \frac{b(a-c)}{2a+b-c} \right\}$ |
| $_tT_C$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \frac{a(b-c)}{2a+b+c} \leq x \leq b \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq b \right\}$ |
| $_tT_D$ | —— | $\left\{ (x,y) \in U : b \leq x \right\}$ |
| $_cT_{A1}$ | —— | $\left\{ (x,y) \in U : -\frac{(2a+b+c)x}{(a+b)(b-c)} \leq y \right\}$ |
| $_cT_{A1'}$ | —— | $\left\{ (x,y) \in U : -\frac{2x}{b-c} \leq y \leq -\frac{(2a+b+c)x}{(a+b)(b-c)} \right\}$ |
| $_cT_{A2}$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a(b-c)}{2a+b+c} \text{ and } -\frac{4x}{b-c} \leq y \leq -\frac{2x}{b-c} \right) \text{ or } \left( \frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } -\frac{(2a-b+3c)x}{(a+c)(b-c)} - \frac{a}{a+c} \leq y \leq -\frac{2x}{b-c} \right) \right\}$ |

## TABLE 3 CONTINUED

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $cT_{A2}$ | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -\frac{(2a+b-3c)x}{(a-c)(b-c)} \leq y \leq -\frac{2x}{b-c} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -\frac{(2a-b+3c)x}{(a+c)(b-c)} - \frac{a}{a+c} \leq y \leq -\frac{2x}{b-c} \right) \right\}$ |
| $cT_{A2}'$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{b-c}{4} \text{ and } -\frac{4x}{b-c} \leq y \leq -\frac{(2a+b-3c)x}{(a-c)(b-c)} \right) \text{ or } \left( \frac{b-c}{4} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{1}{2} - \frac{2x}{b-c} \leq y \leq -\frac{(2a+b-3c)x}{(a-c)(b-c)} \right) \right\}$ |
| $cT_{B1}$ | —— | $\left\{ (x,y) \in U : -\frac{(2a+b-c)x}{(a+b)(b+c)} - \frac{c}{b+c} \leq y \right\}$ |
| $cT_{B1}'$ | —— | $\left\{ (x,y) \in U : -\frac{2x+c}{b+c} \leq y \leq \frac{-(2a+b-c)x}{(a+b)(b+c)} - \frac{c}{b+c} \right\}$ |
| $cT_{B2}$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( x < \frac{a(b-c)}{2a+b+c} \text{ and } -\frac{4bx}{(b+c)(b-c)} - \frac{c}{b+c} \leq y \leq -\frac{2x+c}{b+c} \right) \text{ or } \left( \frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } -\frac{(2a-b+c)x}{(a+c)(b+c)} - \frac{c}{b+c} - \frac{a}{a+c} \leq y \leq -\frac{2x+c}{b+c} \right) \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -\frac{(2a+b-c)x}{(b+c)(a-c)} - \frac{c}{b+c} \leq y \leq -\frac{2x+c}{b+c} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -\frac{(2a-b+c)x}{(a+c)(b+c)} - \frac{c}{b+c} - \frac{a}{a+c} \leq y \leq -\frac{2x+c}{b+c} \right) \right\}$ |
| $cT_{B2}'$ | $2a < b+c$ | $\emptyset$ |

## TABLE 3 CONTINUED

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $cT_{B2}'$ | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{b-c}{4} \text{ and } -\frac{4bx}{(b+c)(b-c)} - \frac{c}{b+c} \leq y \leq -\frac{(2a+b-c)x}{(b+c)(a-c)} - \frac{c}{b+c} \right) \text{ or } \left( \frac{b-c}{4} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{1}{b+c}\left( 2x+\frac{b+3c}{2} \right) \leq y \leq -\frac{(2a+b-c)x}{(b+c)(a-c)} - \frac{c}{b+c} \right) \right\}$ |
| $cT_{B3}$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( \frac{b-c}{4} \leq x \leq \frac{b(a-c)}{2a+b-c} \text{ and } -\frac{2}{b+c}(2x+c) \leq y \leq -\frac{1}{b+c}\left( 2x+\frac{b+3c}{2} \right) \right) \text{ or } \left( \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{(2a-b-3c)x}{(a-c)(b+c)} - \frac{b+2c}{b+c} \leq y \leq -\frac{1}{b+c}\left( 2x+\frac{b+3c}{2} \right) \right) \right\}$ |
| $cT_{C1}$ | —— | $\left\{ (x,y) \in U : -\frac{a}{a+b} \leq y \right\}$ |
| $cT_{C1}'$ | —— | $\left\{ (x,y) \in U : \left( x \leq a \text{ and } -\frac{a+x}{a+b} \leq y \leq -\frac{a}{a+b} \right) \text{ or } \left( x \geq a \text{ and } -\frac{2a}{a+b} \leq y \leq -\frac{a}{a+b} \right) \right\}$ |
| $cT_{C2}$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a(b-c)}{2a-b+c} \text{ and } -\frac{(2a+3b-c)x}{(a+b)(a-c)} - \frac{a}{a+b} \leq y \leq -\frac{x+a}{a+b} \right) \text{ or } \left( \frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \leq y \leq -\frac{x+a}{a+b} \right) \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -\frac{(2a+b-c)x}{(a+b)(a-c)} - \frac{a}{a+b} \leq y \leq -\frac{x+a}{a+b} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \leq y \leq -\frac{x+a}{a+b} \right) \right\}$ |

## TABLE 3 CONTINUED

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $cT_{C2}'$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{b-c}{4} \text{ and } -\frac{(2a+3b-c)x}{(a+b)(b-c)} - \frac{a}{a+b} \leq y \leq -\frac{(2a+b-c)x}{(a-c)(a+b)} - \frac{a}{a+b} \right) \text{ or } \left( \frac{b-c}{4} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{1}{a+b}\left( x+\frac{3a+b}{2} \right) \leq y \leq -\frac{(2a+b-c)x}{(a-c)(a+b)} - \frac{a}{a+b} \right) \right\}$ |
| $cT_{C3}$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \leq y \leq -\frac{1}{a+b}\left( x+\frac{3a+b}{2} \right) \right\}$ |
| $cT_{C4}$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( \frac{a(b-c)}{2a+b-c} \leq x \leq a \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \text{ or } \left( a \leq x \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq -\frac{2a}{a+b} \right) \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \text{ or } \left( a \leq x \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq -\frac{2a}{a+b} \right) \right\}$ |
| $cT_{D1}$ | ―― | $\left\{ (x,y) \in U : \frac{x}{a+b} -1 \leq y \right\}$ |

## TABLE 3 CONTINUED

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $cT_{D1}'$ | ―― | $\left\{ (x,y) \in U : \left( x \leq a \text{ and } -1 \leq y \leq \frac{x}{a+b} -1 \right) \text{ or } \left( a \leq x \text{ and } \frac{1}{a+c}\left[ x-(2a+b) \right] \leq y \leq \frac{x}{a+b} -1 \right) \right\}$ |
| $cT_{D2}$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -\frac{2x}{b-c} -1 \leq y \leq -1 \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } \frac{1}{a+c}\left[ x-(2a+c) \right] \leq y \leq -1 \right) \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -\frac{x}{a-c} -1 \leq y \leq -1 \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } \frac{1}{a+c}\left[ x-(2a+c) \right] \leq y \leq -1 \right) \right\}$ |
| $cT_{D2}'$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \left( \frac{b-c}{4} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{3}{2} \leq y \leq \frac{-x}{a-c} -1 \right) \text{ or } \left( x \leq \frac{b-c}{4} \text{ and } -\frac{2x}{b-c} -1 \leq y \leq -\frac{2x}{a-c} -1 \right) \right\}$ |
| $cT_{D3}$ | ―― | $\left\{ (x,y) \in U : \frac{(b-c)(a-c)}{2(2a+b-c)} \leq x \leq \frac{a-c}{2} \text{ and } \frac{x}{a-c} -2 \leq y \leq -\frac{3}{2} \right\}$ |
| $cT_{D4}$ | ―― | $\left\{ (x,y) \in U : \left( x \leq \frac{a-c}{2} \text{ and } -2 \leq y \leq \frac{x}{a-c} -2 \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -2 \leq y \leq \frac{1}{a+c}\left[ x-(2a+c) \right] \right) \text{ or } \left( a \leq x \text{ and } -2 \leq y \leq \frac{1}{a+b}\left[ x-(2a+b) \right] \right) \right\}$ |

## TABLE 4 EXPRESSIONS FOR THE SETS $W_i$

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $W_1$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : x \leq \frac{a(b-c)}{2a+b+c} \text{ and } -\frac{2x}{b-c} \leq y \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : x \leq \frac{b-c}{4} \text{ and } -\frac{2x}{b-c} \leq y \right\}$ |
| $W_2$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : x \leq \frac{a(b-c)}{2a+b+c} \text{ and } -\frac{4x}{b-c} \leq y \leq -\frac{2x}{b-c} \right\}$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : x \leq \frac{b-c}{4} \text{ and } -\frac{4x}{b-c} \leq y \leq -\frac{2x}{b-c} \right\}$ |
| $W_3$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b-c}{4} \leq x \leq \frac{b(a-c)}{2a+b-c} \text{ and } -\frac{2x+c}{b+c} \leq y \right\}$ |
| $W_4$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b-c}{4} \leq x \leq \frac{b(a-c)}{2a+b-c} \text{ and } -\frac{1}{b+c}\left(2x+\frac{b+3c}{2}\right) \leq y \leq -\frac{2x+c}{b+c} \right\}$ |
| $W_5$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a$ | $\left\{ (x,y) \in U : \frac{b-c}{4} \leq x \leq \frac{b(a-c)}{2a+b-c} \text{ and } -\frac{2(2x+c)}{b+c} \leq y \leq -\frac{1}{b+c}\left(2x+\frac{b+3c}{2}\right) \right\}$ |
| $W_6$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left(\frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } -\frac{x+a}{a+b} \leq y\right) \text{ or } \left(a \leq x \leq b \text{ and } -\frac{2a}{a+b} \leq y\right) \right\}$ |

## TABLE 4 CONTINUED

( $\emptyset$ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $W_6$ | $b+c \leq 2a \leq 2b$ | $\left\{ (x,y) \in U : \left(\frac{b(a-c)}{2a+b-c} \leq x \leq a \text{ and } -\frac{x+a}{a+b} \leq y\right) \text{ or } \left(a \leq x \leq b \text{ and } -\frac{2a}{a+b} \leq y\right) \right\}$ |
| | $b \leq a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq b \text{ and } -\frac{x+a}{a+b} \leq y \right\}$ |
| $W_7$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \leq y \leq -\frac{x+a}{a+b} \right\}$ |
| | $b+c \leq 2a \leq 2b$ | $\left\{ (x,y) \in U : \left(\frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{1}{a+b}\left(x-\frac{3a+b}{2}\right) \leq y \leq -\frac{x+a}{a+b}\right) \text{ or } \left(\frac{a-c}{2} \leq x \leq a \text{ and } \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \leq y \leq -\frac{x+a}{a+b}\right) \right\}$ |
| | $b \leq a \leq 2b+c$ | $\left\{ (x,y) \in U : \left(\frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{1}{a+b}\left(x-\frac{3a+b}{2}\right) \leq y \leq -\frac{x+a}{a+b}\right) \text{ or } \left(\frac{a-c}{2} \leq x \leq b \text{ and } \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \leq y \leq -\frac{x+a}{a+b}\right) \right\}$ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq b \text{ and } -\frac{1}{a+b}\left(x-\frac{3a+b}{2}\right) \leq y \leq -\frac{x+a}{a+b} \right\}$ |
| $W_8$ | $2a < b+c$ | $\emptyset$ |
| | $b+c \leq 2a \leq 2(2b+c)$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \leq y \leq -\frac{1}{a+b}\left(x+\frac{3a+b}{2}\right) \right\}$ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq b \text{ and } \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \leq y \leq -\frac{1}{a+b}\left(x+\frac{3a+b}{2}\right) \right\}$ |

## TABLE 4 CONTINUED

( ∅ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $W_9$ | $2a \leq b+c$ | $\left\{ (x,y) \in U : \left( \frac{a(b-c)}{2a+b+c} \leq x \leq a \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \text{ or } \left( a \leq x \leq b \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \right\}$ |
| | $b+c \leq 2a \leq 2b$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \text{ or } \left( a \leq x \leq b \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq -\frac{2a}{a+b} \right) \right\}$ |
| | $b \leq a \leq 2b+c$ | $\left\{ (x,y) \in U : \left( \frac{b(a-c)}{2a+b-c} \leq x \leq \frac{a-c}{2} \text{ and } -\frac{2(a+x)}{a+b} \leq y \leq \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq b \text{ and } -\frac{2(x+a)}{a+b} \leq y \leq \frac{(b-c)x-a(2a+b+c)}{(a+b)(a+c)} \right) \right\}$ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : \frac{b(a-c)}{2a+b-c} \leq x \leq b \text{ and } -\frac{2(a+x)}{a+b} \leq y \leq \frac{(b+c)x}{(a+b)(a-c)} - \frac{2a+b}{a+b} \right\}$ |
| $W_{10}$ | $a \leq b$ | $\left\{ (x,y) \in U : b \leq x \text{ and } \frac{1}{a+b}\left[x-(2a+b)\right] \leq y \right\}$ |
| | $b \leq a$ | $\left\{ (x,y) \in U : (b \leq x \leq a \text{ and } -1 \leq y) \text{ or } \left( a \leq x \text{ and } \frac{1}{a+b}\left[x-(2a+b)\right] \leq y \right) \right\}$ |
| $W_{11}$ | $a < b$ | ∅ |

## TABLE 4 CONTINUED

( ∅ : The Empty Set )

| Code | Restriction on a,b,and c | Expression |
|---|---|---|
| $W_{11}$ | $b \leq a \leq 2b+c$ | $\left\{ (x,y) \in U : b \leq x \leq a \text{ and } \frac{1}{a+c}\left[x-(2a+c)\right] \leq y \leq -1 \right\}$ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : \left( b \leq x \leq \frac{a-c}{2} \text{ and } -\frac{3}{2} \leq y \leq -1 \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } \frac{1}{a+c}\left[x-(2a+c)\right] \leq y \leq -1 \right\}$ |
| $W_{12}$ | $a < 2b+c$ | ∅ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : b \leq x \leq \frac{a-c}{2} \text{ and } \frac{x}{a-c} - 2 \leq y \leq -\frac{3}{2} \right\}$ |
| $W_{13}$ | $a \leq b$ | $\left\{ (x,y) \in U : b \leq x \text{ and } -2 \leq y \leq \frac{1}{a+b}\left[x-(2a+b)\right] \right\}$ |
| | $b \leq a \leq 2b+c$ | $\left\{ (x,y) \in U : \left( b \leq x \leq a \text{ and } -2 \leq y \leq \frac{1}{a+c}\left[x-(2a+c)\right] \right) \text{ or } \left( a \leq x \text{ and } -2 \leq y \leq \frac{1}{a+b}\left[x-(2a+b)\right] \right) \right\}$ |
| | $2b+c \leq a$ | $\left\{ (x,y) \in U : \left( b \leq x \leq \frac{a-c}{2} \text{ and } -2 \leq y \leq \frac{x}{a-c} - 2 \right) \text{ or } \left( \frac{a-c}{2} \leq x \leq a \text{ and } -2 \leq y \leq \frac{1}{a+c}\left[x-(2a+c)\right] \right) \text{ or } \left( a \leq x \text{ and } -2 \leq y \leq \frac{1}{a+b}\left[x-(2a+b)\right] \right) \right\}$ |

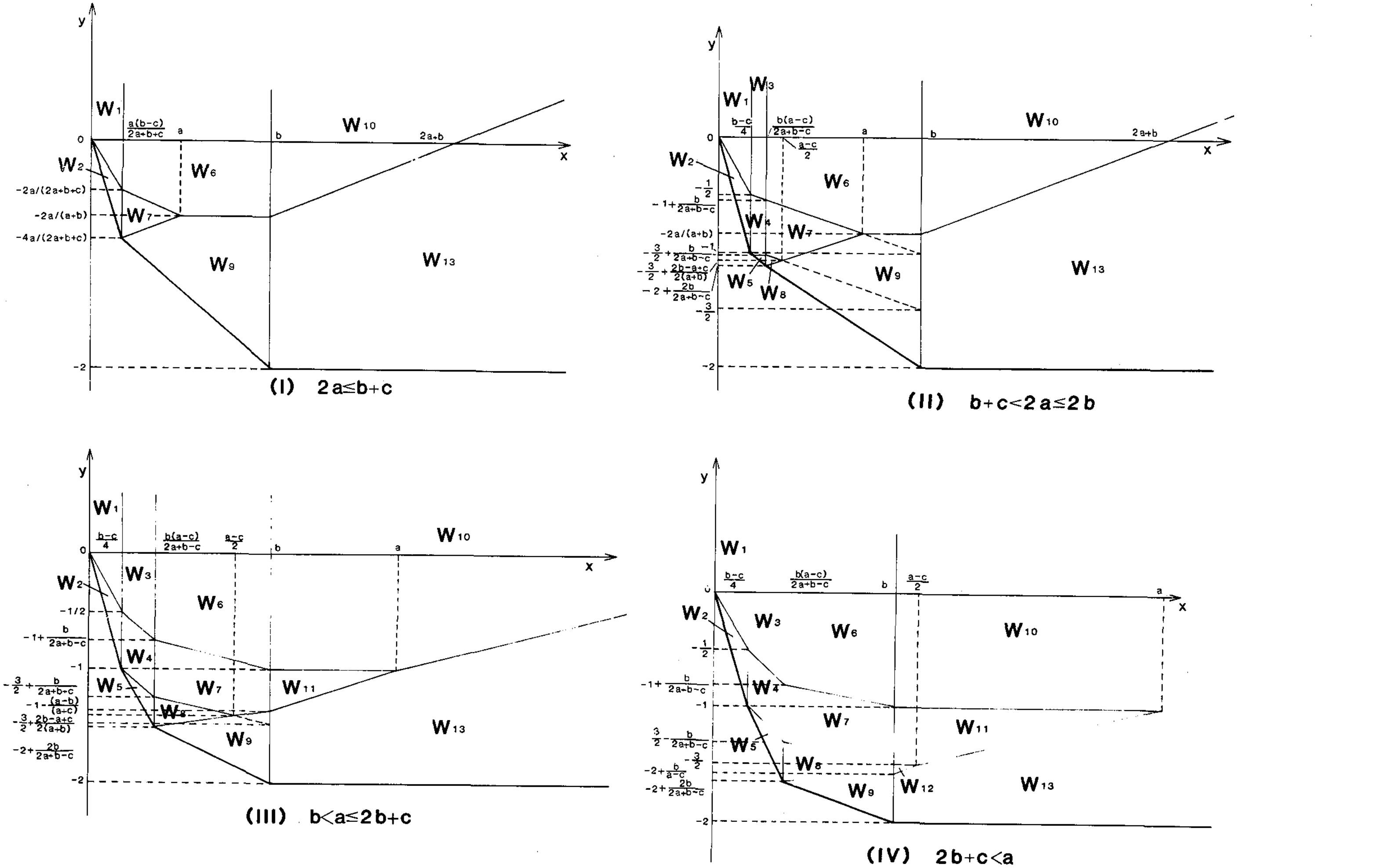



FIG. 5 PROJECTIONS OF x-y-z INTERACTION SURFACE ON x-y PLANE

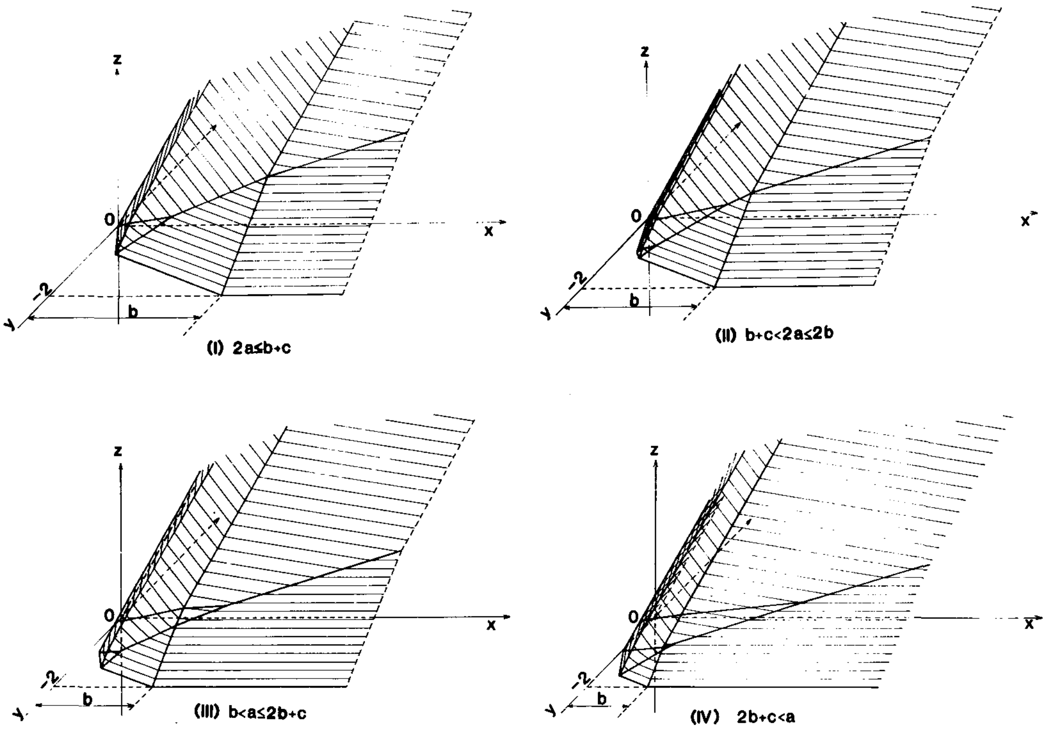
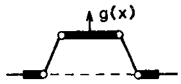
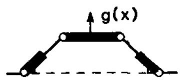
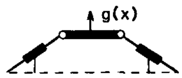
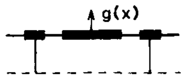



FIG. 6 SCHEMATIC VIEWS OF x-y-z INTERACTION SURFACE

TABLE 5 NORMALIZED TENSILE STRENGTH OF COLUMN BASES

| Code | $2a<b+c$ | $b+c\leq 2a$ | $g(x)$ | Mechanism |
|---|---|---|---|---|
| AA | $x\leq \frac{a(b-c)}{2a+b+c}$ | $x\leq \frac{b-c}{4}$ | $\frac{4x}{b-c}$ |  |
| BB | —— | $\frac{b-c}{4}\leq x\leq \frac{b(a-c)}{2a+b-c}$ | $\frac{2(c+2x)}{b+c}$ |  |
| CC | $\frac{a(b-c)}{2a+b+c}\leq x\leq b$ | $\frac{b(a-c)}{2a+b-c}\leq x\leq b$ | $\frac{2(a+x)}{a+b}$ |  |
| DD | $b\leq x$ | | 2 |  |

o: Plactic Hinge

# TABLE 6 COMPARISON WITH PAST EXPERIMENTAL RESULTS

| Test No. | Base Plates: Dimensions: thickness (cm) | width (cm) | length (cm) | Location and Size of Rigid Zones: a | b | c | Material Property: yield stress ($t/cm^2$) | tensile strength ($t/cm^2$) | Anchor Bolts: n x diameter (inch) (mm) | Material Property: yield stress ($t/cm^2$) | tensile strength ($t/cm^2$) | (1) | Axial Load | (2) (tm) | (3) (tm) | (4) | (5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5 | 30.0 | 38.0 | 0.105 | 0.116 | 0.037 | 2.42 | 4.87 | 2x3/4" | 2.45 | 3.88 | R | | 6.0 | 5.9 | D1 | M. |
| 2 | 2.5 | 30.0 | 38.0 | 0.105 | 0.116 | 0.047 | 2.42 | 4.87 | 2x1" | 3.97 | 6.34 | R | | 13.5 | 12.4 | D1 | M. |
| 3 | 2.5 | 30.0 | 38.0 | 0.105 | 0.116 | 0.037 | 2.42 | 4.87 | 2x3/4" | 8.19 | 9.35 | R | | 12.5 | 11.7 | C1 | M. |
| 4 | 2.5 | 30.0 | 38.0 | 0.105 | 0.116 | 0.047 | 2.42 | 4.87 | 2x1" | 7.14 | 8.49 | R | | 15.0 | 14.0 | C1 | M. |
| 5 | 1.2 | 30.0 | 38.0 | 0.105 | 0.116 | 0.037 | 2.79 | 4.70 | 2x3/4" | 2.87 | 4.92 | R | N=0t | 6.0 | 4.4 | C1 | M. |
| 6 | 1.9 | 30.0 | 38.0 | 0.105 | 0.116 | 0.037 | 2.95 | 4.83 | 2x3/4" | 2.87 | 4.92 | R | | 6.0 | 6.7 | D1 | M. |
| 7 | 1.9 | 30.0 | 38.0 | 0.105 | 0.116 | 0.037 | 2.95 | 4.83 | 2x3/4" | 2.87 | 4.92 | R | | 6.0 | 6.7 | D1 | M. |
| 8 | 1.9 | 30.0 | 38.0 | 0.105 | 0.116 | 0.047 | 2.34 | 4.14 | 2x1" | 2.96 | 4.57 | R | | 9.6 | 8.5 | C1 | M. |
| 9 | 1.9 | 30.0 | 38.0 | 0.105 | 0.116 | 0.047 | 2.34 | 4.14 | 2x1" | 2.96 | 4.57 | R | | 11.0 | 8.5 | C1 | M. |
| 10 | 1.29 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 2.96 | 4.48 | 2x20 | | | S | | 4.0 | 4.4 | C1 | T. |
| 11 | 1.64 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 2.68 | 4.39 | 2x20 | | | S | | 5.4 | 5.5 | C1 | T. |
| 12 | 2.19 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 2.71 | 4.62 | 2x20 | 9.7 t | 13.9 t | S | N=0t | 6.5 | 6.8 | D1 | T. |
| 13 | 2.88 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 3.76 | 5.39 | 2x20 | (tension test | | S | | 7.4 | 8.4 | D1 | T. |
| 14 | 1.28 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 2.96 | 4.48 | 4x20 | results of bolts) | | S | | 7.2 | 6.8 | B1 | T. |
| 15 | 2.27 | 32.0 | 32.0 | 0.141 | 0.111 | 0.047 | 2.71 | 4.62 | 4x20 | | | S | | 9.0 | 11.0 | C1 | T. |
| 16 | 3.2 | 41.0 | 41.0 | 0.073 | 0.110 | 0.037 | 2.56 | 4.18 | 5x22 | 3.17 | 4.54 | R | | 26.0 | 28.3 | D1 | A. |
| 17 | 3.2 | 41.0 | 41.0 | 0.073 | 0.110 | 0.037 | 2.56 | 4.18 | 5x22 | 3.17 | 4.54 | R | | 23.1 | 28.3 | D1 | A. |
| 18 | 3.2 | 41.0 | 41.0 | 0.073 | 0.110 | 0.037 | 2.56 | 4.18 | 5x22 | 4.28 | 4.60 | R | N=40t | 30.0 | 28.7 | D1 | A. |
| 19 | 3.2 | 41.0 | 41.0 | 0.073 | 0.110 | 0.037 | 2.56 | 4.18 | 5x20 | 4.39 | 4.78 | R | | 25.0 | 25.9 | D1 | A. |
| 20 | 1.6 | 15.0 | 30.0 | 0.125 | 0.113 | 0.040 | 2.78 | 4.48 | 3x16 | 2.39 | 3.62 | R | N=0t | 35.3 | 32.4 | C1 | W. |
| 21 | 1.6 | 15.0 | 30.0 | 0.125 | 0.113 | 0.040 | 2.78 | 4.48 | 3x16 | 2.39 | 3.62 | R | only tension | 37.9t | 32.4t | CC | W. |
| 22 | 1.6 | 15.0 | 30.0 | 0.125 | 0.113 | 0.040 | 2.78 | 4.48 | 3x16 | 2.39 | 3.62 | R | N=1.92M/D | 5.7 | 6.5 | C1 | W. |

n : Number of Eather-Side Anchor Bolts

(2) : Maximum Strength Observed in Experiments

(4) : Mechanism Codes in Calculation

(1) Types of Anchorages R : Embeded in Concrete Footings
S : Bolted to Thick Steel Plates

(3) : Calculated Strength

(5) Researchers M. : Masuda,K. T. : Tanaka,H.
A. : Akiyama,H. W. : Wakabayashi,M.

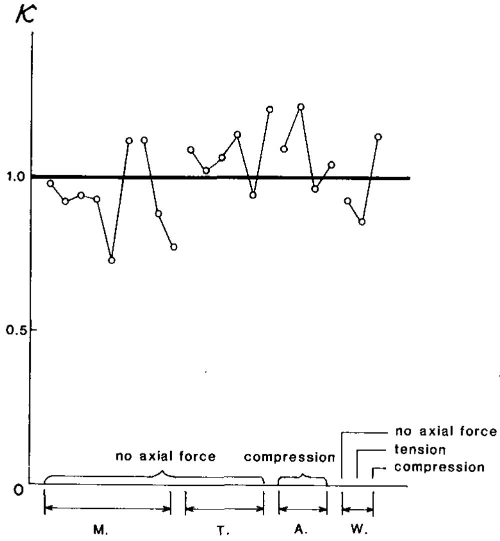



FIG. 7 RATIO OF PREDICTED STRENGTH TO ACTUAL STRENGTH OBSERVED IN PAST EXPERIMENTAL RESULTS

TABLE 7 STATISTIC INFORMATION ABOUT κ

| | |
|---|---|
| Range | 0.73 to 1.23 |
| Mean | 1.01 |
| Standard Deviation | 0.13 |
| Coefficient of Variation | 13 % |

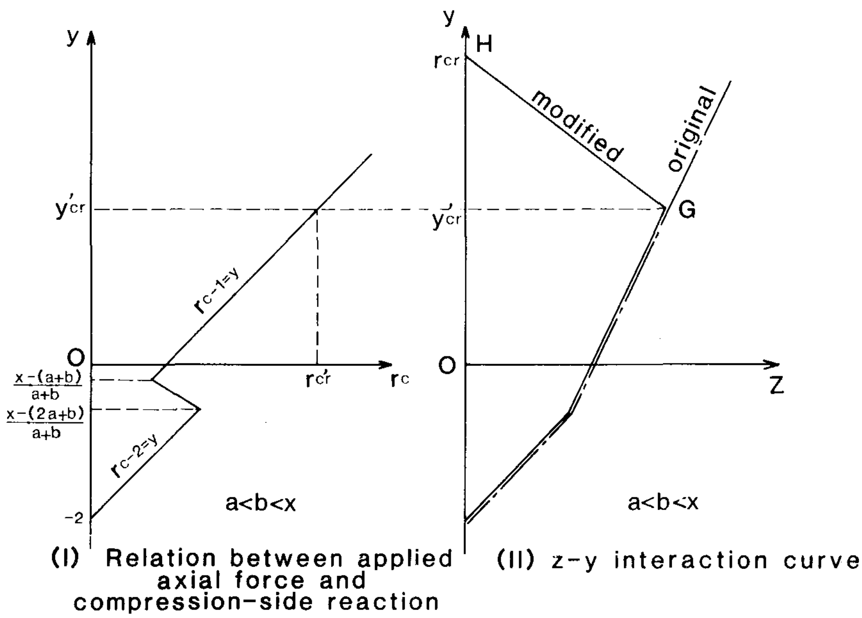



FIG. 8 MODIFICATION OF THE SOLUTION IN CONSIDERATION OF FOOTING FAILURE