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ADDED MASS COEFFICIENT FOR THE DYNAMIC ANALYSIS OF SHELL STRUCTURES SURROUNDED BY WATER

by

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INTRODUCTION

The motion of shell structures surrounded by water belongs to the dynamic interaction problem between fluid and structure. In the analysis of this problem, some analytical solutions are available for simple structures such as a vertical circular column [1] and cylindrical shells [2], while some numerical methods are required for shells with arbitrary configurations [3,4]. In this paper, we formulate some variational principles, which govern the reciprocal motion between fluid and shells, by using the Hamilton's principle. This variational principle enables us to obtain the approximate coupled equations of motion. In the formulation, the following assumptions are used: a) the fluid is considered to be inviscid, incompressible and irrotational, b) the effect of surface waves can be neglected in the dominant region of the response frequency of the shell. By applying the finite element procedure to these variational principle, added mass coefficients are numerically calculated for the dynamic analysis of shell structures surrounded by water.

VARIATIONAL PRINCIPLES

According to the above assumptions, we may express Hamilton's principle by using the velocity potential ϕ and the displacement vector of the shell $\vec{u}$ as follows.

$$\delta\left[\frac{1}{2}\rho\iiint_{\Omega}(\nabla\phi)^2 d\Omega+\frac{1}{2}m\iint_{Su}(\dot{\vec{u}})^2 dS-\iint_{Su}A(\vec{u})\,dS+\iint_{Su}\vec{u}^T\cdot\vec{f}\,dS\right]=0 \qquad (1)$$

where Ω=fluid region of infinite extent, Su=middle surface of the shell, ρ=density of the fluid, m=density of the shell, $\vec{f}$=vector of the external forces, $A(\vec{u})$=strain energy function of the shell and

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[ ]=time integral from t_1 to t_2.

The subsidiary conditions are

$$\nabla\nabla\phi=0 \text{ in } \Omega \qquad (2)$$

$$\phi,_n=\vec{n}^T\cdot\vec{u} \text{ on } Su' \qquad (3)$$

$$\phi,_n=0 \text{ on } Sb \qquad (4)$$

$$\phi=0 \text{ on } Sf \text{ and at infinity} \qquad (5)$$

$$B(\delta\vec{u})=0 \text{ at } C \qquad (6)$$

where $S\acute{u}$, Sb, Sf are interfaces of the shell with fluid, base and free surface, respectively, and C denotes the constrained boundary of the shell, as shown in Fig.1. And $\vec{n}$ is the unit normal vector on $S\acute{u}$ and B represents the linear operator.

The stationary conditions results in the coupled equations of motion:

$$m\ddot{\vec{u}}+D(\vec{u})=-\rho\vec{n}\,\dot{\phi}+\vec{f} \qquad (7)$$

under the condition

$$\delta\iint_{Su}A(\vec{u})\,dS=\iint_{Su}\delta\vec{u}^T\cdot D(\vec{u})\,dS+\int_{C}B(\delta\vec{u})\,dC \qquad (8)$$

where D is a linear operator representing the restored force of the shell.

After calculation with the use of Lagrange multipliers to eliminate the subsidiary conditions of Eqs.(2) to (4), we may obtain the form:

$$\delta[-\frac{1}{2}\rho\iiint_{\Omega}(\nabla\phi)^2 d\Omega+\rho\iint_{S\acute{u}}\phi\vec{n}^T\cdot\dot{\vec{u}}dS+\iint_{Su}(\frac{1}{2}m\dot{\vec{u}}^2-A(\vec{u})+\vec{u}^T\cdot\vec{f})dS]=0 \qquad (9)$$

The stationary conditions gives us Eqs.(2) to (4), (7) under the subsidiary conditions of Eqs.(5) and (6). Because of the infinity of the region Ω, we cannot apply any numerical method to Eq.(9) directly. Hence, we need another variational principle in order to perform the numerical calculation such as the Rayleigh-Ritz method and the finite element method, and two variational principles will be given in the following.

If we can find a set of the eigen functions [5] or an approximate expression for ϕ, which are derived from the boundary integral procedure [6], satisfying Eqs.(2), (4) and (5), we may transform Eq.(9), through the use of Green's identity, into

$$\delta[-\frac{1}{2}\rho\iint_{S\acute{u}}\phi\phi,_n dS+\rho\iint_{S\acute{u}}\phi\vec{n}^T\cdot\dot{\vec{u}}dS+\iint_{Su}(\frac{1}{2}m\dot{\vec{u}}^2-A(\vec{u})+\vec{u}^T\cdot\vec{f})dS]=0 \qquad (10)$$

In this case, the stationary conditions are Eqs.(3) and (7) with the subsidiary conditions of Eqs.(2), (4), (5) and (6).

Let us consider the case where the region Ω is divided into two domains [7]: the finite outer domain Ω_E with the constant depth H and the inner domain Ω_I close to the shell (Fig.1). If the velocity potential $\phi^+(P)$ in Ω_E can be represented [8] by

$$\phi^+(P) = \iint_{S\zeta} G(P;Q)\,\phi^+_{,n}(Q)\,dS \tag{11}$$

where $S\zeta$ is the interface between Ω_E and Ω_I, Eq.(9) can be re-written [9] as

$$\delta\Big[-\frac{1}{2}\rho\iiint_{\Omega_I}(\nabla\phi)^2 d\Omega+\rho\iint_{Su}\phi\vec{n}^T\cdot\dot{\vec{u}}dS+\rho\iint_{S\zeta}\phi\phi^+_{,n}dS$$
$$+\frac{1}{2}\rho\iint_{S\zeta S\zeta}G(Q;Q')\,\phi^+_{,n}(Q)\,\phi^+_{,n}(Q')\,dSdS'+\iint_{Su}\Big(\frac{1}{2}m\dot{\vec{u}}^2-A(\vec{u})+\vec{u}^T\cdot\vec{f}\Big)dS\Big]=0 \tag{12}$$

with the subsidiary conditions of Eqs.(5) and (6). Assuming that $S\zeta$ is a vertical cylinder of circular section, G(Q;Q') becomes

$$G(\theta,z;\theta,z)=\sum_{m=0}^{\infty}\frac{\varepsilon m}{\pi a}\sum_{n=0}^{\infty}\frac{Km(\lambda m\cdot a)}{\lambda n\cdot H\cdot K\acute{m}(\lambda n\cdot a)}\cos\lambda nz\cos\lambda nz\cos m(\theta-\theta) \tag{13}$$

where $\lambda_n=(2n+1)\pi/2H$, $\varepsilon_0=1$, $\varepsilon_m(m\geq 1)=2$ and a is the radius of $S\zeta$. Moreover, Km denotes the modified Bessel function of order m of the second kind.

The above formulation is also valid for the problem of treating groups of shells or variable depth in Ω_I.

NUMERICAL EXAMPLES

Since the variational principles have been established, we can systematically apply the Rayleigh-Ritz method, the finite element method, etc. for obtaining the approximate equations of motion. Here some numerical results by the use of the isoparametric finite element models [10] (Fig.2) are shown for two kinds of shells.

(1) Circular Cylindrical Shell (Fig.3)

Fig.4 shows the variation of the added mass coefficient αmn computed from Eq.(12) by using the modes qmn of free vibration of the shell in the air, where subscripts m and n of qmn denote the number of axial waves and circumferential waves, respectively. The value of αmn at a/R=1 can be obtained from Eq.(10). Two kinds of Gauss integration points: 3×3×3 and 2×2×2 are used to evaluate the integrals with fluid elements. It is shown in Fig.4 that the reduced order integration improves the results.

(2) Elliptical Cylindrical Shell (Fig.5)

Fig.6 and 7 show the relations between μ and αmn for geometrical parameters λ =2.0 and 16.0, which represent the short shell and the long one, respectively.

CONCLUSIONS

In this paper, we have derived variational principles to deal with the dynamic interaction between general shells and fluid with unbounded region, and we also showed some numerical examples of added mass coefficients by means of Eqs.(10) and (12).

For the case of the motion of shells subjected to water waves with a certain period, we can derive the similar equations to Eqs. (10) and (12) by expressing ϕ and $\vec{u}$ as

$$\phi=\mathrm{Re}[i\omega\phi e^{i\omega t}] \quad , \quad \vec{u}=\mathrm{Re}[\vec{\bar{u}}e^{i\omega t}] \tag{14}$$

where ω denotes the angular frequency of waves.

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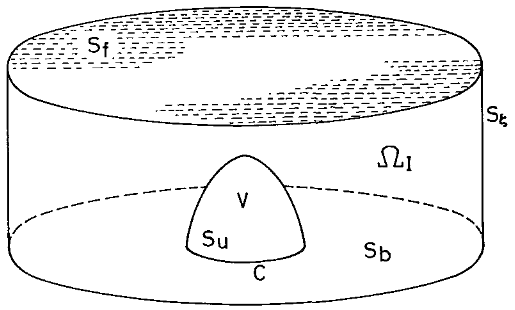

Ω_I : INNER DOMAIN OF THE FLUID
V : SHELL DOMAIN
S_f : FREE SURFACE BOUNDARY
S_ξ : ARTIFCIAL CIRCULAR CYLINDRICAL BOUNDARY
S_b : BOTTOM
S_u : SHELL-FLUID INTERFACE
C : CONSTRAINED BOUNDARY OF THE SHELL

Fig.1 Division of the fluid domain

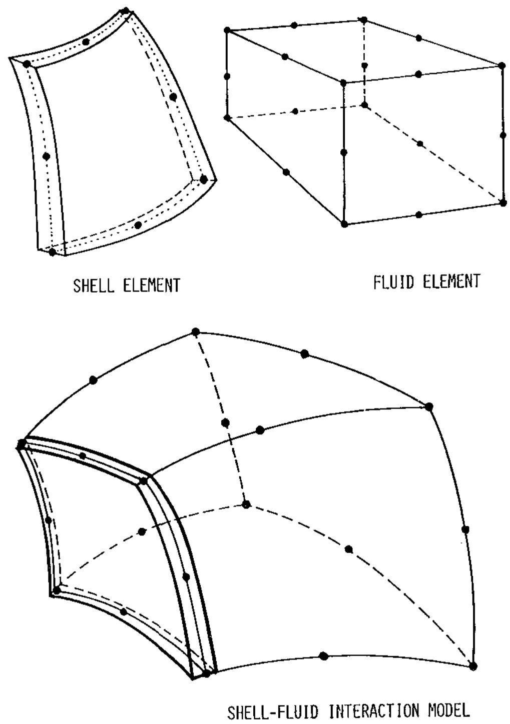

Fig.2 Isoparametric finite element models

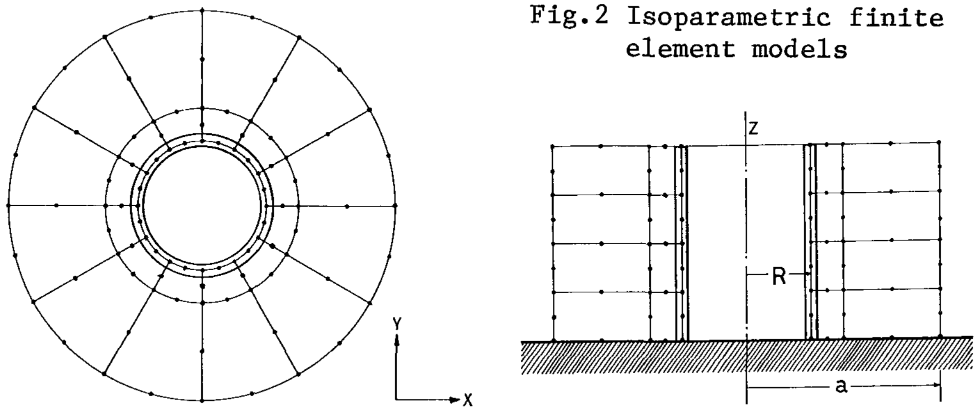

Fig.3 A circular cylindrical shell

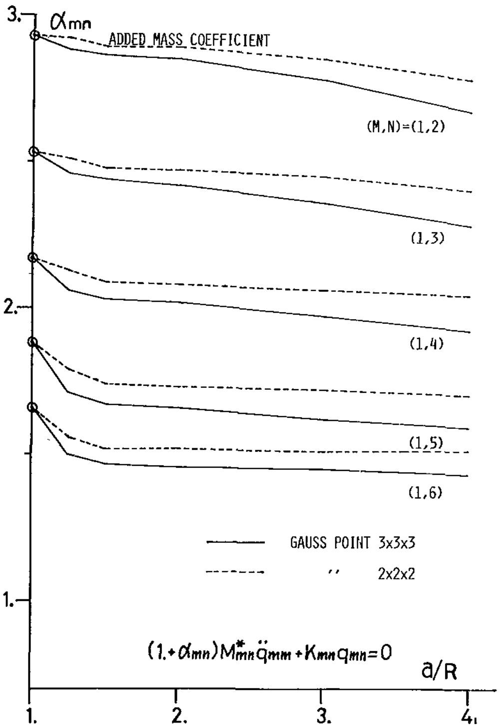

Fig.4 Added mass coefficient for vibration modes of the circular cylindrical shell

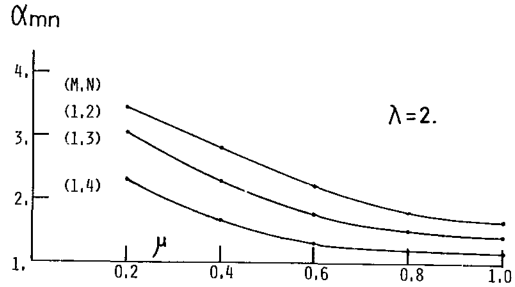

Fig.6 Added mass coefficient for vibration modes of the elliptical cylindrical shell; $\lambda=2.$

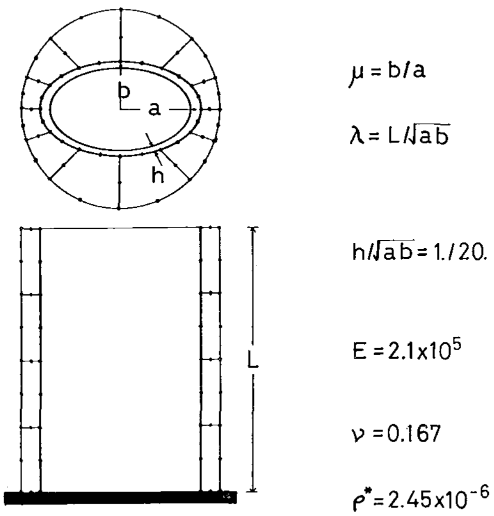

Fig.5 An elliptical cylindrical shell

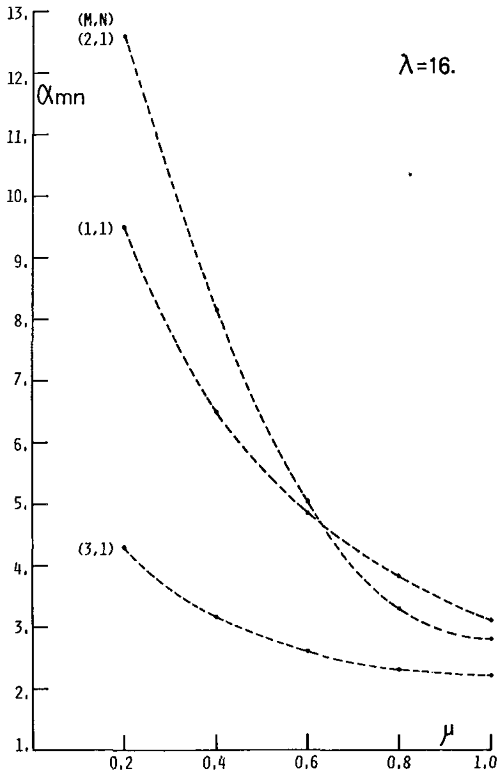

Fig.7 Added mass coefficient for vibration modes of the elliptical cylindrical shell; $\lambda=16.$

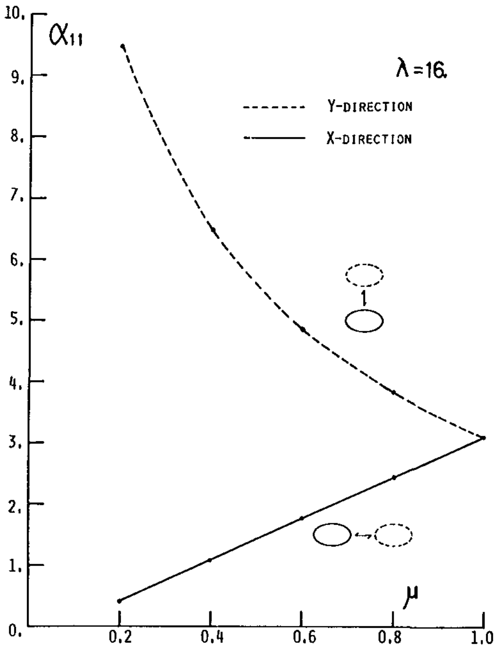

Fig.8 Variation of added mass coefficient for the first mode in X and Y directions.