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BRIEF NOTE

COMPUTER SIMULATION OF TWO DIMENSIONAL PLATE TECTONICS PROBLEMS BY A NEW DISCRETE METHOD OF ANALYSIS

by

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Summary

Plate tectonics theory in seismology explains that the compression-failure-spring back motion accompanied by the relative movement of plates (rock plates about 100 km in thickness) which cover the surface of the earth is the mechanism of occurrence of strong earthquakes.

In this note the above behavior of a two-dimensional simple fault model will be simulated numerically by using a new discrete model which is suitable for elasto-plastic plane strain problems including a contact effect.

1. A New Discrete Model in Plane Strain Problem

A new discrete element for plane strain problems used in present analysis will be described briefly.

Consider two rigid triangular plates connected by three different types of springs k_d, k_s, and k_r which resist relative displacements δ_V, δ_H, and φ respectively as shown in Fig. 1.

The element equilibrium equation is expressed as follows:

$$[k]\{u\} = \{f\} \quad \cdots\cdots \quad (1)$$

where $[k]$ is the element stiffness matrix given in Table 1, and $[u]$ and $\{f\}$ are the centroidal displacement and external force vector respectively given as follows:

$$\left.\begin{aligned} \{u\}^t &= \lfloor u_1\ v_1\ \theta_1\ u_2\ v_2\ \theta_2 \rfloor \\ \{f\}^t &= \lfloor X_1\ Y_1\ M_1\ X_2\ Y_2\ M_2 \rfloor \end{aligned}\right\} \quad \cdots\cdots \quad (2)$$

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In case of isotropic material which has Young's modulus E and Poisson's ratio ν, the above three spring constants can be determined by using finite difference approximation of strain as follows:

$$\left.\begin{aligned} k_d &= \frac{E(1-\nu)\ell_{35}}{(1+\nu)(1-2\nu)(d_1+d_2)} \\ k_s &= \frac{E\,\ell_{35}}{(1+\nu)(d_1+d_2)} \\ k_r &= \frac{k_d\,\ell_{35}^2}{12} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \quad (3)$$

where d_1 and d_2 are shown in Fig. 2.

To show validity of this new element, elasto-plastic analyses of the punch problem and a slit notch specimen under tensile load were made in Ref. (1). In these analyses the rotational component φ was neglected and yet the ultimate loads and slip lines obtained were in good agreement with the results obtained by using the so-called "slip line theory".

In this element the springs are arranged on the boundaries of the element so that it will be conveniently used in elasto-plastic plane contact problems.

2. Numerical Analysis of a Simple Fault Model

A two-dimensional fault model and the mesh division of the upper plate are shown in Fig. 3. The lower plate is assumed to be rigid and moves down with a constant speed in the direction parallel to the contact surface BC. As for the external force the body force due to gravity is considered. The surface AB is assumed to be fixed and the boundary condition on the contact surface BC is as follows:

$$\left.\begin{aligned} &\left.\begin{aligned} \dot{u}_t &= U \\ \dot{u}_n &= O \end{aligned}\right\} \text{ when } f < \mu N \\ &\left.\begin{aligned} \dot{u}_t &\ \text{free} \\ \dot{u}_n &= O \end{aligned}\right\} \text{ when } f = \mu N \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \quad (4)$$

where

$\dot{u}_t$: tangential velocity

$\dot{u}_n$: normal velocity

f : tangential stress

N : normal pressure

μ : coefficient of static friction

Figs. 4 and 5 show results of elastic analysis. In Fig. 4 the deformation of the earth surface and the state of the contact surface are shown where the numbers (1-5) imply the sequence of slip movement on the contact surface with the corresponding deformed shape. Fig. 5 shows time variation of the vertical displacement at six points (a-f) on the earth surface which are indicated in Fig. 4.

Fig. 6 shows results of elasto-plastic analysis. In this analysis the maximum shearing stress theory was used as the yield criterion. The numbers in the figure have the same meaning as in Fig. 4 and the thick line implies slip due to yielding.

3. Conclusion

Numerical simulation of pre-seismic behavior of a two-dimensional simple fault model was carried out by using a new discrete element in plane strain problems.

It is expected that numerical method such as the finite element method will be a powerful and indispensable tool for the prediction of a future great earthquake.

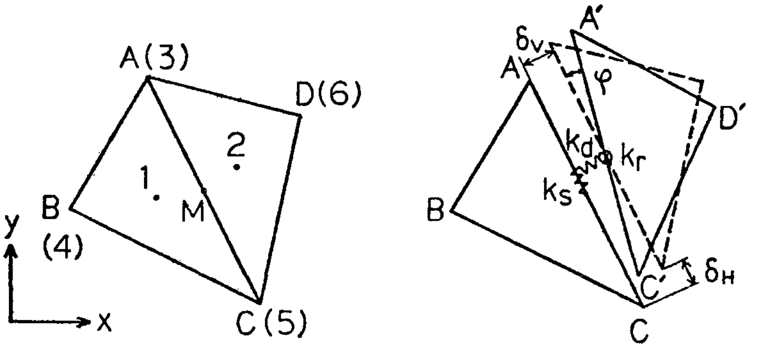
Numerical investigations of a whole process of earthquake behavior including elasto-plastic wave propagation analysis in two-dimensional medium is a future work.

References

(1) T. Kawai and Y. Toi, "A New Element in Discrete Analysis of Plane Strain Problems", Seisan Kenkyu, Vol. 29, No. 4 (1977)

(2) T. Kawai, "New Discrete Models and Their Application to Seismic Response Analysis of Structures", Nuclear Engineering and Design (to be published)

(3) K. Shimazaki, "Pre-Seismic Crustal Deformation by an Underthrusting Oceanic Plate, in Eastern Hokkaido, Japan", Physics of the Earth and Planetary Interiors 8 (1974)



(a) before deformation (b) after deformation

$(x_i,\ y_i)$: coordinates of a point i (i=1,---,6)

$$x_{ij} = x_i - x_j\ ,\quad y_{ij} = y_i - y_j$$

points 1 and 2 : centroids of two triangles

ℓ_{35} : length of $\overline{AC}$

Fig.1. A New Plane Strain Element

Table 1 Stiffness matrix of a new plane strain element

	u_1	v_1	θ_1	u_2	v_2	θ_2
X_1	$k_d y_{53}^2 + k_s x_{53}^2$					
Y_1	$-(k_d - k_s) x_{53} y_{53}$	$k_d x_{53}^2 + k_s y_{53}^2$				
M_1	$k_d y_{53} \Delta_{11} - k_s x_{53} \Delta_{21}$	$-(k_d x_{53} \Delta_{11} + k_s y_{53} \Delta_{21})$	$k_d \Delta_{11}^2 + k_s \Delta_{21}^2 + k_r l_{35}^2$		SYM.	
X_2	$-(k_d y_{53}^2 + k_s x_{53}^2)$	$(k_d - k_s) x_{53} y_{53}$	$-(k_d y_{53} \Delta_{11} - k_s x_{53} \Delta_{21})$	$k_d y_{53}^2 + k_s x_{53}^2$		
Y_2	$(k_d - k_s) x_{53} y_{53}$	$-(k_d x_{53}^2 + k_s y_{53}^2)$	$k_d x_{53} \Delta_{11} + k_s y_{53} \Delta_{21}$	$-(k_d - k_s) x_{53} y_{53}$	$k_d x_{53}^2 + k_s y_{53}^2$	
M_2	$k_d y_{53} \Delta_{22} - k_s x_{53} \Delta_{12}$	$-(k_d x_{53} \Delta_{22} + k_s y_{53} \Delta_{12})$	$k_d \Delta_{11} \Delta_{22} + k_s \Delta_{21} \Delta_{12} - k_r l_{35}^2$	$-(k_d y_{53} \Delta_{22} - k_s x_{53} \Delta_{12})$	$k_d x_{53} \Delta_{22} + k_s y_{53} \Delta_{12}$	$k_d \Delta_{22}^2 + k_s \Delta_{12}^2 + k_r l_{35}^2$

$$\left.\begin{aligned} 2\Delta_{11} &= x_{53}(x_{31}+x_{51}) + y_{53}(y_{31}+y_{51}) \\ 2\Delta_{12} &= x_{53}(y_{32}+y_{52}) - y_{53}(x_{32}+x_{52}) \\ 2\Delta_{21} &= -x_{53}(y_{31}+y_{51}) + y_{53}(x_{31}+x_{51}) \\ 2\Delta_{22} &= -x_{53}(x_{32}+x_{52}) - y_{53}(y_{32}+y_{52}) \end{aligned}\right\}$$

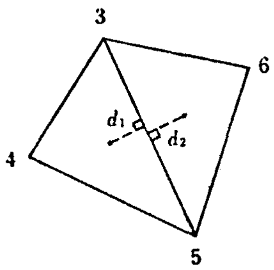


Fig.2. Determination of the Spring Constant

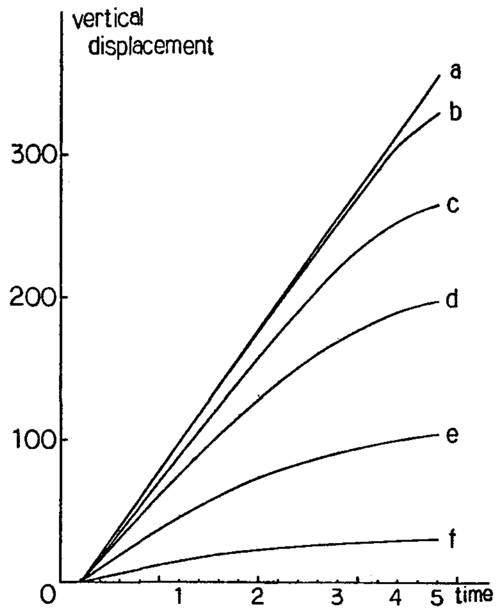


Fig.5. Time Variation of the Vertical Displacement (Elastic Analysis)

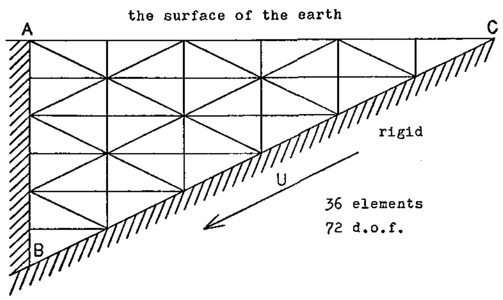


Fig.3. Finite Element Representation of a Two- dimensional Fault Model

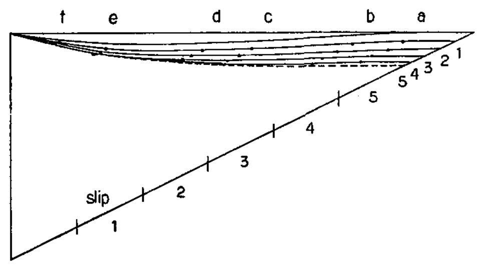


Fig.4. Pre-seismic Behavior of the Upper Plate (Elastic Analysis)

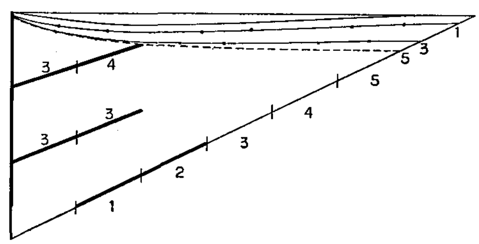


Fig.6. Pre-seismic Behavior of the Upper Plate (Elasto-plastic Analysis)