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# INELASTIC RESPONSE OF H-SHAPED COLUMNS TO TWO DIMENSIONAL EARTHQUAKE MOTIONS

by

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## SYNOPSIS

H-shaped steel columns subjected to two components of earthquake ground motions were analyzed by Computer-Actuator (On-line) Hybrid System. Bi-axial bending behavior was examined with regard to the many combinations of the scaled intensities in the NS and EW components of earthquake ground motions. A numerical analysis based on a simple bi-axial bending theory of beam-columns were also carried out for the identical ground motions used in the hybrid analysis. The results obtained by the numerical analysis shows a fairly good coincidence with that by the hybrid analysis. Some tentative conclusions on the response behavior were drawn on the basis of both numerical and hybrid analysis results.

## INTRODUCTION

In practical design methods of building frames, a planar model of the structure is usually selected and designed to resist one component of expected major earthquake motions. Three-dimensional behavior, however, takes place in the building frame due to three components of a earthquake ground motion and an asymmetric distribution of stiffness relative to the mass center. Many researches have been reported on such behaviors. In these literatures it is concluded that response calculations based on one horizontal component of the ground motion in a given direction result sometimes in unconservative estimates of the same direction displacements compared with that from an analysis where interaction due to both horizontal components of the ground motion is considered (1, 2,3). But the conclusion will be more complicated in real structures, since the interaction effects depend significantly on the distribution of column stiffness and the column strength in addition to the types of the excited ground motions. Moreover, H-shaped columns widely used in steel building frames have much difference in the bending strength and the stiffness with respect to major axis and minor axis, respectively. The bi-axial

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behavior of the H-shaped columns caused by bi-axial bending in the presence of thrust has been examined by many researchers. The results and conclusions are summarized by Chen in his paper and his recently published book (4,5).

Research works in Japan are also very active (6,7,8). But most of such works are focussed on response behavior to earthquake ground motions which should be considered acting in arbitrary directions in general. These base excitation inputs to steel frames result in bi-axial bending and torsion of the columns. The conclusions shown in the papers can be summarized as follows:

1) Torsion of columns with small slenderness ratios can be neglected in the case that the eccentric distance between the mass center and the stiffness center on the floor is small.
2) Torsion effect should be considered in case the eccentric distance is not small and the slenderness ratios of columns are large.
3) The ultimate strength of a H-shaped column is controlled by the strength resistance against the minor-axis bending.
4) The response displacement caused by the minor axis bending starts to drift in a direction as the strength of the column deteriorates.

Response behaviors of H-shaped columns to two horizontal components of recorded earthquake ground motions were analyzed by so-called Computer-Actuator (On-line) Hybrid System. The H-shaped columns analyzed were considered the columns of assumed simple frame models. The results of the analysis were described in comparison with the results of a numerical analysis which is based on a very simple bi-axial bending theory of the beam-column problem.

## USE OF COMPUTER-ACTUATOR (ON-LINE) HYBRID SYSTEM

It is possible to apply Computer-Actuator (On-line) Hybrid System to two dimensional response analysis of steel frames. The principle and analytical procedure of the system have been reported (9,10), and several analyses done by the system were also described in the previous papers (11,12). Brief review on the analytical procedure of the system is represented here intending the emphasis on the specific feature of the two dimensional response behavior.

Principal directions of a H-shaped column section are initially paralled to the east-west and the north-south directions, respectively. In the case that the mass is lumped on the top of the column and the rotation about the column axis is neglected, the equations of motion predicting the response at the top of the column can be expressed as follows;

$$M\ddot{U}^{i} + Q_X^{i} = -M\ddot{U}_0^{i}$$
$$M\ddot{V}^{i} + Q_Y^{i} = -M\ddot{V}_0^{i} \qquad (1)$$

where M is the mass. $\ddot{U}^i$, $\ddot{V}^i$ denote the response accelerations, $Q_X^i$, $Q_Y^i$ the restoring forces, and $\ddot{U}_0^i$ , $\ddot{V}_0^i$ the ground accelerations in the X, Y direction, respectively. The superscript, i , denotes the variables at the time, $t=i\Delta t$, where $\Delta t$ is the time increment used in the step-by-step numerical integration. The response accelerations, $\ddot{U}^i$ and $\ddot{V}^i$ can be expressed by the following approximate formulas:

$$\ddot{U}^{i} = (U^{i+1} - 2U^{i} + U^{i-1}) / (\Delta t)^2$$
$$\ddot{V}^{i} = (V^{i+1} - 2V^{i} + V^{i-1}) / (\Delta t)^2 \qquad (2)$$

In the hybrid system analysis, the restoring forces, $Q_X^i$ and $Q_Y^i$ , are estimated by the load test on the column specimen which is controlled by the computer in parallel to the numerical integration. Then, the response displacements, $U^{i+1}$ and $V^{i+1}$ , at time $t=(i+1)\Delta t$, can be calculated from Eq.(1) by use of $Q_X^i$ and $Q_Y^i$ measured in real time and a finite difference formulas Eq.(2). The response displacement, $U^{i+1}$ and $V^{i+1}$ , are imposed in turn on the column specimen to measure new restoring force, $Q_X^{i+1}$ and $Q_Y^{i+1}$ at time $t=(i+1)\Delta t$. This procedure is continued successively until a run of analysis is completed.

## PROCEDURE OF THE HYBRID ANALYSIS

### Analytical models

An analytical model used in the hybrid analysis is a one-bay square single-story building model which comprises a rigid floor and four H-shaped steel columns as shown in Fig.1. In addition, the following assumptions are adopted:

1) A symmetric distribution of stiffness relative to the mass center of the floor is presumed.
2) The slenderness ratios of the H-shaped columns are relatively small.
3) The pricipal directions of the column sections are placed in parallel to the EW and the NS directions.
4) The twisting moment about the column axis due to the earthquake motion is small to be compared with the strong axis bending and the weak axis bending.

5) Therefore, the rigid floor connecting the tops of columns always moves horizontally without showing twist.

Column specimens and test set-up

By these assumptions, the analysis of a single column is considered enough to predict the response behavior of the building model if the four columns are identical. Therefore, the analytical model can be reduced to a most fundamental vibration model in Fig.2. In the hybrid analysis, bi-axial bending tests must be carried out to measure restoring force characteristics. Eventually the tests were conducted on welded built-up column specimens, H-70X70X6X6, as shown in Fig.3. In the figure, the intended loading arrangement is also expressed, even though it shows only a elevation paralled to the strong-axis bending plane. Two identical column specimens are placed on and under the load distributor, the sides of which two actuators (hydraulic jacks) are connected to. The lower column is considered a specimen. The mechanical properties of specimens are summarized in Table 1. The two dimensional displacement at the top of the specimen is provided by the two actuators which are controlled to impose horizontally the exact response displacement components after the response analysis. The thrust, P, of the specimen is provided by an another jack which is also controlled to keep constant compressive force. The thrust was set 30% of the yield force $P_Y$ ($=\sigma_Y A$:$\sigma_Y$ = the measured yield stress, A= the section area). An elevation of the overall loading system is shown in Fig.4. The close-up photo of a specimen under experiment is shown in Fig.5.

Assumed frames and scaled ground acceleration records

A series of hybrid analyses were carried out for the frame models listed in Table 2. In the load tests, the use of the column specimens with the same section and the same length were intended. Then, the almost same masses were determined after the pre-determined fundamental elastic period T (the period with respect to the strong axis bending motion is fixed 0.5 sec). The variables considered were the ground motion characteristics (EW and NS components of 1968 HACHINOHE,and EW and NS components of 1940 EL CENTRO accelerograms) and the scaled intensities were scaled on the basis of the yield acceleration $\alpha_{pc}$ ($\alpha_{pc} = M_{pc}/(ML/2)$, $M_{pc}$ = the full plastic moment under thrust). Fig.6 shows the combinations of the intensities of ground accelerations used.

## SOME OF THE RESULTS BY THE HYBRID ANALYSIS

Some results are selected and shown in Figs.7 to 10. The time histories of the relative displacements U in X direction (parallel to the weak axis bending plane), the relative displacements V in Y direction (parallel to the strong axis bending plane), the restoring force $Q_X$ in X direction and $Q_Y$ in Y direction are represented for DBC-A-4, A-5,B-1 and C-1. In the tests, the column specimens were always placed to coincide the strong principal axis with X axis and the weak principal axis with Y axis, respectively. The maximum and minimum values of the response displacements are summarized in Table 3. In the case that the large drift occurred, such a drift is always observed in X direction. This behavior is of a same kind as reported in the literatures. The combinations of the ground intensities circled in Fig.6 show that the large drifts were not observed due to these intensities. It seems that the large drifts were mainly caused by the greater magnitudes of the intensities in Y direction. This is worthy of noting in the design of steel rigid frames with H-shaped columns.

## A COMPARISON WITH A COMPUTER ANALYSIS

### Description of a computer analysis

The hybrid analysis is considered to simulate the real elastic-plastic response behavior to earthquake ground motions. To predict the hybrid analysis results shown above, a computer analysis was carried out. The computer analysis described was originally proposed by Fujita et al. and its results were compared with the monotonically increasing load test results (13). The basic assumptions taken for granted there are

(1) Only the stress and strain component normal to the section are considered.
(2) The twisting moment and deformation are neglected.
(3) The Bernoulli-Navier law is admitted.
(4) The axial force is constant along the axis of a beam-column.
(5) The small deflection theory is applied. The incremental calculation is conducted.
(6) The premature local buckling and torsional buckling do not occur.

In the analysis described later, an assumption for the stress and strain relationship is added. The relationship used can be represented by Fig.11. This tri-linear model was determined so that it can predict also well the cyclic behavior of a beam-column due to bi-axial bending moments and thrusts.

### An analytical model

The assumed frames used in the hybrid model are single-story models with columns built into the rigid floors. The forces and the moments acting on a column is shown in Fig.12(a). By symmetry, the calculation on a canti-lever in Fig.12(b), a half of a column, can represent the overall behavior. To know response displacements to horizontal forces induced by earthquake ground motions at the top of the canti-lever, the differential equation of the beam-column was solved approximately by a finite difference method. In the calculations the flexural rigidities at the locations indicated by Fig.13 were evaluated.

A discrete model was developed for the evaluation of rigidity in the plastic range. This model assumes that a H-shaped section consists of uniaxially stressed filaments along its long axis as shown in Fig.14. The filaments are assumed uniaxially stressed within a filament and the stress obeying the tri-linear stress-strain model can be obtained from the strain of the filament centroid.

Results by the computer analyses

The computer analyses were conducted on the same columns as used in the hybrid analyses. The masses and the intensities of ground accelerations were also identical to the corresponding columns in the hybrid analyses. The results for DBC-A-4, A-5, B-1 and C-1 are represented by dashed lines in Figs.7 to 10 for comparison.

## CONCLUSIONS

In this study, a rigid floor structural idealization has been used, and its translational displacements without rotations was assumed in both the hybrid analysis and the computer analysis. In spite of the limitations of these analytical models, the tentative conclusions can be drawn as below:

1) Two dimensional behavior of a square single-story frame subjected to the two components of earthquake acceleration records can be simulated by Computer-Actuator (On-line) Hybrid System.
2) The large drifts in the response displacements seem to occur mainly by the greater intensities of ground accelerations in Y direction (in the strong axis bending place) than the level, say, 1.5 $\alpha_{pc}$ .
3) The large drifts always occur in X direction (in the weak axis bending plane). This tendency is not affected by the type of acceleration records.
4) A simple calculation for beam-columns under biaxial bendings can predict the response bahavior of inelastic columns to two dimensional ground motions, even though twisting deformations are neglected in the calculation.
5) A tri-linear model is assigned to the stress-strain relationship in the above calculation. The same calculation with the proposed tri-linear model is approved to predict the cyclic behavior due to bi-axial loadings.

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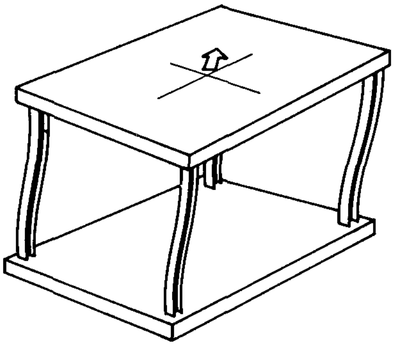


Fig.1 A square single-story building model

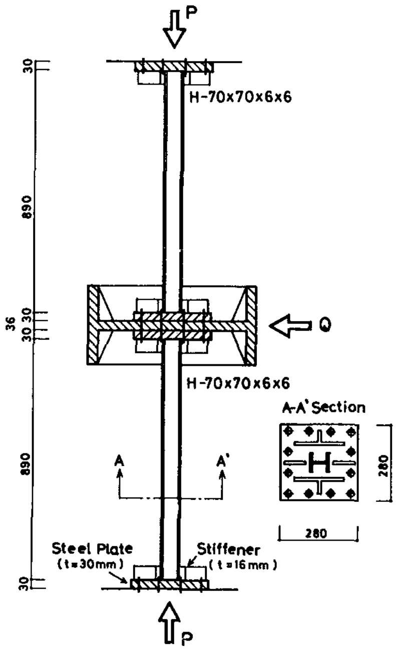


Fig.3 A schematic view of the loading arrangement

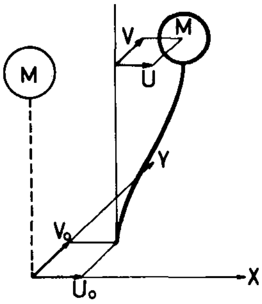


Fig.2 A lumped mass model

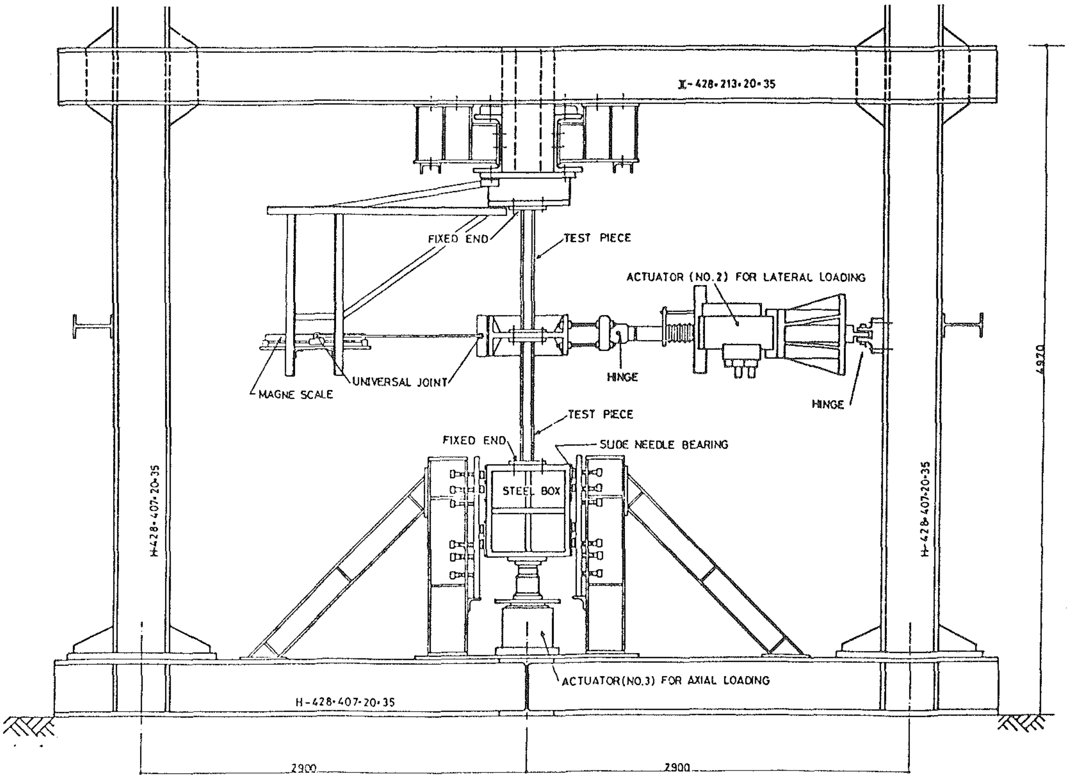


Fig.4 The overall loading system

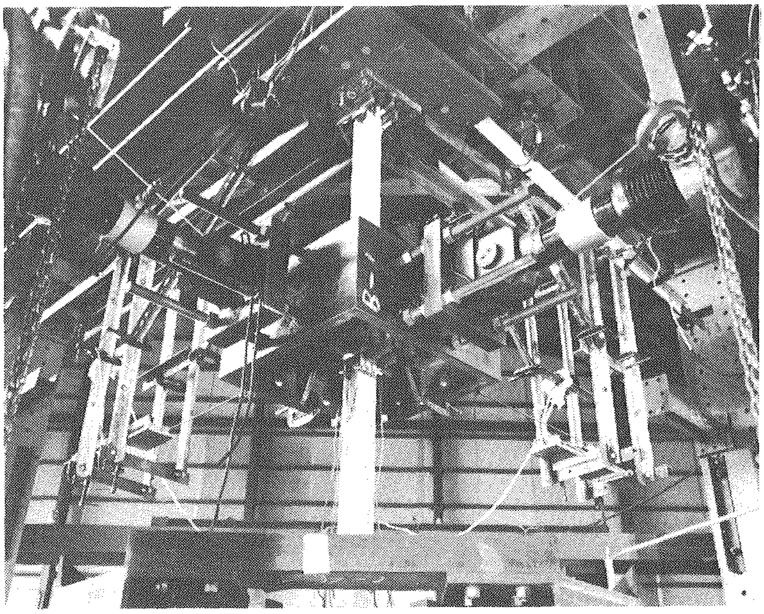


Fig.5 A close-up of a specimen under experiment

## Table 1 The mechanical properties of specimens

| SPECIMEN | Ix ($cm^4$) | Iy ($cm^4$) | λx | λy | Mpcx (t·cm) | Mpcy (t·cm) | Upcy (cm) | Upcx (cm) |
|---|---|---|---|---|---|---|---|---|
| DBC-A-1 | 98.4 | 36.2 | 31.6 | 52.2 | 90.9 | 51.1 | 0.572 | 0.874 |
| A-2 | 97.3 | 35.9 | 31.7 | 52.2 | 90.4 | 50.8 | 0.578 | 0.881 |
| A-3 | 98.3 | 36.2 | 31.7 | 52.2 | 91.2 | 51.2 | 0.576 | 0.880 |
| A-4 | 97.5 | 35.9 | 31.7 | 52.3 | 90.5 | 50.9 | 0.578 | 0.884 |
| A-5 | 97.5 | 35.7 | 31.6 | 52.3 | 90.5 | 50.6 | 0.575 | 0.880 |
| DBC-B-1 | 96.8 | 36.0 | 31.9 | 52.3 | 90.3 | 51.0 | 0.580 | 0.880 |
| B-2 | 97.6 | 35.6 | 31.7 | 52.5 | 90.7 | 50.6 | 0.579 | 0.887 |
| DBC-C-1 | 98.5 | 35.9 | 31.6 | 52.4 | 91.3 | 51.0 | 0.577 | 0.882 |
| C-2 | 98.7 | 36.4 | 31.7 | 52.2 | 91.5 | 51.5 | 0.578 | 0.881 |
| C-3 | 98.5 | 36.3 | 31.6 | 52.0 | 91.4 | 51.4 | 0.572 | 0.874 |
| C-4 | 100.2 | 36.3 | 31.8 | 52.0 | 93.0 | 52.9 | 0.578 | 0.905 |

Ix (Iy): Moment of inertia

λx (λy): Slenderness ratio

Mpcx (Mpcy): Plastic Moment under axial load

$U_{pcx(y)} = Q_{pcx(y)} \cdot L^3 / (12 \cdot EI_{y(x)})$

Qpcx (Qpcy)=Mpcx (Mpcy)/(L/2)

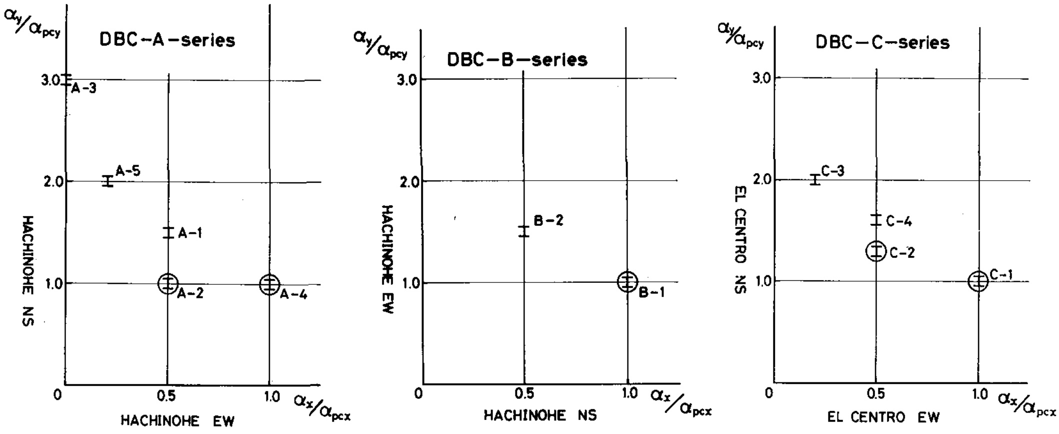
$\sigma_y = 3.30$ t/cm

## Table 2 The frame models analysed

| SPECIMEN | DIRECTION | STRUCTURE |  |  |  | GROUND MOTION |  |  |
|---|---|---|---|---|---|---|---|---|
|  |  | Q pc (t) | K e (t/cm) | T (sec) | M (t·sec²/cm) | α/αpc | α (gal) | INPUT EARTHQUAKE |
| DBC-A-1 | x | 1.15 | 1.10 | 0.807 | 0.0181 | 0.5 | 31.9 | HACHINOHE-EW |
|  | y | 2.05 | 2.85 | 0.5 |  | 1.5 | 170.0 | -NS |
| A-2 | x | 1.15 | 1.09 | 0.809 | 0.0181 | 0.5 | 31.6 |  |
|  | y | 2.04 | 2.86 | 0.5 |  | 1.0 | 112.5 |  |
| A-3 | x | 1.16 | 1.12 | 0.793 | 0.0183 | 0.0 | 0.0 |  |
|  | y | 2.06 | 2.82 | 0.5 |  | 3.0 | 345.9 |  |
| A-4 | x | 1.15 | 1.11 | 0.813 | 0.0186 | 1.0 | 61.6 |  |
|  | y | 2.04 | 2.93 | 0.5 |  | 1.0 | 109.7 |  |
| A-5 | x | 1.14 | 1.12 | 0.806 | 0.0184 | 0.2 | 12.4 |  |
|  | y | 2.04 | 2.90 | 0.5 |  | 2.0 | 222.2 |  |
| DBC-B-1 | x | 1.15 | 1.11 | 0.798 | 0.0179 | 1.0 | 63.3 | HACHINOHE-NS |
|  | y | 2.03 | 2.82 | 0.5 |  | 1.0 | 113.9 | -EW |
| B-2 | x | 1.14 | 1.09 | 0.808 | 0.0180 | 0.5 | 31.6 |  |
|  | y | 2.04 | 2.84 | 0.5 |  | 1.5 | 169.9 |  |
| DBC-C-1 | x | 1.15 | 1.11 | 0.806 | 0.0182 | 1.0 | 63.0 | EL CENTRO-EW |
|  | y | 2.06 | 2.88 | 0.5 |  | 1.0 | 112.8 | -NS |
| C-2 | x | 1.16 | 1.11 | 0.807 | 0.0183 | 0.5 | 31.6 |  |
|  | y | 2.06 | 2.89 | 0.5 |  | 1.3 | 146.1 |  |
| C-3 | x | 1.16 | 1.12 | 0.808 | 0.0186 | 0.2 | 12.5 |  |
|  | y | 2.06 | 2.93 | 0.5 |  | 2.0 | 222.4 |  |
| C-4 | x | 1.19 | 1.13 | 0.795 | 0.0181 | 0.5 | 32.8 |  |
|  | y | 2.09 | 2.86 | 0.5 |  | 1.6 | 184.9 |  |

$Q_{pc} = M_{pc}/(L/2)$ Ke :Elastic stiffness M :mass $T = 2\pi\sqrt{M/Ke}$

$\alpha_{pc} = Q_{pc}/M$



(a) (b) (c)

Fig.6 The combinations of the intensities of ground accelerations

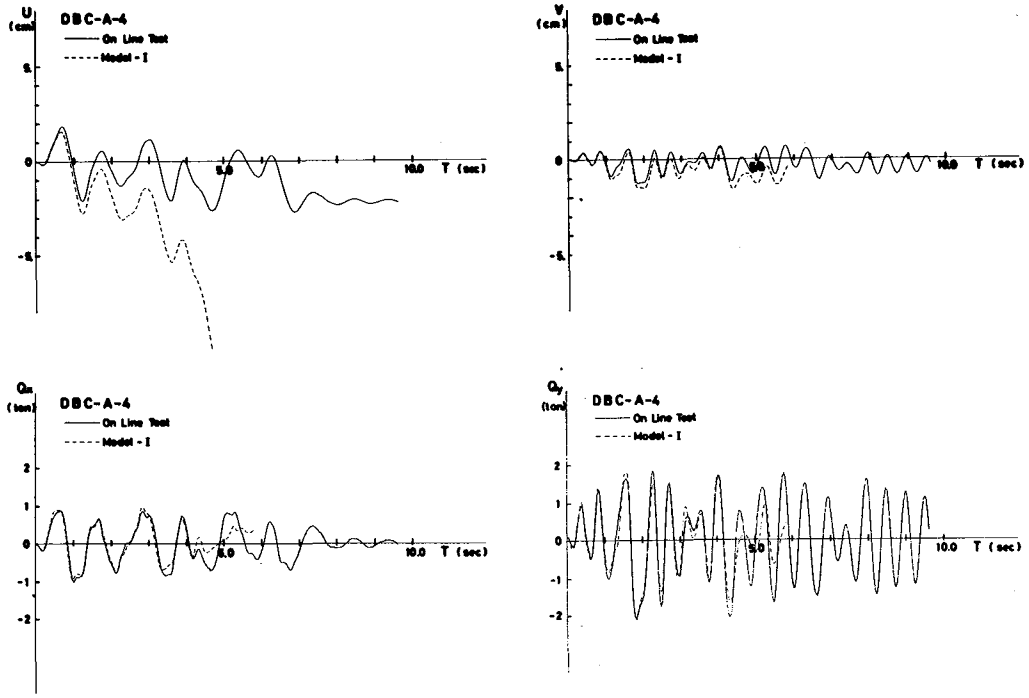


Fig.7 The time histories of response displacements and restoring forces (DBC-A-4)

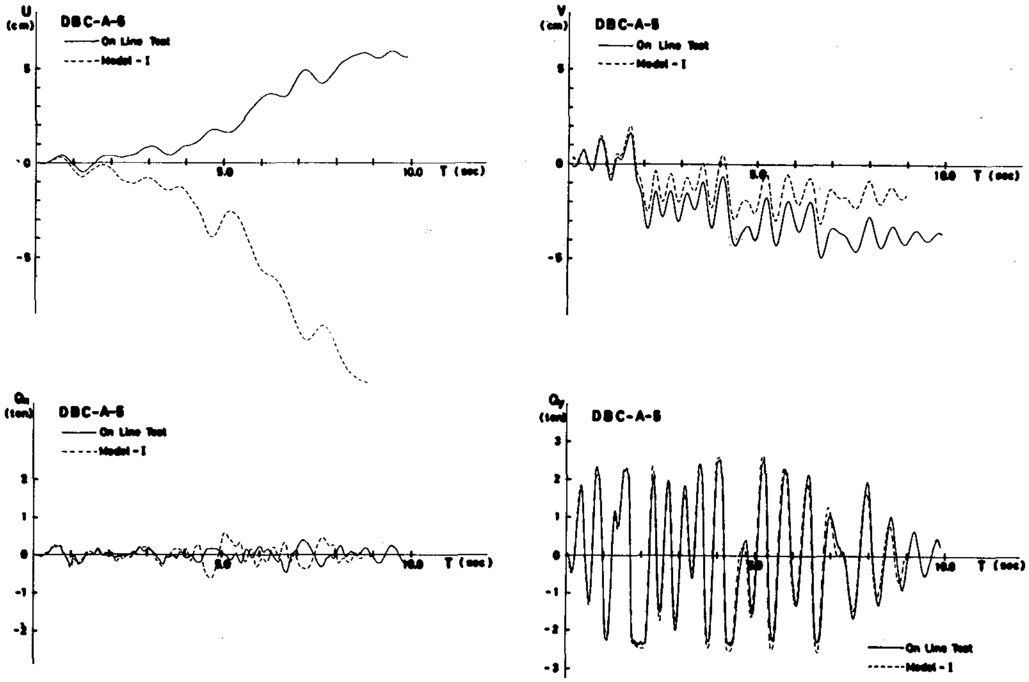


Fig.8 The time histories of response displacements and restoring forces (DBC-A-5)

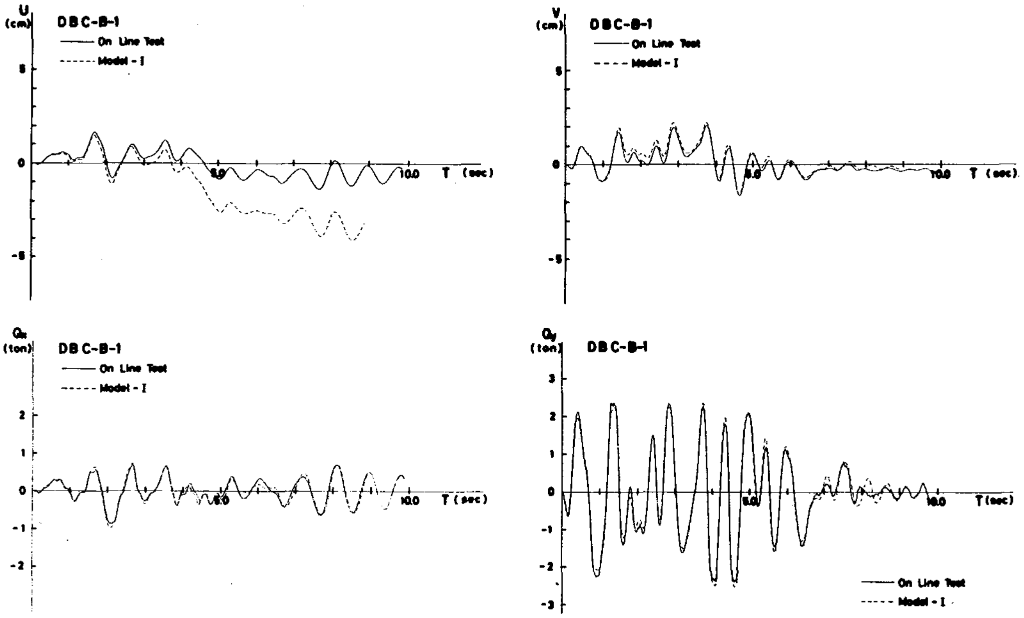


Fig.9 The time histories of response displacements and restoring forces (DBC-B-1)

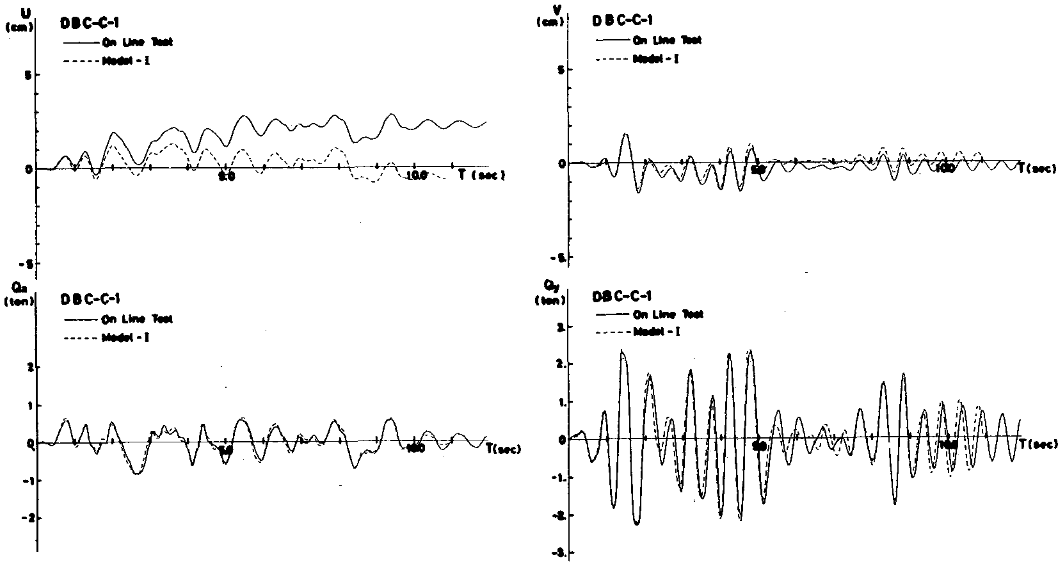


Fig.10 The time histories of response displacements and restoring forces (DBC-C-1)

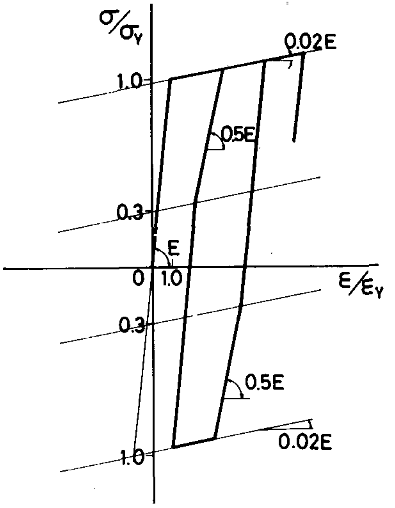
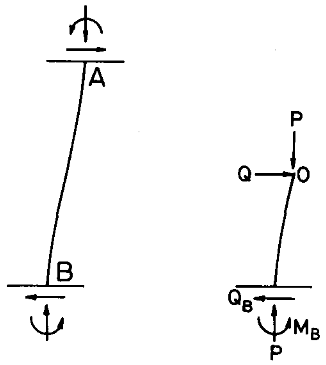


Fig.11 The stress-strain relation used in the computer analysis



(a) (b)

Fig.12 The forces and the moments acting on a column

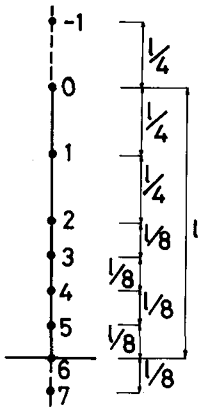


Fig.13 The locations where the variables be evaluated in the finite difference method

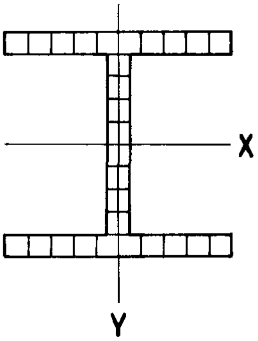


Fig.14 The uniaxially stressed filament model

Table 3 The maximum and minimum values of the response displacements by the hybrid analyses

| SPECIMEN FOR EXPERIMENT | $\alpha_x/\alpha_{pcx}$ | $\alpha_y/\alpha_{pcy}$ | Umax | ~ | Umin (cm) | Vmax | ~ | Vmin (cm) |
|---|---|---|---|---|---|---|---|---|
| DBC-A-1 | 0.5 | 1.5 | 5.96 | ~ | -1.12 | 0.93 | ~ | -2.65 |
| 2 | 0.5 | 1.0 | 0.81 | | -1.33 | 0.59 | | -1.66 |
| 3 | 0.0 | 3.0 | 9.85 | | -0.09 | 5.21 | | -1.80 |
| 4 | 1.0 | 1.0 | 1.87 | | -2.71 | 0.66 | | -1.27 |
| 5 | 0.2 | 2.0 | 5.96 | | -0.45 | 1.66 | | -4.92 |
| DBC-B-1 | 1.0 | 1.0 | 1.66 | ~ | -1.35 | 2.08 | ~ | -1.66 |
| 2 | 0.5 | 1.5 | 10.01 | | -0.02 | 2.00 | | -4.28 |
| DBC-C-1 | 1.0 | 1.0 | 2.80 | ~ | -0.33 | 1.52 | ~ | -1.63 |
| 2 | 0.5 | 1.3 | 0.85 | | -0.42 | 2.23 | | -1.72 |
| 3 | 0.2 | 2.0 | 7.50 | | -0.01 | 2.71 | | -2.34 |
| 4 | 0.5 | 1.6 | 10.06 | | -0.05 | 2.61 | | -1.86 |