DEVELOPMENT OF A VIBRATION CONTROL SYSTEM FOR STRUCTURES BY MEANS OF "MASS PUMPS"

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SYNOPSIS

As one of the subjects included in the research project "Disaster Prevention in Urban Area" which has been carried out in the Institute of Industrial Science, University of Tokyo since 1971, a new device of vibration control system to be applied to structures is being developed for the purpose of minimizing the structural response to ground motion during earthquake.

The basic concept of the vibration control system is implementation of the structure with a device called "mass pumps" in which the oscillation of very high velocity is caused in the contained liquid by the vibration of the structure. It was verified both theoretically and experimentally that most part of the kinetic energy transmitted to the structure is absorbed in the form of the oscillation of the liquid contained in the "mass pump", resulting a drastic decrement of the response of the structure. This can be explained as the result of the mass and the damping effects of the "mass pumps".

It was also proved and demonstrated that this device can be a powerful isolator of mechanical vibration.

This paper describes the outline of the theoretical basis and the result of the experiments developed and carried out with regard to the system designed for single degree-of-freedom systems.

1. Introduction

The earthquake resistant design of structures requires the structural engineers to devote every effort in the directions of:

1) minimizing the vibratory response of the structure to the ground motion, and
2) to provide the structure with strength and ductility necessary to cope with the induced response.

To attain the first objective, it is necessary to provide the means of optimizing the vibration characteristics of the structure against the expected disturbances. The purpose of this

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research is to develop a new mechanism of controlling the structural vibration, which is not only effective but also economically practicable in real structures and reliable in its action against the incidental occurrence of earthquakes.

Under this objective, a new mechanism of vibration control was proposed: a mechanism composed of a framework and a pair of expansible vessels which are connected with one another by flexible tubes and filled by a liquid, either water or oil. These vessels are so installed in the framework that the vibration of the frame induces the pumping action of the liquid with the amplified velocity, the effective mass being produced. So let us call this vessel unit a "mass pump".

It was proved by the analysis of a simple model of this system that, because of the mass effect and the damping effect accompanied by the pumping action of the vessels, most of the kinetic energy put in the framework was absorbed in the form of the high velocity oscillation of the liquid in the pump.

The vibration tests of steel portal frames provided with such "mass pump" verified the drastic reduction of response.

In the following the principle of the vibration control system is described and the typical results of the tests are presented as a demonstration.

2. The Principle of the Vibration Control

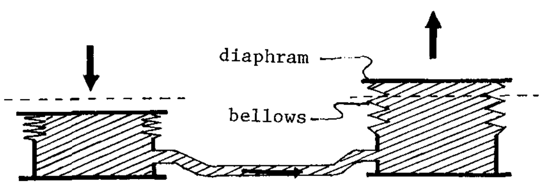

Fig. 1 "Mass Pump"

As shown in Fig. 1, let a pair of closed vessels having bellows be connected with one another by a narrow tube and filled with liquid such as water or oil. When alternating cyclic tensile and compressive forces are applied to the diaphrams of the both vessels, the liquid contained in the tube oscillates due to the difference of pressure at the both ends of the tube. Since this device, installed in a framework, exerts mass effect during vibration of the whole system, let it be called a "mass pump".

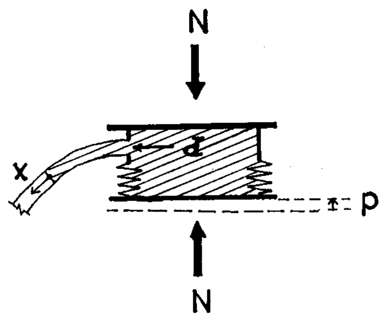

Fig. 2 Deformation of Bellows and Movement of Liquid

Referring to Fig. 2, it is apparent that the velocity of the liquid in the tube, $\dot{x}$, takes the value the velocity of the top diaphragm, $\dot{d}$, multiplied by the ratio of the effective sectional area of the bellows to the sectional area of the conduct tube, i.e.

$$\dot{x} = \beta \dot{d} \tag{1}$$

where $\beta = A / a$ (2)

A: effective sectional area of bellows
a: sectional area of conduit

Also, there is the following relation between the total force, N, acting on the diaphrams of the vessel and the total pressure, P, acting on the total mass of the liquid contained in the conduit to produce acceleration:

$$N = \beta P \tag{3}$$

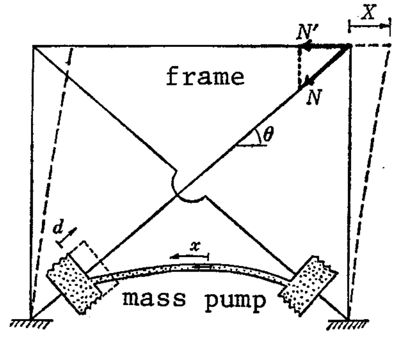

Fig.3 Installation of Mass Pump in Portal Frame

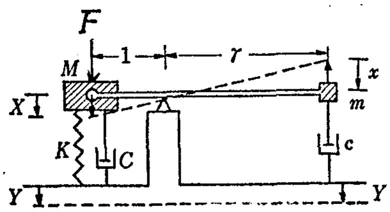

Fig.4 Model of Vibrating System

Now, let us consider a portal frame shown in Fig. 3, where the mass pump is installed through the diagonal members. Provided that the deformation of the diagonal members is neglegible, we have the following relation between the lateral displacement, X , of the top of the frame and the elongation of the bellows, d:

$$d = \alpha X, \qquad \alpha = \cos\theta \tag{4}$$

where θ is the angle of the diagonal member axis.

Also, the horizontal component of the axial force of the diagonal member is expressed as

$$N' = \alpha N, \quad \alpha = \cos\theta \tag{5}$$

and this component becomes the horizontal force acting to the top of the frame.

Consequently, it can be concluded that the liquid in the conduit moves under the condition of

$$x = \beta d = \alpha \beta X = \gamma X \tag{6}$$

where
$$\gamma = \alpha \beta \tag{7}$$

and also that the inertia force of the liquid moving in the conduit, $-m\ddot{x}$, produces the horizontal force acting on the top of the frame which is given by

$$N' = \alpha N = -\alpha \beta (m\ddot{x}) = -\gamma (m\ddot{x}) \tag{8}$$

where m denotes the mass of the liquid.

A model of the vibrating system shown in Fig. 4 can be conceived as the one which satisfies the both conditions given by Eqs. (6) and (8). In Fig. 4 the following notation is used:

M,K,C : mass, spring constant and damping coefficient, respectively, for the frame

m,c : mass and damping coefficient, respectively, for the liquid moving in the conduit

It must be noted the spring constant K of the frame includes the stiffness of the bellows of the mass pump.

In the following, the equation of motion for this model is derived and the basic character of the response to the external disturbance is discussed.

1) Response to the External Lateral Force

When a dynamic horizontal force F is applied to the top level of the frame, consideration of the balance of moments at the supporting point of the lever arm in Fig. 4 and introduction of the relations in Eqs. (6) and (8) leads to the following equation of motion:

$$(M+\gamma^2 m)\ddot{X}+(C+\gamma^2 C)\dot{X}+KX=F \tag{9}$$

where the damping for the moving liquid represented by the coefficient c was assumed to be viscous.

Using the notation of

$$\bar{M}=M+\gamma^2 m, \quad \bar{C}=C+\gamma^2 C \tag{10}$$

Eq. (9) is reduced to

$$\bar{M}\ddot{X}+\bar{C}\dot{X}+KX=F \tag{11}$$

This is nothing but the equation of motion for a one mass system having an effective mass $\bar{M}$, an effective damping coefficient $\bar{C}$ and the original stiffness K. It is easily understood that $\gamma^2 m$ and $\gamma^2 c$ in Eq. (10) represent the mass and the damping effect, respectively, of the liquid moving in the conduit.

From Eq. (11), the undamped natural frequency of the system is obtained as

$$\omega_o=\sqrt{K/\bar{M}} \tag{12}$$

which means the original natural frequency of the frame, $\sqrt{K/M}$, is reduced by a ratio of $\sqrt{M/\bar{M}}$ by the effect of the mass pump.

Also, the effective critical damping coefficient of the system becomes

$$\bar{h}=\bar{C}/2\bar{M}\bar{\omega}_o \tag{13}$$

The ratio of the effective damping coefficient, $\bar{h}$, to the original one, h, is dependent on the ratio of the damping effect, $\gamma^2 C$, to the mass effect, $\gamma^2 m$. As described later, the exprimental results show apparent increase of the critical damping coefficient by the incorporation of the mass pump.

In consequence of Eqs. (12) and (13), the response to an external dynamic force can be obtained by treating the system as the one having the effective values of mass and critical damping coefficient, $\bar{M}$ and $\bar{h}$ respectively, subjected to the original external force F.

For example, the amplitude of displacement caused by harmonic excitation from a vibration generator of rotating unbalance mass, whose moment of mass is $m_0 r$, is given by

$$X_d = \frac{m_0 \gamma}{\bar{M}} \frac{(\omega/\bar{\omega}_0)^2}{\sqrt{\{1-(\omega/\bar{\omega}_0)^2\}^2 + 4\bar{h}^2 (\omega/\bar{\omega}_0)^2}} \quad (14)$$

in contrast with the case of the original frame without the mass pump, given by

$$X_d = \frac{m_0 \gamma}{M} \frac{(\omega/\omega_0)^2}{\sqrt{\{1-(\omega/\omega_0)^2\}^2 + 4h^2 (\omega/\omega_0)^2}} \quad (15)$$

Comparing Eqs. (14) and (15), it can be concluded that the incorporation of the mass pump gives rises to the following effects on the response of the system:

1) the original resonant frequency ω_0 decreases to $\bar{\omega}_0$, $\bar{\omega}_0/\omega_0$ being equal to $\sqrt{M/\bar{M}}$ (mass effect),
2) the original response amplitude decreases by the factor of $M/\bar{M}$ for the whole range of the forcing frequency (mass effect), and
3) the original amplification factor of the resonant response decreases according to the increase of the critical damping coefficient from h to $\bar{h}$ (damping effect).

One of the most important aspects of the mass and damping effects of the mass pump is that the effective mass and damping terms for the moving liquid are represented by $\gamma^2 m$ and $\gamma^2 c$, having the factor of γ^2, instead of γ, the factor by which the velocity of the frame is amplified for the liquid in the conduit.

As an example, let us evaluate the mass effect for the liquid in the following case:

effective sectional area of bellows:	A = 100 cm
sectional area of conduit:	a = 1.0 cm
length of conduit:	ℓ = 100 cm
angle of diagonal member axis:	θ = 45°

Then, we have

$$\beta = A/a = 100$$

$$\alpha = \cos\theta = \cos 45° = 1/\sqrt{2}$$

$$\gamma^2 = (\alpha\beta)^2 = 100^2/2 = 5{,}000$$

The real mass of the liquid in the conduit, assumed to be water, is given by

$$m = \rho a l = 1\,\text{gram}/\text{cm}^3 \times 1\,\text{cm}^2 \times 100\,\text{cm} = 100\,\text{gram}$$

The effective mass of the moving water is then

$$\gamma^2 m = 5{,}000 \times 100\ \text{gram} = 5 \times 10^5\ \text{gram} = 500\,\text{kg}$$

Thus, it is shown that the contribution of the 100 grams of contained water to the effective mass of the vibrating system is 0.5 ton!

Another point to be noted is the fact that the mass effect of the pump is of a passive nature, i.e. the effective mass is caused merely by the vibration of the frame, and does not cause any active body force effect to which the frame will be subjected on the occasion of the ground motion.

2) Response to Ground Motion

The horizontal displacement of the ground motion in the frame system is represented by the vertical displacement Y of the support of the model in Fig. 4, X being the relative displacement between the frame and the support.

Then, we have

$$\begin{aligned} &\text{absolute displacement of the frame:} && X+Y \\ &\text{absolute displacement of the liquid:} && x+Y=-\gamma X+Y \end{aligned} \qquad (16)$$

Using the expressions of Eqs. (16), the following equation of motion is obtained:

$$(M+\gamma^2 m)\ddot{X} + (C+\gamma^2 C)\dot{X} + KX = -(M-\gamma m)\ddot{Y} \qquad (17)$$

Introducing Eq. (10) and dividing by the mass term, Eq. (17) is reduced to

$$\ddot{X} + \frac{\bar{C}}{\bar{M}}\dot{X} + \frac{K}{\bar{M}}X = -\frac{M-\gamma m}{M+\gamma^2 m}\ddot{Y} > -\frac{M}{\bar{M}}\ddot{Y} \qquad (18)$$

in contrast to the equation for the case without the mass pump,

$$\ddot{X} + \frac{C}{M}\dot{X} + \frac{K}{M}X = -\ddot{Y} \qquad (19)$$

It must be noted that the absolute magnitude of the excitation force on the right side in Eq. (18) is reduced by a factor smaller than $M/\bar{M}$ compared to Eq. (19). Hence, it can be concluded that the three different effects, caused by the incorporation of the mass pump for the case of the excitation by force, hold again for the case of the ground motion.

3. Results of Experiments

In order to demonstrate the effects of the mass pump which were theoretically derived hitherto, a series of tests was carried out. Representative results obtained by the forced vibration tests of a system of steel portal frame will now be presented.

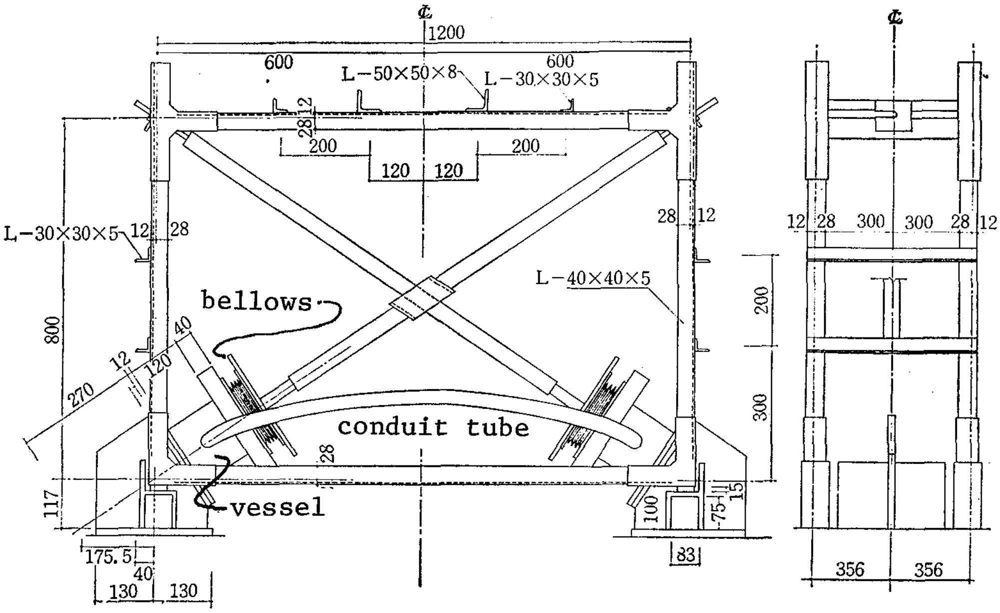

Fig.5 Test Frame (unit in mm)

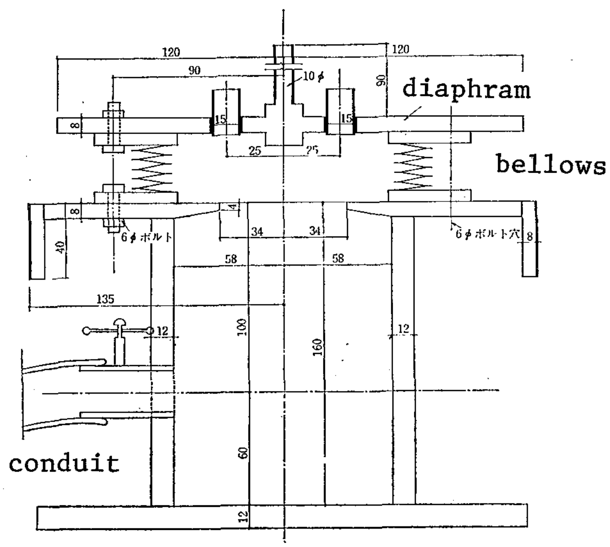

Fig.6 Section of Vessel in Mass Pump

A steel frame having the form and the dimensions shown in Fig. 5 was subjected to two different tests of forced vibration, i.e. application of the harmonic excitation of force by means of a rotating unbalance vibration generator set on the beam of the frame, and the application of the harmonic excitation of displacement by means of a shaking table.

The mass of the frame and the vibrator, interpreted as that of the one mass system, was 53 kg. The spring constant of the frame was 744 kg/cm. Fig. 6 shows the section of the vessels of the mass pump. 12 units of stainless steel bellows were used. The properties of these bellows were as follows: 105 mm and 135 mm iner and outer diameters, 25 mm neutral length, 113 cm^2 effective sectional area, and 3.6 kg/mm in spring constant. Fig. 7 is the photograph showing the test system being excited by the vibration generator.

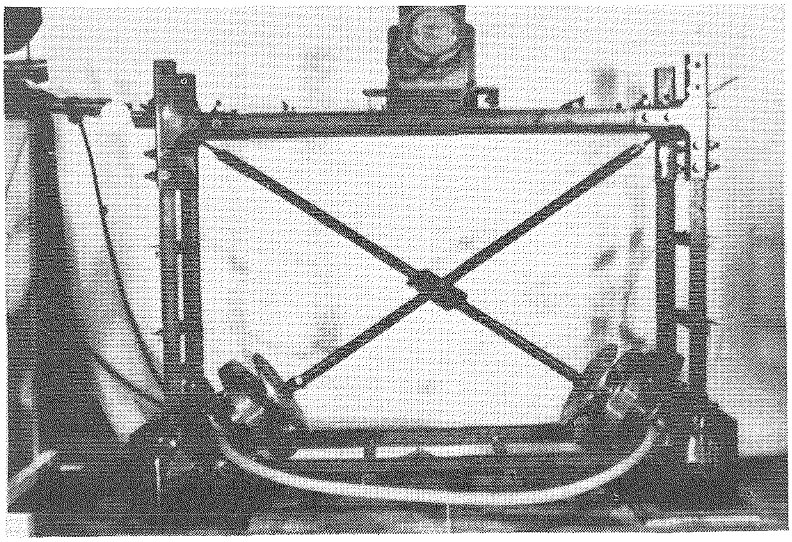

Fig. 7 Test Frame during Vibrator Test

At first, the test was done in a state where the connection between the frame and the diagonal members was made loose in order to see the vibration characteristics of the original frame. Then, the mass pump, filled with water or oil, was fastened to the frame. The response to the excitation was observed for several cases of different combination of diameter and length of the conduit tubes.

Fig. 8 shows the results of the forced vibration test by the vibration generator of 2.0 kg·cm in moment of mass, where the amplitude of the lateral displacement of the top of the frame was plotted with regard to the forcing frequency.

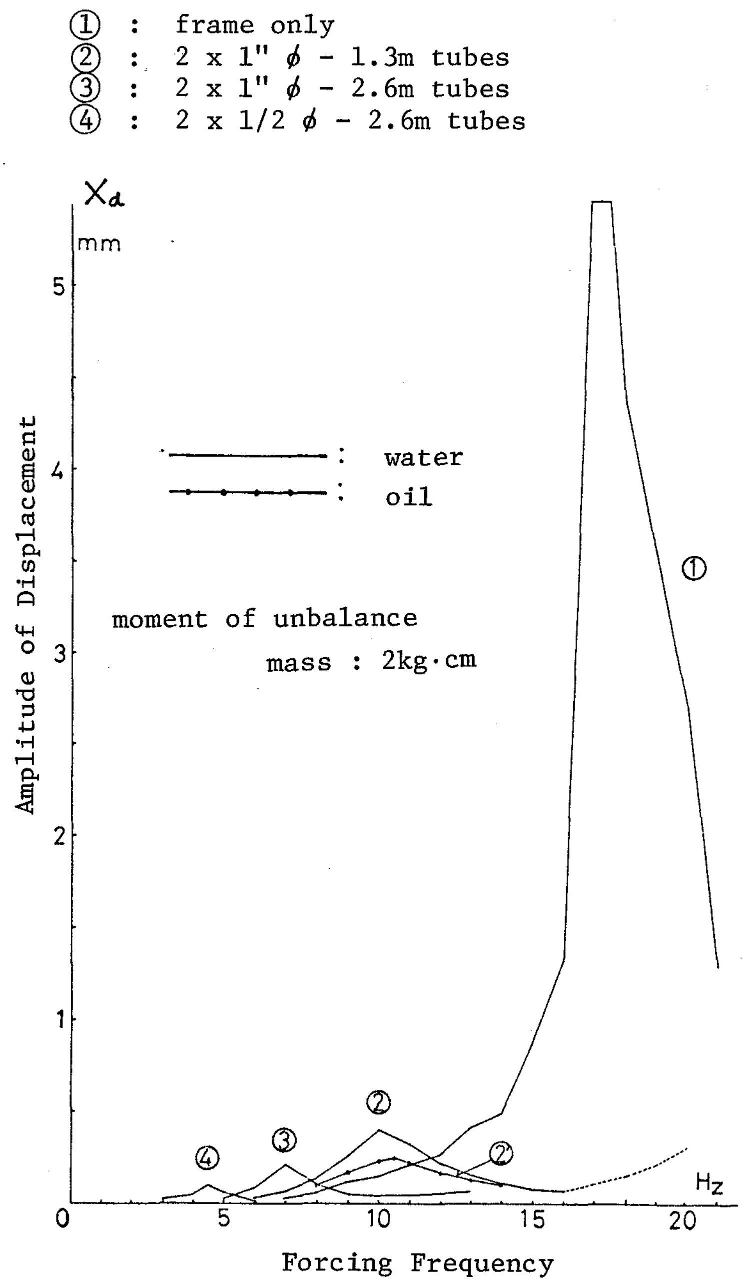

Fig.8 Result of Vibrator Test
—— Amplitude of Displacement

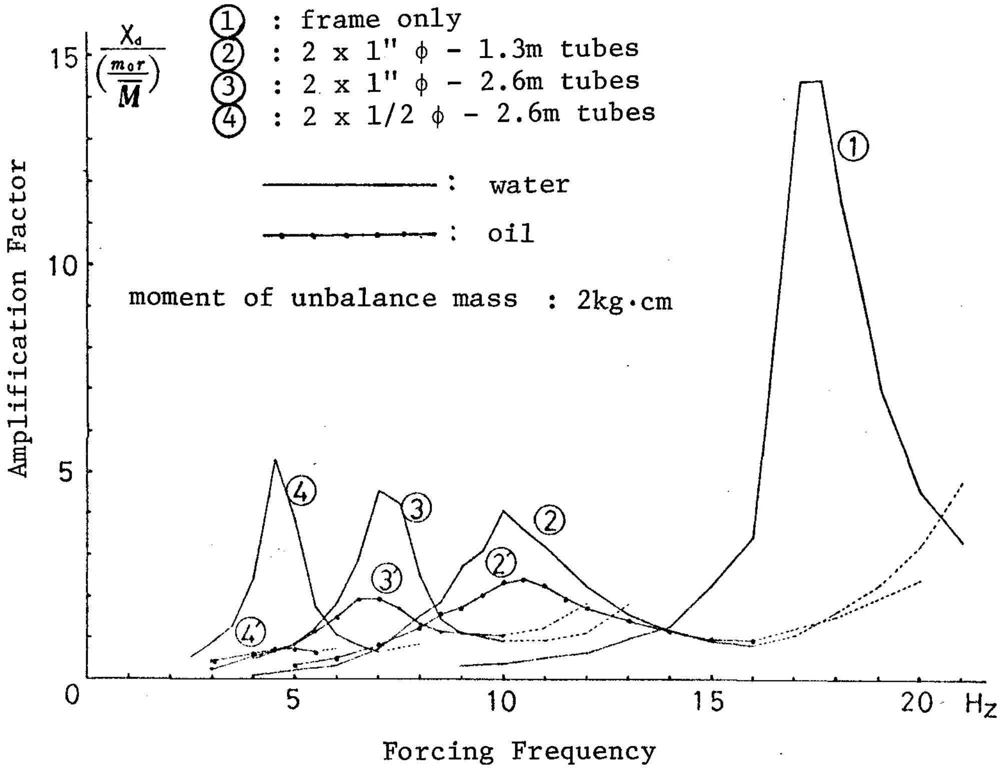

Fig.9 Result of Vibrator Test

—— Amplification Factor of Displacement

The curve ① shows the response of the original frame without mass pump, curves ②, ③ and ④, the responses of the frame with the mass pump filled with water having conduit tubes of various dimensions. The curve ②' represents the response of the system having the same conduit tube as the case ② but filled with oil instead of water.

From these results, it can be observed that the resonant frequency of approximately 17 Hz for the original frame was reduced, by the mass effect of the pump, to the range from 4.5 to 10 Hz and that there was apparent reduction of the amplitude as the result of the combined mass and damping effects of the pump. The reduction of the resonant amplitude should be considered drastic even when the decrement of the exciting force from the generator in accordance with the reduction of the resonant frequency is taken into account.

In Fig. 9, the same amplitude of displacement is represented in the amplification factor, the factor by which the static displacement due to the exciting force at the each resonant point is magnified. The degree of reduction of this amplification factor indicates the magnitude of the damping effect of the mass pump. While the critical damping coefficient of the original frame, corresponding to the peak value of 15 in the amplification factor, is assessed to be h = 0.025, the peak values of other curves yield higher damping, i.e. approximately h = 0.1 for the cases of the pump filled with water and values from h = 0.2 to h = 0.6 for the cases of the pump filled with oil.

In Figs. 8 and 9, the portion of the each curve represented by a dotted line suggests the existence of another peak of resonance corresponding to the higher mode of vibration of the system with mass pump. Based on this fact, it was supposed that the more realistic characteristics of the vibrating system might be represented by the model shown in Fig. 10. When the excitation frequency is so high as the dashpot for the liquid comes to a state of overdamping, the displacement of the lever arm of the model diminishes, the mass effect decreasing and leading to prevailing deformation of the upper spring. In this model, the upper spring with the constant of K_1 corresponds to the stiffness of the system when the movement of the liquid in the conduit is completely blocked by, for example, closing the valves installed at the ends of the conduit.

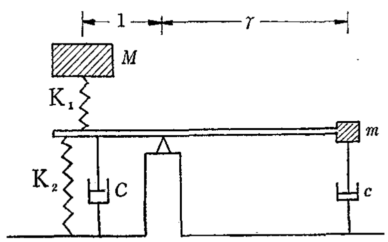

Fig.10 3-Parameter Model with Mass Effect

Fig. 11 shows the results of the shaking table test where the harmonic excitation of displacement of ±0.1 mm in amplitude was applied. In this figure the amplification factor of displacement, ratio of the amplitude of the frame to that of the shaking table, was plotted with regard to the forcing frequency. Here, the curve 1 represents the response of the original frame, curves 3 and 4, the response of the system with the mass pump filled with oil, while curve 2 corresponds to the case where the movement of the oil in the conduits was blocked by closing the valves. It is clear that the response of the system to the harmonic ground motion drastically decreased due to the effect of the mass pump incorporated in the frame. It must be noted that the amplification factor for the higher mode resonance as well as the one for the fundamental mode was depressed very low.

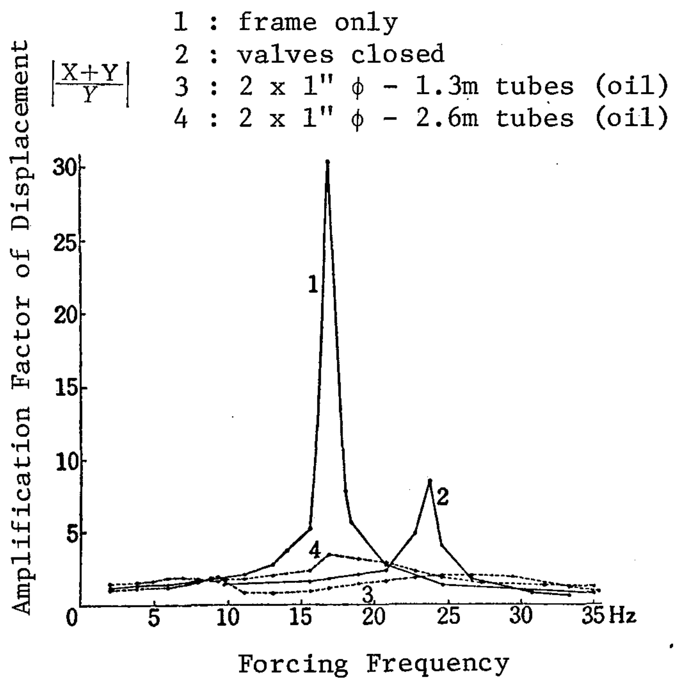

Fig.11 Result of Shaking Table Test

—— Amplification Factor of Displacement

In consequence the curve 3, for example, shows the reduced amplification factor not larger than 2.0 for the whole range of the forcing frequency tested, in contrast to the maximum factor of 30 for the original frame.

4. Conclusions

It has been demonstrated that the new system described in this paper has outstanding functions which can be utilized in different phases of vibration control, namely:

1) control of natural frequency of structures,
2) reduction of the amplitude of vibratory response of structures for any forcing to natural frequency ratio, ω / ω_0, and
3) reduction of the amplification factor for resonance.

It can be concluded that the appropriate combination of these effects of the "mass pump" will lead to minimization of the structural response to earthquakes or the mechanical vibration.

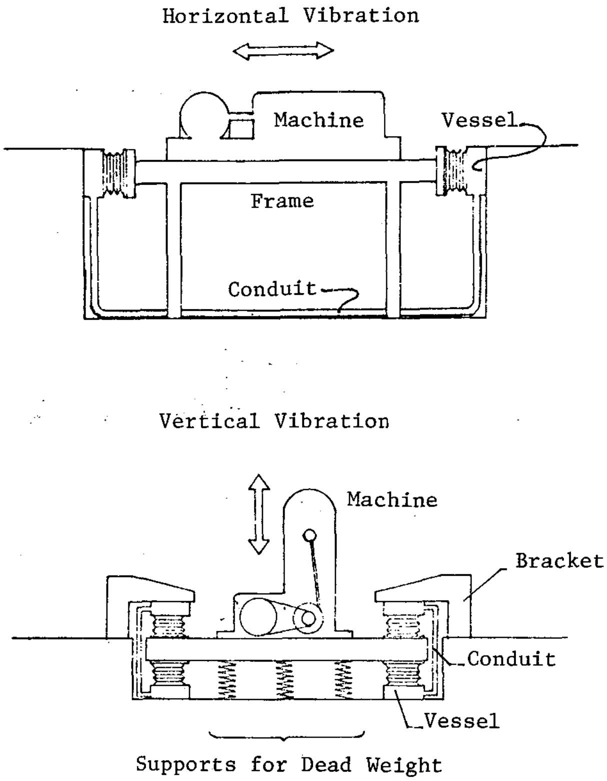

Fig. 12 Examples of Machine Basement

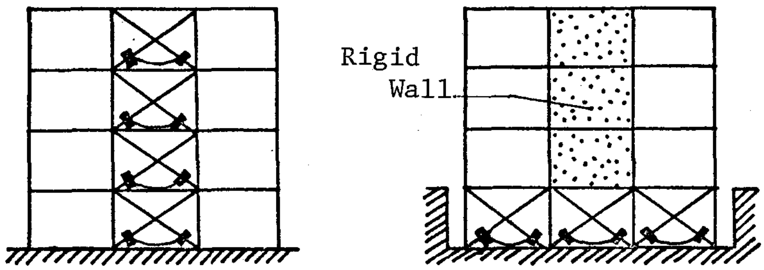

Fig. 13 Examples of Multi-Storied Building

Fig. 12 shows examples of the scheme of application of the device as isolators of mechanical vibration. Fig. 12 illustrates two different applications of the system to multi-storied building frames, to reduce the response to ground motion.

The conditions of economy and reliability, which were considered to be essential to this development, can be fulfilled because of the simplicity of the mechanism and the ease of maintenance of the mass pump system.

Future development requires the establishment of a method of optimum design of the mass pumps, especially in relation to the damping characteristics of the liquid moving in the mass pump units.

Acknowledgements

Appreciation is expressed to Miss K. Kanazawa and Mr. N. Matsui, university of Tokyo, for their valuable assistance in the experimental works and also to Prof. R.H. Gallagher, Cornell University, for his kindness of correcting the manuscript.

THE END

(December, 1973)