NON-LINEAR EARTHQUAKE RESPONSE ANALYSIS OF STRUCTURES BY A COMPUTER-ACTUATOR ON-LINE SYSTEM

(Detail of the System)

by

Koichi TAKANASHI,[I] Kuniaki UDAGAWA,[II] Matsutaro SEKI,[II]
Tsuneo OKADA[I] and Hisashi TANAKA[III]

Introduction

The research project of "Dynamic Failure of Structures and Structural Elements" was launched in 1971 by the members of Earthquake Resistant Structure Research Center (ERS) at the Institute of Industrial Science, University of Tokyo. The objective of this project was to investigate the dynamic behaviour of structures and structural elements during earthquake for establishing an advanced method of earthquake resistant design of structures such as buildings, bridges, dams and other structures.

For this purpose, the dynamic testing facilities including a data analyzing system have been installed at the Chiba Experimental Station of the Institute of Industrial Science.[1),2)]

The facilities consist of the following three sub-systems:

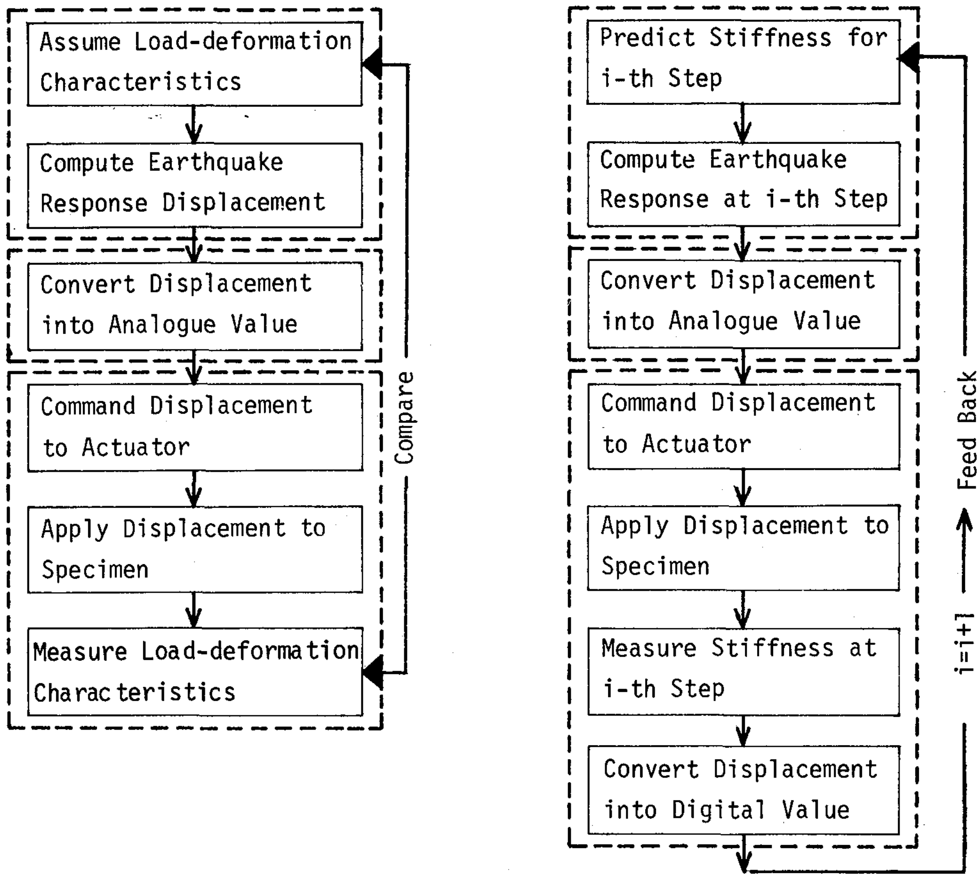
a) Sub-system for dynamic loading
.... Electro-hydraulic actuators and testing floor.

b) Sub-system for data analysis
.... Digital computer and Data convertors

c) Sub-system for vibration test
.... Shaking table

Two systems for dynamic testing on structures and structural elements have been developed since 1971, making loop systems between a) sub-system for dynamic loading and b) sub-system for data analysis. The block diagrams of the developed loop systems are shown in Fig. 1.[1),2),3)]

I) Associate Professor, II) Research Associate, III) Professor;
Institute of Industrial Science, University of Tokyo



a) Off-line System

b) On-line System

Fig. 1 Loop Systems for Earthquake Response Analysis of Structures

The specifications of the equipments of the sub-systems for dynamic loading and for data analysis are:

1) Electro-hydraulic actuators (2) ..... Loading capacity is ± 30 tons for Static and ± 20 tons for dynamic. Maximum stroke is ± 150 mm.. Control parameter is displacement or load. Arbitrary displacement or loading history can be commanded. Function generator for sinusoidal, triangular and rectangular waves is also installed. Pumping capacity of hydraulic power supply is 300 ℓ/min. at the pressue of 210 kg/cm^2.

2) Digital computer (HITAC 10 II) ..... Mini computer with 12k words of central memory and 64k words of extended core memory in magnetic drum.

3) D/A convertor .... 4 channels convertor with minimum sampling time of 1 ms. per one sample without limitation of sample size. Level of maximum and minimum output command is ± 2.5 V..

4) A/D convertor .... 16 channels convertor with minimum sampling time of 1 ms. per one sample without limitation of sample size. Level of input command is within ± 10.0 V..

The first test series was carried out by the "Off-line system" on steel and reinforced concrete structural elements. Earthquake response deflection of the structural elements which had been calculated previously by b) Sub-system for data analysis was applied to the structural elements by a) Sub-system for dynamic loading and the measured load-deflection relationships were compared with the assumed analytical models which had been adopted in the calculation.[4),5)]

Recently, the second loop system "Computer-Actuator On-line system" has been developed and the test series on steel and reinforced concrete buildings was carried out.[3),6)] The system consists of a) Sub-system for dynamic loading linked with b) Sub-system for data analysis and simulates the non-linear earthquake response of structures without assuming the non-linear analytical model of load-deformation characteristics of the structures or structural elements but with directly using the real behaviour obtained from the on-line dynamic testing.

This paper is mostly concerned with the principle of this developed "Computer-Actuator On-line System" and the detail of the procedure to perform the response analysis by the system.

Objective and Scope

Non-linear hysteresis loops such as bi-linear, tri-linear, Ramberg-Osgood type etc. have been widely adopted as analytical models for the earthquake response analysis of structures, while the necessity to develop more realistic analytical model which can represents the stiffness and strength deterioration of the structure during earthquake has been strongly recognized. Because, a simplified analytical model sometimes could not represent the real non-linearity of the structure which depends much upon a lot of uncertain factors such as non-linearity of materials, local failure, loss of structural stability etc..

Recently, numerous analytical models taking into account the stiffness and strength degradation have been proposed for reinforced concrete and steel members.[5),7),8),9),10),11)] However, a difficulty to evaluate their appropriateness for the response analysis is due to a lack of the informations on the real dynamic behaviours of structures during earthquake, because most of them have been derived from the data obtained under idealised and limited conditions such as statically reversed loading tests, although some trials to improve the disadvantages have been done including the authors' works by the "Off-line System".[4),5),8),12),13),14),15)]

The objective to develop the Computer-Actuator On-line System was to realize a simulation of the non-linear earthquake response taking into account the actual non-linear behaviours of structures and to help developing realistic analytical models for the non-linear restoring force characteristics of structures and structural elements.

A pioneering work in this field was done by M. Hakuno in 1969.[16)] He tested cantilever steel beams by the on-line system consisting of an analogue computer and an electro-magnetic actuator. His work has encouraged the authors to develop this system. In 1974, T. Tajime et al[17)] tested the composite structural system of a building and foundation piles by the computer-actuator on-line system which was similar to the authors' system.

The on-line system has been expected to improve the disadvantages both of the computer analysis and of the vibration test by shaking table for the earthquake response analysis.

The advantage of the on-line system are summarized as follows:

1) Earthquake response simulation can be performed taking into account the real non-linear restoring force characteristics of structures or structural elements without assuming the non-linear analytical models for them.
   This advantage may help not only to realize the simulation of earthquake response of the structure of which non-linear restoring force characteristics are quite uncertain but also to check the adequacy of simplified analytical models for structure and structural elements and to improve them so that they can represent more realistic behaviours.

2) Dynamic or pseudo dynamic failure test on large size or real size specimen can be performed by electro-hydraulic actuator with a comparatively small capacity. Because, a test specimen for dynamic or pseudo dynamic test by this system is not always a whole structure but a part of the structure or structural elements.
   This may solve the problem of similarity of failure mechanism between a model specimen and a prototype of the structure, which is usually rather hard to obtain for dynamic model test by shaking table.

3) The observation of the failure mode of the structural elements, monitoring of the failure mechanism of the structure

and the collection of the data can be easily done by the modification of the response period. Because, a test need not be performed on real time but can be performed on enlarged non-real time controlled by the computer, i.e., a response of the structure can be sustained at any moment for the observation and the collection of the data if the displacement is chosen for the control parameter. The influence of the modification of the response period to the response of the structure depends mostly upon the effect of loading rate to the restoring force characteristics, however, an appropriate modification may be permissible so far as the earthquake response is concerned.[18)]

4) Gravity load can be easily applied to the specimen by actuator previous to dynamic test in order to consider the stress due to the gravity load such as axial stress in column, bending stress in beam, etc., while a difficulty is sometimes due to a heavy weight attached to the specimen for dynamic test by shaking table. Similarity of mass or frequency is considered in the computer program.

Earthquake Response Analysis of One-story One-bay Building Frame

A principle of the response analysis by the computer-actuator on-line system is to solve the non-linear differential equation to express the earthquake response of the structure considering the stiffness change which is obtained by the dynamic loading test performed in parallel with the analysis.

The structural stiffness at each step required for the analysis was input directly into the computer from the structural experiment. Predicted deformation of the structure or structural members by the analysis at each step was given to the specimens by the hydraulic actuators and the stiffness anticipated by the experiment was taken into for the analysis at next step. This procedure was repeated step by step until the response of the structure was terminated. The experiment was also controlled by the digital computer through the D/A convertor and the A/D convertor corresponding to a commulative displacement at each step of the analysis.

Considering both the practicability of the on-line test and the accuracy in the numerical analysis, the first test series was started from the analysis of a single degree of freedom system. Using the developed on-line system, the earthquake response of one-story one-bay steel and reinforced concrete building frames were analyzed based upon the dynamic loading test of the members.

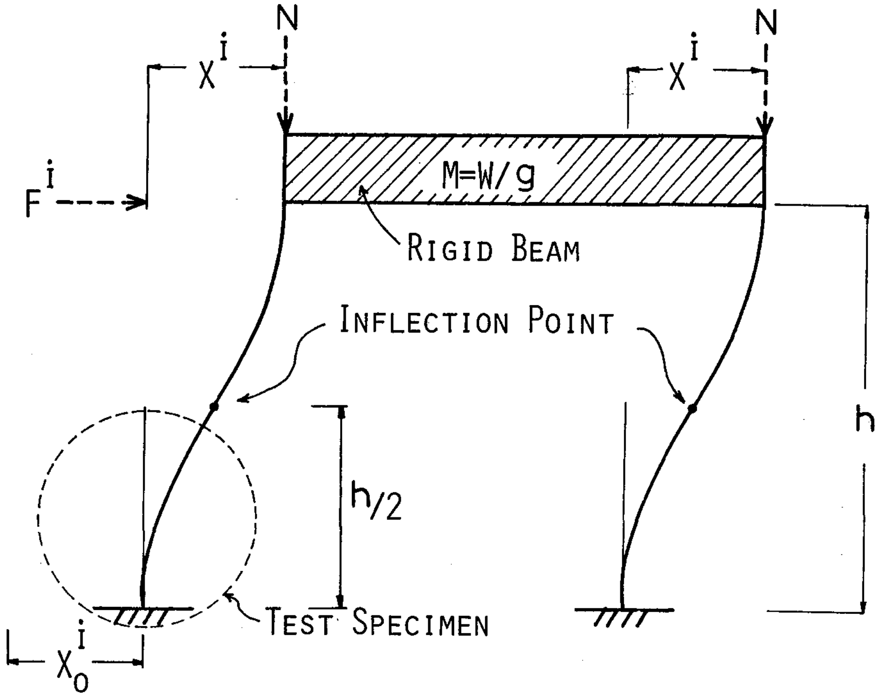


Fig. 2 Analyzed Frame and Coordinates

a) Analyzed building frame and frame coordinates

As shown in Fig. 2, the prototype of the analyzed building frame consists of two flexible columns connected with a rigid beam.

Adopting the frame coordinates shown in Fig. 2 and assuming the constant mass is concentrated at the tips of the columns, the equilibrium equation during earthquake is

$$M\Delta\ddot{x}^i + C\Delta\dot{x}^i + \Delta F^i = -M\Delta\ddot{x}_0^i \qquad (1)$$

where,
- M : Concentrated mass W/g
- W : Weight of the building frame
- g : Acceleration of gravity
- C : Viscous damping coefficient. A constant ratio to the critical damping was assumed in the analysis
- ΔF^i : Increment of the restoring force at i-th step
- $\Delta\ddot{x}^i$: Increment of the acceleration at i-th step
- $\Delta\dot{x}^i$: Increment of the velocity at i-th step
- $\Delta\ddot{x}_0^i$: Increment of the ground acceleration at i-th step

The evaluation of ΔF^i in the equation (1) is the most important to obtain a satisfactory accuracy for the earthquake response analysis by the on-line system. So, an appropriate numerical analysis method to predict the new incremental restoring force based upon the data obtained at the previous steps should be adopted.

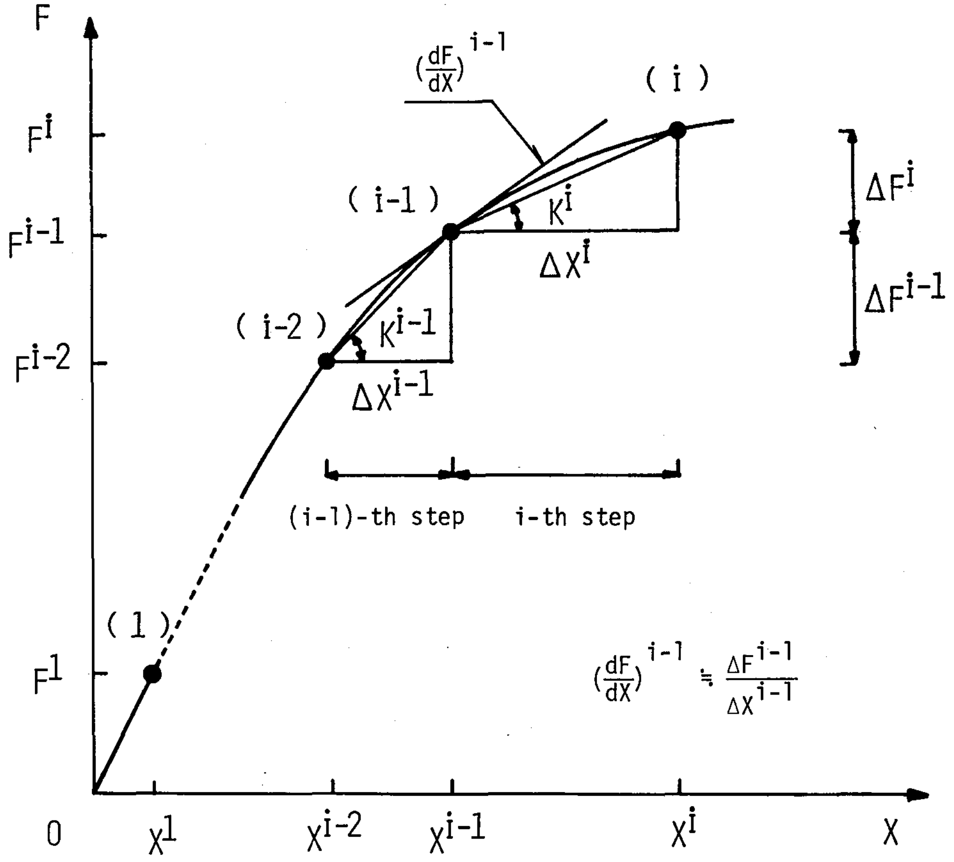


Fig. 3 Definition of Notations

<u>b) Evaluation of incremental restoring force</u>

Incremental restoring force ΔF^i at i-th step which was adopted in this on-line test was determined as follows:

The force-displacement characteristics of a single degree of freedom is shown in Fig. 3 to illustrate the definition of the notations.

Incremental restoring force ΔF^i is expressed by the equation (2).

$$\Delta F^i = F^i - F^{i-1} = F(x^{i-1} + \Delta x^i) - F(x^{i-1}) \qquad (2)$$

Considering the first term of the Taylor series expansion of F(X) about the point x^{i-1}, as the linearized approximation of ΔF^i, the equation (3) is obtained.

$$\Delta F^i \fallingdotseq \left(\frac{dF}{dx}\right)^{i-1} \Delta x^i \qquad (3)$$

where, Δx^i : Increment of the displacement at i-th step

$\left(\frac{dF}{dx}\right)^{i-1}$: Tangential stiffness at (i-1)-th point

Accordingly, the equation (1) is transformed into the equation (4).

$$M\Delta\ddot{x}^i + C\Delta\dot{x}^i + \left(\frac{dF}{dx}\right)^{i-1} \Delta x^i = -M\Delta\ddot{x}_0^i \qquad (4)$$

The tangential stiffness $\left(\frac{dF}{dx}\right)^{i-1}$ should be calculated based upon the measured values obtained from the experiment, but the function F(x) in the equation (4) beyond the (i-1)-th step has not been measured yet. Therefore, some approximation should be employed. As the first approximation of the tangential stiffness $\left(\frac{dF}{dx}\right)^{i-1}$ at (i-1)-th point, the secant stiffness at the previous (i-1)-th step was employed.

$$\left(\frac{dF}{dx}\right)^{i-1} \fallingdotseq \frac{F^{i-1} - F^{i-2}}{\Delta x^{i-1}} = \frac{\Delta F^{i-1}}{\Delta x^{i-1}} \qquad (5)$$

Consequently, the equilibrium equation is as follows:

$$M\Delta\ddot{x} + C\Delta\dot{x}^i + K^{i-1}\Delta x^i = -M\Delta\ddot{x}_0^i \qquad (6)$$

where, $K^{i-1} = \dfrac{\Delta F^{i-1}}{\Delta x^{i-1}}$

c) Test specimen and member coordinates

In order to develop the on-line system for the earthquake response analysis of the building frame based upon the loading test of the member, a specimen with a half of a column height was adopted, assuming that 1) the inflection point of each column was located at the mid-height and 2) the change of the axial force during earthquake was negligible.

These assumptions helped to perform the loading test by a small specimen.

The member coordinates satisfying the assumptions mentioned above is shown in Fig. 4.

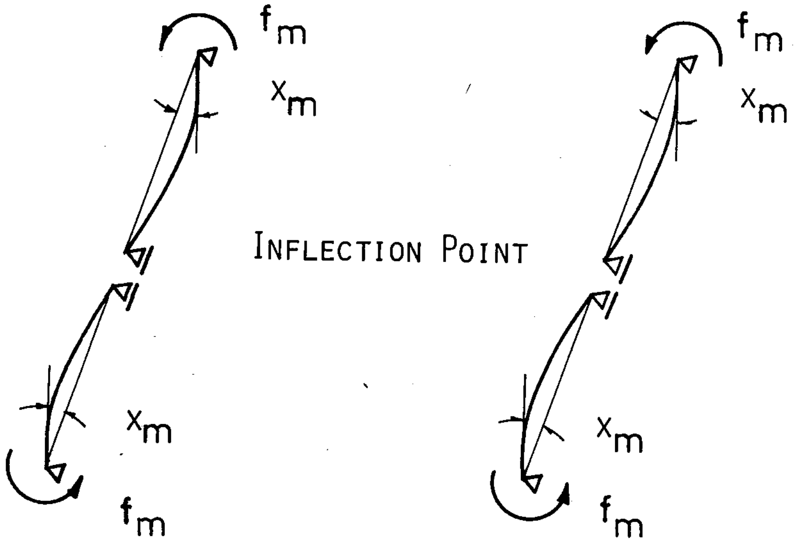


Fig. 4 Member Coordinates

The frame stiffness K^{i-1} in the equation (6) is synthesized with the stiffness at the member coordinates and the compatibility condition between the member coordinates and the frame coordinates. The equilibrium equation is

$$\Delta f_m^i = K_m^i \ \Delta x_m^i \quad (7)$$

where, suffix m : Index for the member coordinates

Δf_m^i : Increment of the force at i-th step

Δx_m^i : Increment of the displacement at i-th step

K_m^i : Stiffness at the member coordinates

The compatibility condition between the member coordinates and the frame coordinates is

$$\Delta x_m^i = \frac{1}{h} \Delta x^i \tag{8}$$

The equilibrium condition considering the N-Δ effect is

$$\Delta F_m^i = \left(\frac{4}{h} - \frac{2N}{K_m^i}\right) \Delta f_m^i \tag{9}$$

The frame stiffness is

$$K^i = \left(\frac{4}{h} - \frac{2N}{K_m^i}\right) K_m^i \cdot \frac{1}{h} = \frac{4}{h^2} K_m^i - \frac{2N}{h} \tag{10}$$

d) Loading system and transformation between loading system coordinates and frame coordinates

Two different types of the loading system were prepared to perform the analysis as shown in Fig. 5.

In the loading system-A, a specimen with the length of a half of the story height of the frame was supported at the rotating hinge-A and the rotating and sliding hinge-B. A rigid arm (A-C) with the length L was connected at the support-A and the lateral force P_L was applied by the actuator at the free end of the rigid arm for yielding the moment f_m at the end of the specimen. The loading test was controlled by the displacement x_L measured at the end of the rigid arm.

The stiffness at the loading system coordinates-A was defined as follows:

$$K_L^i = \Delta p_L^i / \Delta x_L^i \tag{11}$$

where, K_L^i : Stiffness at the loading system coordinates-A at the i-th step

ΔP_L^i : Increment of the force at i-th step

Δx_L^i : Increment of the displacement at i-th step

The compatibility condition and the equilibrium condition between the member coordinates and the loading system coordinates are:

$$\Delta x_L^i = L \Delta x^i \tag{12}$$

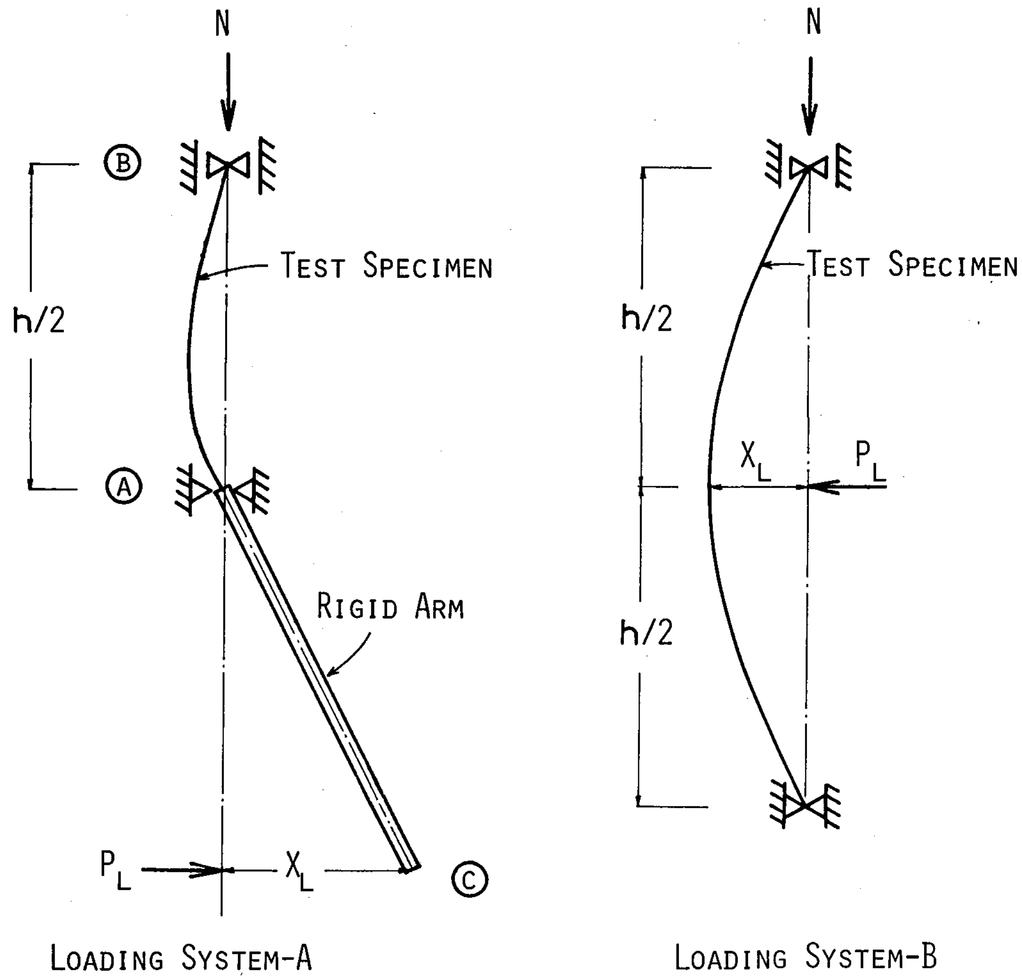


Fig. 5 Loading System and Coordinates

$$\Delta f_m^i = L \, \Delta p_L^i \tag{13}$$

So, the member stiffness K_m^i in the equation (10) is

$$K_m^i = L^2 \, K_L^i \tag{14}$$

Substituting the equation (14) into the equation (10), the frame stiffness is expressed by the stiffness at the loading coordinates-A.

$$K^{i} = \frac{4L^2}{h^2} K_L^{i} - \frac{2N}{h} \qquad (15)$$

where, the approximations of sin(Δx/h) ≒ (Δx/h) and cos(Δx/h) ≒ 1 are adopted.

The compatibility condition between the loading system coordinates-A and the frame coordinates is

$$\Delta x_L^{i} = L\,\Delta x_m^{i} = \frac{L}{h}\Delta x^{i} \qquad (16)$$

In the loading system-B, a specimen with length of the height of the frame was adopted and the lateral force P_L was applied at the mid-span. The lateral force was controlled by the displacement x_L at the mid-span.

The same consideration for the transformation can be applied to the loading system-B.

The stiffness at the loading system coordinates-B was defined as follows:

$$K_L^{i} = \Delta p_L^{i} / \Delta x_L^{i} \qquad (17)$$

The compatibility condition and the equilibrium condition between the loading coordinates and the frame coordinates are

$$\Delta x_L^{i} = \frac{h}{2}\Delta x_m^{i} \qquad (18)$$

$$\Delta f_m^{i} = \left(\frac{h}{4} + \frac{N}{K_L^{i}}\right)\Delta p_L^{i} \qquad (19)$$

The member stiffness K_m^{i} is

$$K_m^{i} = \left(\frac{h}{4} + \frac{N}{K_L^{i}}\right) K_L^{i} \cdot \frac{h}{2} = \frac{h^2}{8} K_L^{i} + \frac{Nh}{2} \qquad (20)$$

Substituting the equation (20) into the equation (10), the frame stiffness is

$$K^{i} = \frac{1}{2} K_L^{i} \qquad (21)$$

The compatibility condition between the loading system-B coordinates and the frame coordinates is

$$\Delta x_L^i = \frac{h}{2} \Delta x_m^i = \frac{1}{2} \Delta x^i \qquad (22)$$

e) Detail of the procedure

The flow diagram of the analysis using the loading system-A is shown in Fig. 6. The procedure of the analysis was as follows:

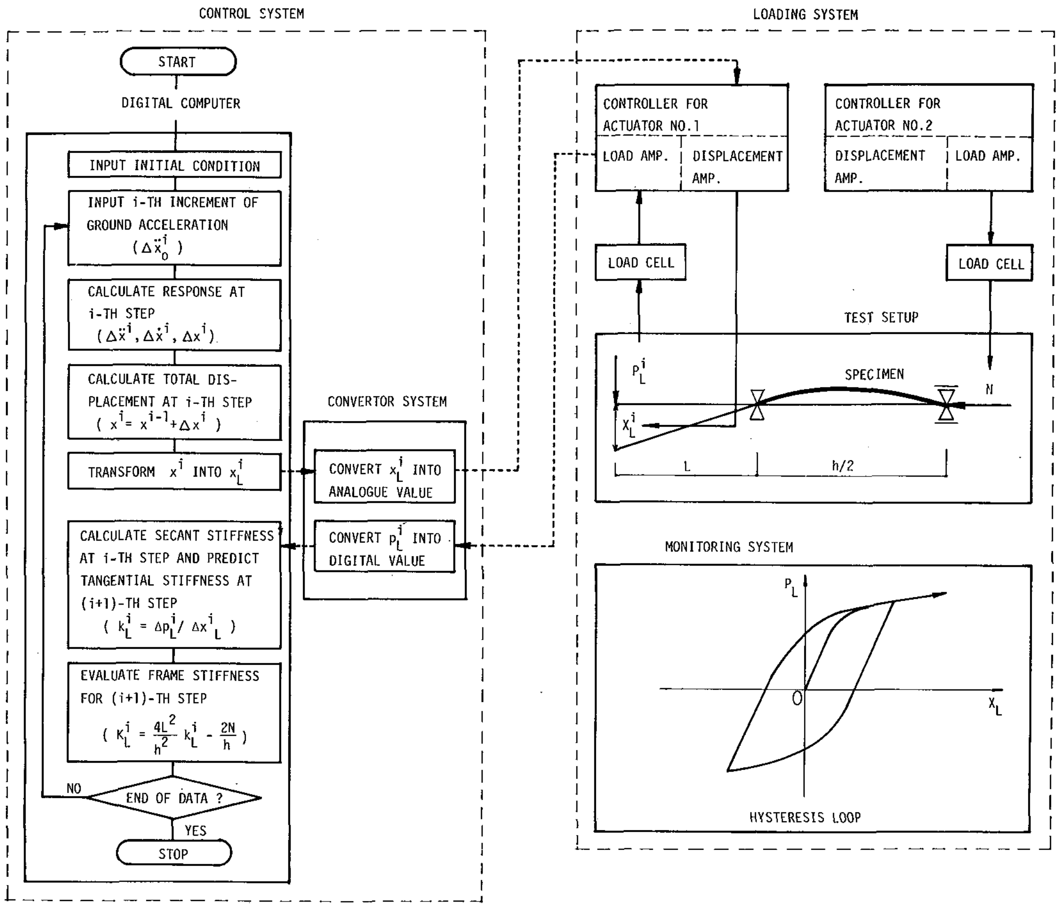


Fig. 6 Block Diagram of On-line System

1) Input the initial conditions:

Properties of the frame and the specimen: M,C,N,L,h
Initial conditions for analysis: $K^0=K_{LE}$, $\ddot{x}_0^1$,
$x^0=\dot{x}^0=\ddot{x}^0=0$, where, K_{LE} is the initial stiffness obtained from the static test in the elastic range performed prior to the on-line test.
Control data for the actuator and the convertors:
Initial values and ranges of the load and the displacement measuring devices.
Sampling time and sample size of the D/A and A/D convertors.

2a) Solve the equation (6) and calculate the response at the first step:

$$\Delta\ddot{x}^1,\ \Delta\dot{x}^1,\ \Delta x^1$$

2b) In general, calculate the response at i-th step:

$$\Delta\ddot{x}^i,\ \Delta\dot{x}^i,\ \Delta x^i,\ \ddot{x}^i=\ddot{x}^{i-1}+\Delta\ddot{x}^i,\ \dot{x}^i=\dot{x}^{i-1}+\Delta\dot{x}^i,\ x^i=x^{i-1}+\Delta x^i$$

If the sign of the velocity $\dot{x}^i$ has changed, repeat the step 2b) with the initial frame stiffness K_E replacing K^{i-1}.

3) Transform the displacement at the frame coordinates x^i into the displacement at the loading system coordinates x_L^i by the equation (16).

4) Convert the digital value of x_L^i into the analogue value through the D/A convertor.

5) Apply the displacement x_L^i to the specimen and measure the lateral force P_L^i.

6) Convert the analogue value of P_L^i into the digital one and calculate the increment of the force.

$$\Delta P_L^i = P_L^i - P_L^{i-1}$$

7) Calculate the secant stiffness at the loading system coordinates and evaluate the frame stiffness at the (i+1)-th step.

$$K_L^i \fallingdotseq \frac{\Delta P_L^i}{\Delta x_L^i}$$

$$K^i = \frac{4L^2}{h^2} K_L^i - \frac{2N}{h}$$

8) Repeat the step 2b) with (i+1) replacing i.

Concluding Remarks

The principle and the procedure of the earthquake response analysis by the computer-actuator on-line system were described. It has been approved by the first test series, which results will be reported later, that the developed system can realize to perform the non-linear response analysis of frame structures taking account the non-linear behavior of the members. Further improvement has been doing on the accuracy to predict the stiffness at the new step especially for the multi-degrees of freedom system and on the accuracy to measure the load and the displacement obtained from the test.

Acknowledgments

The reported work was performed as a part of the project on "The Defence System of Urban Functions against Environmental Disturbances" sponsored by the Ministry of Education, Japanese Government, at the Institute of Industrial Science, University of Tokyo. Sincere appreciation is due to Professor K. Kubo and other members of ERS for their guidance and encouragement. Mr. T. Shigenobu's contribution in developing the computer system program is especially acknowledged.

References

1) Kubo K., Okada T., Kawamata S., "Studies on Aseismicity of Civil Structures" Journal of IIS, Vol. 24, No. 3, 1972 (in Japanese)

2) ERS "A Closed Loop System for Dynamic Testing of Structures and Structural Elements" The Japanese National Committee of The International Association For Earthquake Engineering, Tokyo, Japan, May 1973

3) Takanashi K. et al "Non-linear Earthquake Response Analysis of Structures by A Computer-Actuator On-line System" Trans. of AIJ, March 1975, (in Japanese with abstract in English)

4) Takanashi K. et al "Failure of Steel Beams due to Lateral Buckling under Repeated Loads" LABSE Symposium, Lison, September 1973

5) Okada T., Seki M. et al "Restoring Force Characteristics of Reinforced Concrete Columns" Journal of IIS, Vol. 25, No. 2, 1973 (in Japanese)

6) Takanashi K. et al "Seismic Failure Analysis of Structures by

Computer-Pulsator On-line System" Journal of IIS, Vol. 26, No. 11, 1974 (in Japanese)

7) Wakabayashi M., "Studies on Damping and Energy Absorption of Structures" Introductory Report of LABSE Symposium, Lisbon, 1973

8) Takeda, T., Sozen, M.A., Nielsen, N.N., "Reinforced Concrete Response to Simulated Earthquakes," Journal of the Structural Division, ASCE, Vol. 96, No. ST12, December 1970

9) Umemura, H., Aoyama, H., and Takizawa, H., "Analysis of the Behavior of Reinforced Concrete Structure during Strong Earthquakes Based on Empirical Estimation of Inelastic Restoring Force Characteristic of Members," Proceedings, Fifth World Conference on Earthquake Engineering, Rome, Italy, June, 1973, Session 6B, Paper No. 275.

10) Park R., Kent C.D., Sampson R.A., "Reinforced Concrete Members with Cyclic Loading." Proceeding of the ASCE, ST 7, July 1972

11) Popov E.P., Pinkney R.B., "Reliability of Steel Beam-to-Column Connections under Cyclic Loading" Proc. of 4th World Conference on Earthquake Engi., Santiago, January, 1969

12) Tanaka H. et al "Inelastic Lateral Buckling of Steel Beams Subjected to Repeated and Reversed Loadings" Journal of IIS, Vol. 25, No. 2, 1973 (in Japanese)

13) Shigenobu T., Tanaka H., "Research on Restoring Force Characteristics of Steel Members under Repeated Axial Forces" Journal of IIS, Vol. 26, No. 2, 1974 (in Japanese)

14) Udagawa K., Takanashi K., Tanaka H.,"Flexural Torsional Failure of Steel Columns under Repeated Loads" Pre-Print of Annual Conference, AIJ, 1974 (in Japanese)

15) Otani S., "Inelastic Analysis of R/C Frame Structures," Journal of the Structural Division, ASCE, Vol. 100, No. ST7, July, 1974

16) Hakuno M. et al "Dynamic Test of Beams Controlled by Computer" Trans. of JSCE, No. 171, 1969

17) Tajime T. et al "On-line Real-time Test on The Earthquake Response of Building-Foundation Pile Structural System" Synthetic Symposium on Disaster-Science Nagoya, 1974

18) Hanson R.D., "Post-Elastic Response of Mild Steel Structure" Proc. 3rd Japan Earthquake Engineering Symposium, 1970

IIS : Institute of Industrial Science, University of Tokyo
AIJ : Architectural Institute of Japan
ASCE: American Society of Civil Engineers
JSCE: Japan Society of Civil Engineers