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A DISCRETE ANALYSIS ON DYNAMIC COLLAPSE OF CLAMPED BEAMS AND RECTANGULAR PLATES LOADED IMPULSIVELY

By

Tadahiko KAWAI I) and Yutaka TOI II)

SYNOPSIS

In the previous papers by the present authors 2), 3) a new discrete method of analysis was proposed on the dynamic collapse of beams and plates subjected to transverse impulsive loads. In the present paper, this method is applied to the dynamic collapse analysis of clamped beams and rectangular plates under distributed impulsive loading. Numerical results obtained are compared with the experimental ones by N. Jones in order to verify the validity of the present method.

NOTATION

B ; beam breadth or semi-width of plate

C_b ; spring compliance

E ; Young's modulus

G ; elastic shear modulus

H ; beam or plate thickness

L ; half span of beam or semi-length of plate

M ; bending moment

M_p ; plastic moment ($\sigma_0 H^2/4$)

t ; time

V_0 ; initial impulsive velocity

w ; transverse displacement

w_m ; permanent transverse deflection at midspan of beam or at center of plate

x, y ; cartesian coordinate

I) Professor, Institute of Industrial Science, Univefsity of Tokyo.

II) Graduate Student, University of Tokyo.

η_b ; viscous coefficient of dashpot

η_G ; viscous coefficient for shearing deformation

λ ; dimensionless initial kinetic energy ($\rho V_0{}^2 L^2/M_p$)

ρ ; density of specimens

σ_0 ; yield stress in simple tension

ν ; Poisson's ratio

INTRODUCTION

New beam and plate bending elements (termed "the rigid body-spring model") were proposed by one of authors in Ref. 1), based on the experimental evidences for the plastic collapse behavior of these structures under lateral loading. These discrete models consist of rigid elements (rigid bars for a beam and rigid triangles for a plate) connected by a rotational spring, in which the number of nodal degrees of freedom is only one (lateral displacement) and the concept of plastic hinges or plastic hinge lines can be easily introduced. So that they are conveniently used in the limit load analysis of beam and plate structures with considerable reduction of computing time.

These discrete elements were applied to the dynamic collapse analysis in Ref. 2) where the dynamic collapse behavior of a long beam and a simply supported square plate under concentrated lateral impulsive loading was simulated successfully within a very short computing time. The numerical result obtained for a long beam was in good agreement with the theoretical elasto-plastic solution for a beam with infinite length.

The rigid body-spring models were extended to the elasto-viscoplastic analysis in Ref. 3) by replacing a rotational spring with a rheological elasto-viscoplastic system which was composed of a spring, a slider and a dashpot as shown in Fig. 1. By using these models, the dynamic elasto-plastic analysis considering a strain rate effect was carried out in Ref. 3) for impulsively loaded beams and plates.

In this note the validity of the present discrete method is examined especially from the quantitative viewpoint through comparison of numerical results with theoretical rigid-plastic solutions and experimental results given in the literature5) for the dynamic collapse behavior of clamped beams and rectangular plates under distributed impulsive loading.

DYNAMIC COLLAPSE OF CLAMPED BEAMS AND RECTANGULAR PLATES

Numerical analyses are carried out according to the procedure described in Refs. 2) and 3), based on the following assumptions:

(1) The lateral deflections are infinitesimal.

(2) The shearing deformations and the rotatory inertial forces are neglected.

(3) The static relation between the bending moment and the curvature is bilinear with no strain hardening and the yield condition is

$$M = M_p$$

Newmark's β method with $\beta = 1/6$ is applied to the numerical time integration of the equation of motion.

Calculations were made by using the domestic computer HITAC 8800/8700 which is approximately comparable to the IBM 370-158.

CLAMPED ALUMINUM BEAMS UNDER DISTRIBUTED IMPULSIVE LOADING: The dynamic collapse analysis neglecting a strain rate effect is conducted for a number of clamped aluminum 6061T6511 beams with a rectangular cross section which are subjected to a triangular or a rectangular shaped initial velocity field as shown in Fig. 2.

The material constants are written below.

$$E = 10.5 \times 10^6 \text{ lb/in}^2$$

$$\rho = 0.000251 \text{ lb sec}^2/\text{in}^4$$

$$\sigma_0 = 49296 \text{ lb/in}^2$$

Six cases (No.1 ∿ No.6) presented in Table 1 are analyzed with 10 uniform elements idealization for the whole length and using 1.87 μsec as the time increment.

The obtained results are shown in Figs. 3 and 4, from which it is seen that the present numerical solutions are in good agreement with the experimental results.5) CPU time was about 11 seconds for one case.

CLAMPED MILD STEEL RECTANGULAR PLATES UNDER UNIFORMLY DISTRIBUTED IMPULSIVE LOADING: The dynamic elasto-viscoplastic analysis as well as the usual elasto-plastic analysis is carried out for a number of clamped mild steel rectangular plate specimens under a uniform initial velocity field, since mild steel is more sensitive than aluminum to a strain rate.

The material constants of mild steel are given below.

$E = 30.0 \times 10^6$ lb/in^2

$\nu = 0.3$

$\rho = 0.000723$ lb sec^2/in^4,

$\sigma_0 = 36800$ lb/in^2

$k = G/\eta_G = 0.6 \times 10^6$ sec^{-1}

The dimensions of rectangular plates are as follows (See Fig. 5):

$L = 2.53125$ in $B = 1.5$ in

Due to symmetry only one quadrant is discretized as shown in Fig. 5 with the following input data:

Number of elements = 60

Number of springs = 98

Total degrees of freedom = 30

Six cases (No.1 ∿ No.6) presented in Table 2 are analyzed with the time increment 0.7 μsec.

The obtained results are shown in Figs. 6 and 7 with the theoretical rigid-plastic and exprimental results.5) It can be seen from these figures that numerical results are highly improved by considering a strain rate effect. CPU time was about 43 seconds for one case.

CONCLUSIONS

The effectiveness of the previously proposed discrete element method in the dynamic collapse analysis has been demonstrated through comparison of numerical and experimental results for clamped beams and rectangular plates under uniformly distributed impulsive loading.

By using this method, the complicating dynamic collapse behavior of beam and plate structures accompanied by the elasto-plastic bending wave propagation can be simulated successfully with small computing cost and sufficiently accurate solutions can be obtained.

Ref. 4) is the detailed version of this note.

REFERENCES

1) T. Kawai: New Element Models in Discrete Structural Analysis, J. of the Society of Naval Architects of Japan, Vol. 141 (1977)

2) T. Kawai and Y. Toi: New Discrete Analysis on Transient Response of Beams and Plates Subjected to Transverse Impact, Transactions of the Japan Society of Mechanical Engineers, Vol. 45, No. 389 (1979) (in Japanese)

3) Y. Toi and T. Kawai: A New Discrete Analysis on Dyanmic Collapse of Structures, J. of the Society of Naval Architects of Japan, Vol. 143 (1978)

4) Y. Toi and T. Kawai: A New Discrete Analysis on Dynamic Collapse of Structures (Further report), ibid., Vol. 145 (1979) (to appear)

5) See literatures cited in Ref. 4)

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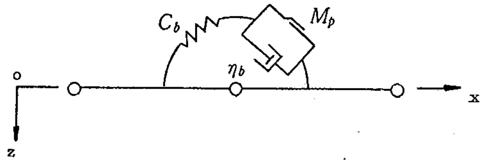
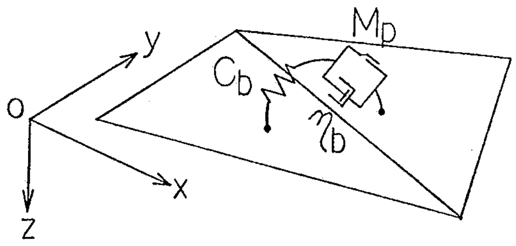
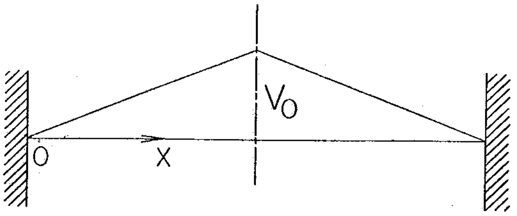


(a) Beam bending element

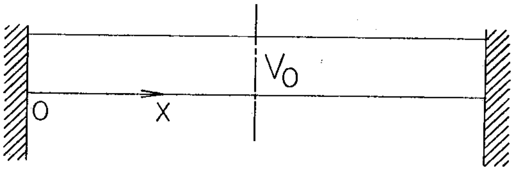


(b) Plate bending element

Fig. 1. Rigid Element Models for Elasto-viscoplastic Analysis in Beam and Plate Bending Problems



(a) Triangular



(b) Rectangular

Fig. 2. Initial Impulsive Velocity Fields for Clamped Beams

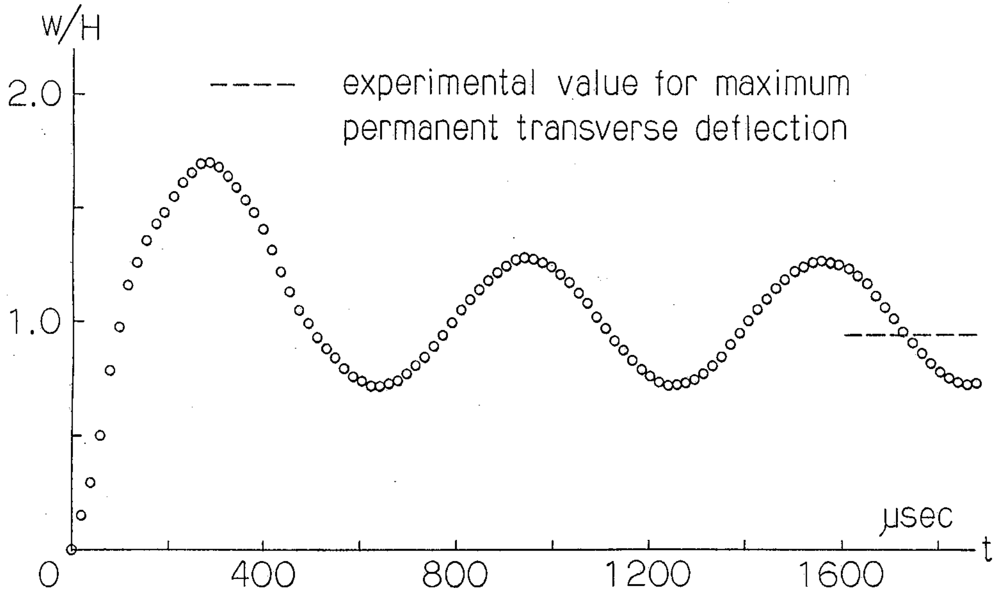


Fig.3. Transverse Displacement-time Histories at the Center of Clamped Beam Specimen No. 3

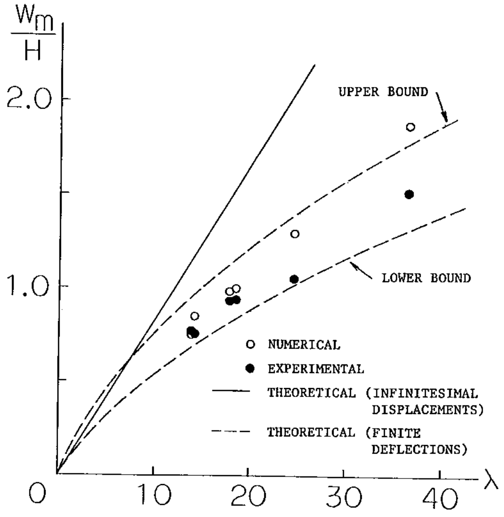


Fig.4. Maximum Permanent Transverse Displacements for Clamped Beams

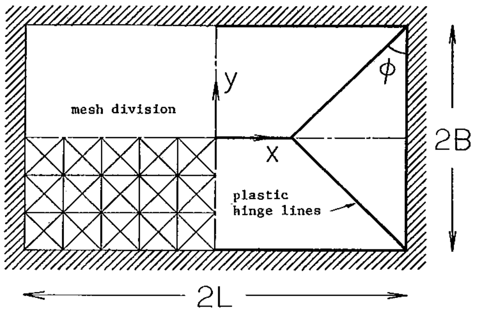


Fig.5. Clamped Rectangular Plate Subjected to a Uniformly Distributed Transverse Impulsive Load

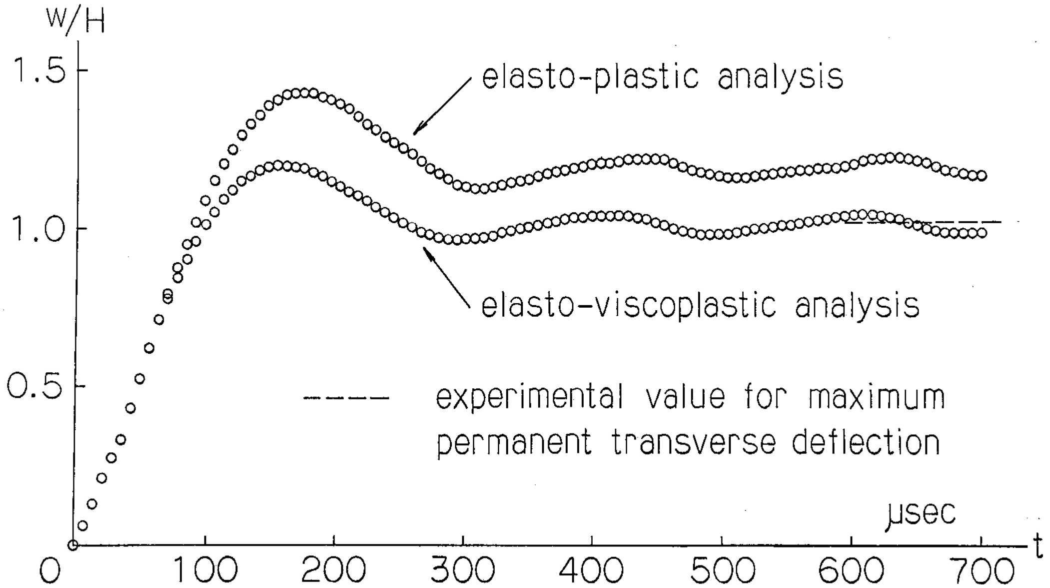


Fig.6. Transverse Displacement-Time Histories at the Center of Mild Steel Plate Specimen No. 3

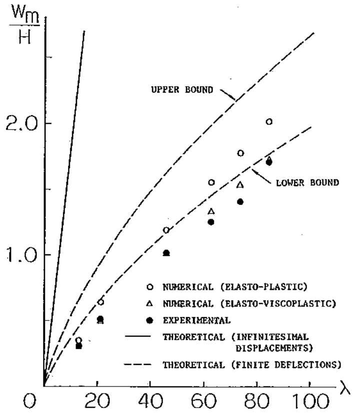


Fig.7. Maximum Permanent Transverse Displacements for Mild Steel Plates

Table 1. Data for Aluminum 6061T6511 Beam Specimens

Test No.	B (in)	H (in)	2L (in)	Explosive shape	V_0 (in/sec)	λ	w_m/H experimental	w_m/H numerical
1	0.3751	0.2007	5.0045	rectangular	2258	36.32	1.510	1.873
2	0.3752	0.2003	5.0075	rectangular	1851	24.53	1.053	1.294
3	0.3753	0.1996	5.0080	rectangular	1599	18.45	0.941	0.994
4	0.3752	0.2006	5.0045	rectangular	1405	14.09	0.755	0.846
5	0.3742	0.2008	5.0055	triangular	2084	13.74	0.770	0.752
6	0.3757	0.2001	5.0040	triangular	2358	17.70	0.934	0.984

Table 2. Data for Hot-Rolled Mild Steel Plate Specimens

Specimen No.	H (in)	V_0 (ft/sec)	λ	w_m/H experimental	w_m/H numerical(E-P)	w_m/H numerical(E-VP)
1	0.1728	69.69	12.92	0.310	0.348	0.306
2	0.1728	88.85	20.92	0.515	0.640	0.498
3	0.1725	130.90	45.6	1.022	1.195	1.019
4	0.1729	153.36	62.25	1.257	1.557	1.337
5	0.1728	166.26	73.25	1.411	1.783	1.541
6	0.1727	178.02	84	1.715	2.018	1.734