The Response Spectrum Analysis of Building and Building-Appendage Structure System to an Artificial Earthquake with Two Ground Predominant Periods

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1. Introduction

The response spectrum analysis of one-degree-of-freedom system and two-degrees-of-freedom system which simulate building and building- appendage system respectively is conducted for artificial earthquake with two ground predominant periods. The results are compared with those for actual earthquake motions. The artificial earthquake is assumed as stationary random vibration with Gaussian distribution which has band limited white power spectrum at base rock and ground layer characteristic can be described by two one-degree-of-freedom systems. For the description of the maximum of the random vibration corresponding to the earthquake and the response of the structure system for it the probability density function of the extreme by Rice[1] is made use of.

The same sort of analysis for the ground layer characteristic of one-degree-of-freedom system has been made.[2),3)] The table of the acceleration amplification factor was given for the convenience of the structural design. It showed that the response spectrum by the simulation covered well those by earthquake motions as an envelope. However, the precise comparison indicated the discrepancy of the shape of the spectrum, that is, the spectrum for the simultation had one peak in spite that those for the earthquake motions had generally several ones, and also indicated the difference of the spectrum value that the simulated one was easy to be larger than those by earthquake motions. Especially for the appendage system the acceleration amplification factor at the natural period which was equal to that of the building system and the ground predominant period was much larger than that by earthquake motions in case of the comparable system parameters. The study in the present paper is carried out to improve these difficulties observed in the original spectrum and simulated one.

Another reason for this study is that the additive existence of the second ground predominant period may explain the extraordinary large displacement response[4)] in long period range for earthquake motions caused by large magnitude scale as Niigata earthquake (June 16, 1964).

The investigation makes it obvious that the questions above mentioned are well solved by assuming the two ground predominant periods. The shape of the acceleration response spectrum for the artificial earthquake can be made correspond to that for the earthquake motions. The maximum of the acceleration amplification factor of the simulated spectrum of the building system is reduced and the number of the peak increases from one to two by assuming the two ground predominant periods. The amount of the reduction is conspicuous for the appendage system and the factors for the artificial earthquake and the earthquake motion show satisfactorily good coincidence. The displacement response spectrum suggests that the existence of another ground predominant period in long period range which seems to happen for earthquake motions of large magnitude causes large displacement response in longer period range than the second ground predominant period.

However, the analysis made for one ground predominant period does not loose the importance yet in that the acceleration response spectrum value at the peak gives the severest condition for the structural design.

2. Basic Equations for the System

Fig. 1 shows a model scheme of the system dealt with. The structure system is composed by two-degrees-of-freedom system which simulates building structure system B and the appended structure such as equipment, piping and general machine structure system M the mass of which is usually much smaller than that of the building system.[2),5),6)] If the mass ratio is made zero, the behaviour of the main system can describe that of the building system itself. It was shown by Kanai[7)] and Tajimi[8)] that the ground characteristic G might be represented by one-degree-of-freedom system in sense of engineering approach.

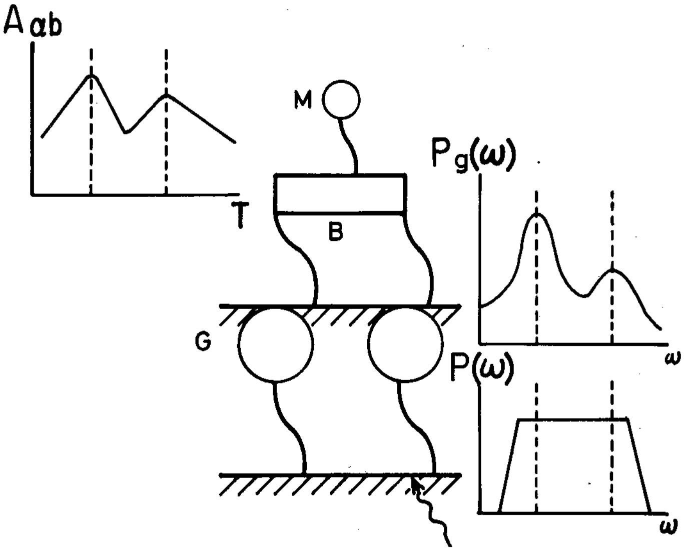


Fig. 1. Model scheme of the ground and the structure system

In this paper this is assumed two-degrees-of-freedom system and its transfer function for the acceleration is given as follows,

$$H_g(s)=\frac{2\omega_{g1}h_{g1}s+\omega_{g1}^2}{s^2+2\omega_{g1}h_{g1}s+\omega_{g1}^2}+\lambda\frac{2\omega_{g2}h_{g2}s+\omega_{g2}^2}{s^2+2\omega_{g2}h_{g2}s+\omega_{g2}^2} \tag{1}$$

where $\lambda=0$ reduces the system to one-degree-of freedom system by which the various analysis has been conducted. ω_g is the circular frequency obtained by $\omega_g=2\pi/T_g$ where T_g is the ground predominant period, h_g is the equivalent damping ratio and s is Laplace operater. The suffix 1 and 2 for ω_g and h_g indicate the two systems respectively.

The power spectrum of the ground motion is described by $P_g(\omega)$, The transfer function of the acceleration response of the building and the appendage structure system to the ground acceleration is described respectively as

$$H_b(s) = \frac{1}{\triangle}(2\omega_b h_b s + \omega_b^2)\ (2\omega_m h_m s + \omega_m^2) \tag{2}$$

$$H_m(s) = \frac{1}{\triangle}(2\omega_m h_m s + \omega_m^2)\ (2\omega_b h_b s + \omega_b^2) \tag{3}$$

where

$$\triangle = s^4 + \{2\omega_b h_b + (1+\gamma)2\omega_m h_m\}\ s^3 + \{\omega_b^2 + (1+\gamma)\ 2\omega_b h_b \cdot 2\omega_m h_m\}\ s^2 + (\omega_b^2 \cdot 2w_m h_m + \omega_m^2 \cdot 2\omega_b h_b)\ s + \omega_b^2 \cdot \omega_m^2 \tag{4}$$

The suffix b and m denote the building and the appended machine structure system, γ is the mass ratio, that is, the ratio of the mass of the appendage to that of the building system. When $\gamma = 0$, which means that even if the machine structure system is appended to the building system, the mass of the former system is small enough comparing to that of the latter and the motion of the appendage does not influence the behavior of the building at all, (2) can be given as

$$H_b(s) = \frac{2\omega_b h_b s + \omega_b^2}{s^2 + 2\omega_b h_b s + \omega_b^2} \tag{5}$$

This is equal to the transfer function of the building system only. As for the spectrum of the building system mentioned below, this relation is utilized. The motion of the base under the ground layer is represented by stationary random vibration. Its power spectrum characteristic is assumed to be band limited white. Shaping function of the characteristic is given by

$$H_f(s) = \frac{1}{(1+\psi_1 s)^2} \frac{s^2}{(1+\psi_2 s)^2} \tag{6}$$

Evaluating the maximum of the simulated earthquake motion and that of the response of the structure system for it, the probability density function of extreme p(y), which is derived by Rice, for stationary random vibration with Gaussian distribution is utilized. p(y) is represented as follows,

$$P(y)=\frac{1}{\sqrt{2\pi}}\cdot\frac{\sqrt{I_0I_4-I_2^2}}{\sqrt{I_0I_4}}\exp\left\{-\frac{I_0I_4}{2(I_0I_4-I_2^2)}y^2\right\}$$

$$+\frac{I_2}{2\sqrt{I_0I_4}}y\exp\left(-\frac{y^2}{2}\right)\left\{1+\mathrm{erf}\ \frac{I_2}{\sqrt{2(I_0I_4-I_2^2)}}y\right\} \qquad (7)$$

where

$$I_0=\frac{1}{2\pi}\int_0^\infty|H(s)|^2kd\omega \quad I_2=\frac{1}{2\pi}\cdot\frac{1}{4\pi^2}\int_0^\infty|sH(s)|^2kd\omega \qquad (8)$$

$$I_4=\frac{1}{2\pi}\cdot\frac{1}{16\pi^4}\int_0^\infty|s^2H(s)|^2kd\omega$$

$$y=I(t)/\sqrt{I_0} \qquad (9)$$

I (t) means the random vibration and k is related to the constant of white spectrum. p(y) for the ground acceleration, its response of the building and the appendage is obtained by putting

$$H(s)=H_g(s)\cdot H_f(s) \qquad (10)$$

$$H(s)=H_b(s)\cdot H_g(s)\cdot H_f(s) \qquad (11)$$

$$H(s)=H_m(s)\cdot H_g(s)\cdot H_f(s) \qquad (12)$$

into (7) respectively, the integral is carried out by making use of the formula for residue integral given by Newton Jr. and others.[9] If the maximum corresponds to the point where p(y) is small enough, p(y)= 0.01 can be chosen as a representative point. Then the amplification factor $A_{\alpha b}$ and $A_{\alpha m}$ for the building and the appendage structure system are respectively provided by

$$A_{\alpha b}=\frac{\sqrt{I_{ob}}\,y_b}{\sqrt{I_{og}}\,y_g} \qquad (13)$$

$$A_{\alpha m}=\frac{\sqrt{I_{om}}\,y_m}{\sqrt{I_{og}}\,y_g} \qquad (14)$$

where y is the point where p(y) =0.01 is satisfied and the suffix g, b and m indicate the ground, the building and the appendage system respectively.

The procedure is quite same as that investigated by the previous study. In this paper the characteristic of the spectrum under the condition of (1) is studied.

The transfer function of the relative velocity and displacement response of the building system are given as

$$H_{bv}(s) = -\frac{s}{\triangle}\left\{ s^2 + (1+\gamma)\,2\omega_m h_m s + (1+\gamma)\,\omega_m^2 \right\} \tag{15}$$

$$H_{bd}(s) = -\frac{1}{\triangle}\left\{ s^2 + (1+\gamma)\,2\omega_m h_m s + (1+\gamma)\,\omega_m^2 \right\} \tag{16}$$

The similar transfer functions of the relative velocity and displacement of the appendage system to the building system are

$$H_{mv}(s) = -\frac{s}{\triangle}(2\omega_b h_b s + \omega_b^2) \tag{17}$$

$$H_{md}(s) = -\frac{1}{\triangle}(2\omega_b h_b s + \omega_b^2) \tag{18}$$

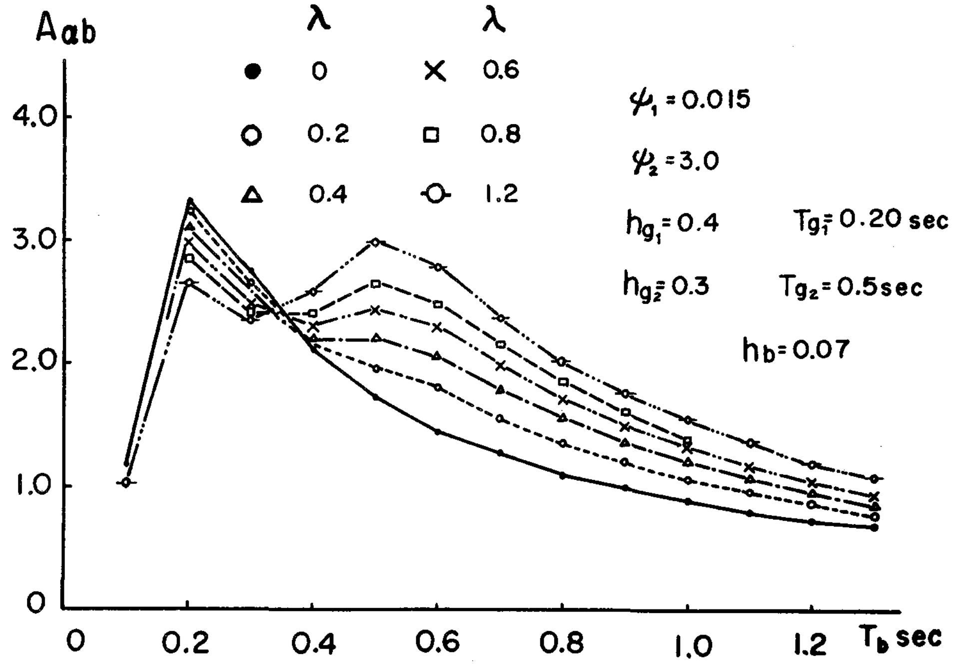
The input to the these system is the ground acceleration. These can be substituted into $H_m(s)$ in (11) and $H_b(s)$ in (12) in place of (2) and (3) to obtain the respective response.

3. The Analysis for the Building Structure System

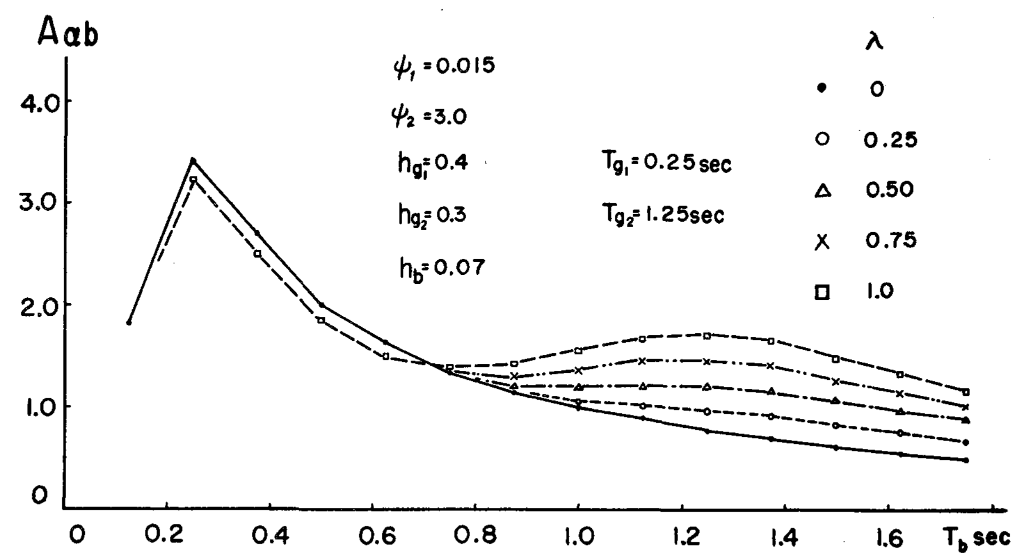
Fig. 2 shows examples of the response spectra of the acceleration amplification factor which are obtained by (13). T_{g1} = 0.2s and T_{g2} = 0.5s for Fig. 2 (a) are adopted for the sake of comparison with the spectrum for earthquake motion which will be mentioned below. h_{g1} = 0.4 and h_{g2} = 0.3 are used as the equivalent damping ratio. The former is the value recommended as a standard for the case of single predominant period and the latter is an example for the computation. ψ_1 = 0.008s and ψ_2 = 30.0 s are equal to break point frequency f_1 = 10.6 Hz and f_2 = 0.531 Hz respectively, h_b = 0.07 is used as damping ratio of the structure model.

λ = 0 means that only a ground predominant period exists and the response spectrum is equal to that of one-degree-of-freedom system, that is, the spectrum has a single peak at the point where the natural period of the structure coincides with the ground predominant period. The shape of the spectrum can be normalized if the ground predominant period varies for the same damping ratio as long as the predominant period is single. The peak value of the factor at T_b = 0.2s decreases as λ becomes large. This implies that the new addition of the long ground predominant period reduces the acceleration amplification factor at the original peak. As the result the spectrum has two peaks at the natural period of the structure corresponding to the ground predominant periods for certain value of λ. At a λ the magnitude of the peaks becomes equal. The value for the longer period continues to be larger as λ increases.

Looking at the magnitude of the peak of the spectrum, it is not larger than that for the single ground predominant period and the magnitude of either peak is not smaller than that both peaks give equal height of the amplification factor which should be taken into account in aseismic design of the structure system in case that the natural period and the



(a)



(b)

Fig. 2. The acceleration amplification factor of the building system (a) & (b)

ground predominant period are not given definitely.

Even if T_{g2} becomes larger, the tendencies as above mentioned do not change. Fig. 2 (b) shows another example of the combination of two ground predominant periods. In this case the longer predominant period exists at five times as much as the short one, however, the sensitivity that the amplification factor decreases and increases as λ increases depends on the ratio of the ground predominant periods. The amplification factor at the peaks of the spectrum for the building system is plotted for the abscissa of λ taking the ratio T_{g2}/T_{g1} as the parameter in Fig. 3. This shows that the cross point of both curves, that is, the amplification factor at

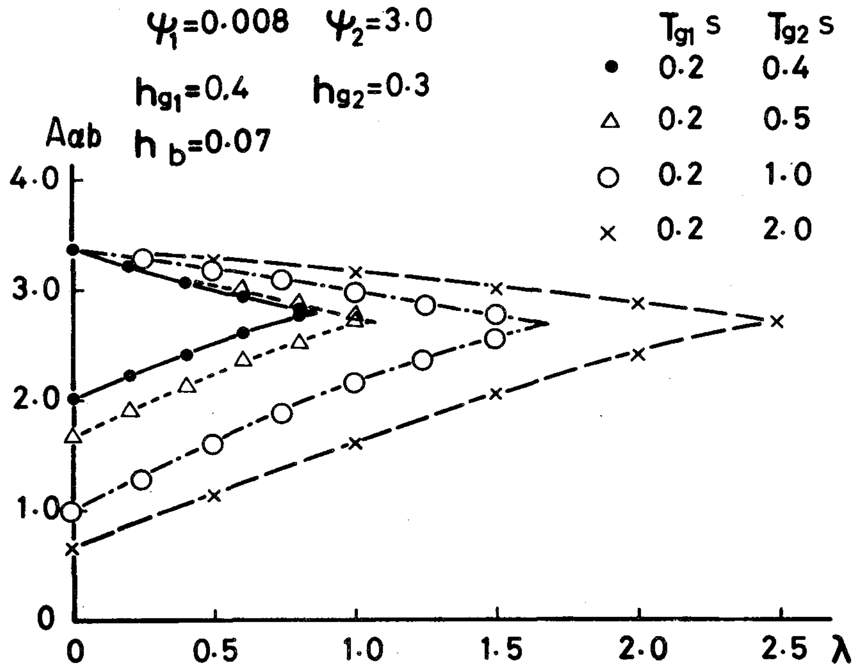


Fig. 3. The relation between and the amplification factor at the peak

which the both peaks are made equal does not almost vary even if the ratio T_{g2}/T_{g1} changes. However, λ at which the both peaks are equal becomes large for large value of the ratio T_{g2}/T_{g1}.

Fig. 4 (a) and (b) shows the displacement response spectra for the artificial earthquake, which are depicted by taking the factor of the maximum of the displacement response to that of the ground acceleration A_{db}. The parameters used for these correspond to those in Fig. 2 (a) and (b) respectively. These figures explain that the existence of the longer predominant period simply increases the displacement response in longer period range than the predominant period. The amount of the increase at a certain long natural period of the structure is larger for the case that the second ground predominant period appears at a longer period. This phenomenon is really found about the response spectra for the strong earthquake motions as Niigata earthquake observed in Tokyo.

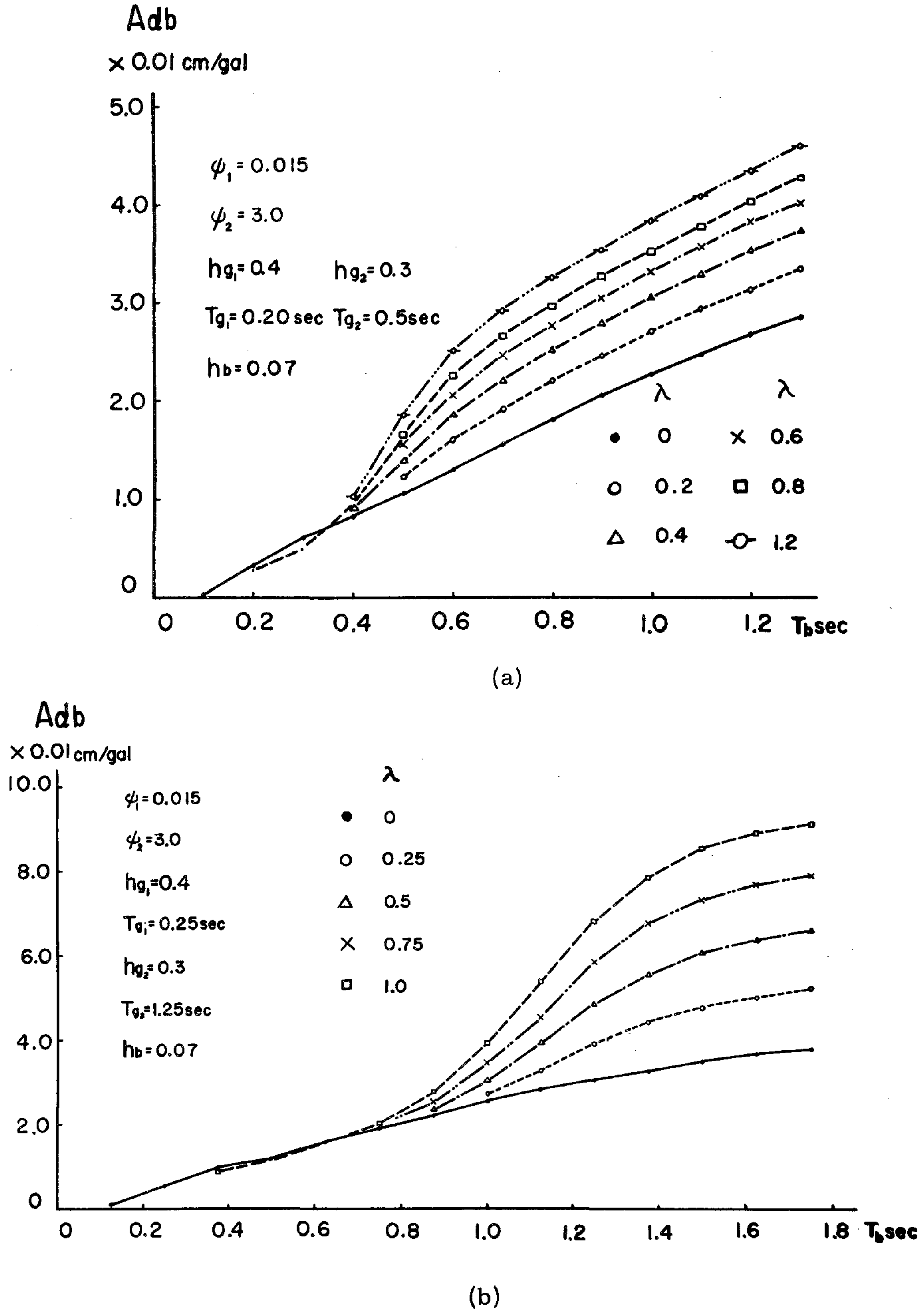


Fig. 4. The displacement response spectrum of the building system (a) & (b)

The analytical spectra and those for earthquake motions such as El Centro (NS, May 18, 1940) and Taft (NS, July 21, 1952) are compared in Fig. 5. Interpolated Curve of λ =0.9 are taken for the analysis so that the analytical spectra

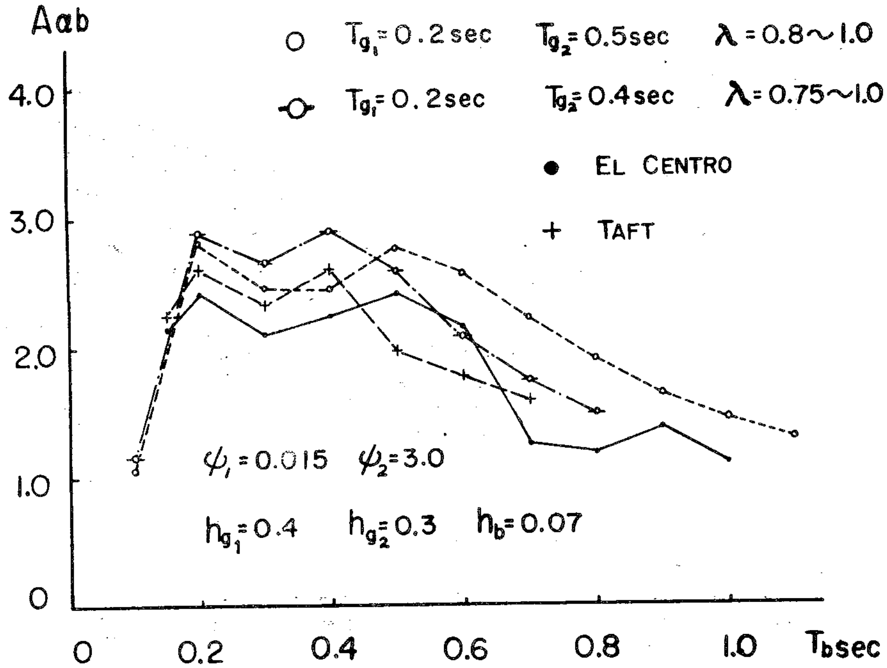


Fig. 5. Comparison of the acceleration amplification factor spectrum for the building system.

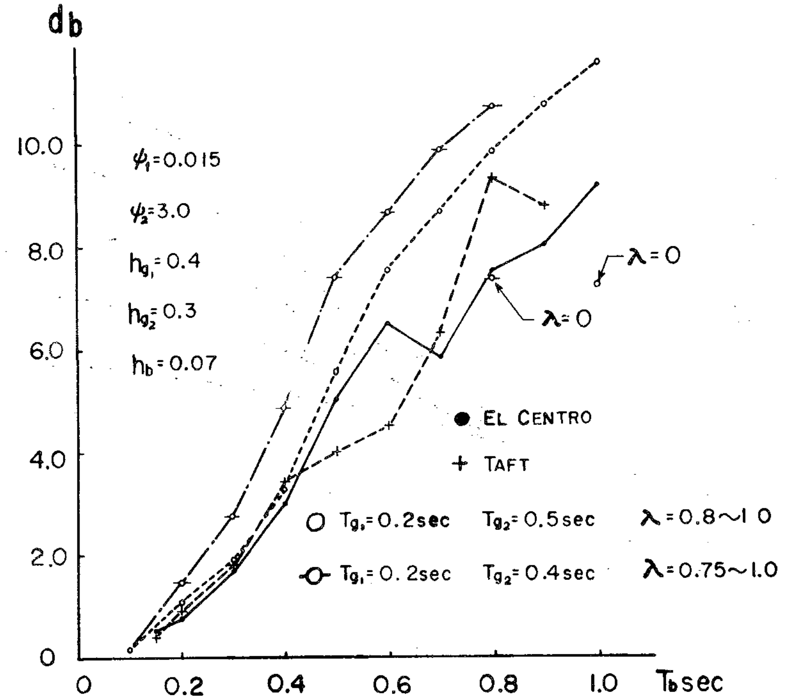


Fig. 6. Comparison of the displacement response spectrum for the building system

may agree with those by the earthquake motions. Although there is still discrepancy between both spectra, the shape of the spectrum shows good agreement. The analytical response spectrum obtained for the single ground predominant period showed a quite good representation for the spectra by earthquake motions in sense of an envelope, however, it never gave better agreement in the value of the amplification factor and the shape of the spectra as is seen in Fig. 4.

Fig. 6 shows same sort of comparison as for the displacement response spectrum. This also tells us that the simulation for the spectrum with two ground predominant periods provides better agreement in the magnitude and the shape than those with single predominant period. The agreement in the short period range is especially conspicuous.

Being based on the damage by San Fernando earthquake (Feb. 9, 1971) a standard design spectrum is recommended on the tripartite diagram.[10] The spectra based on the aforementioned analysis are shown and compared with the recommended spectrum in Fig. 7. This again shows that it seems adequate to assume the two ground predominant periods for the artificial earthquake so as to simulate the standard design spectrum.

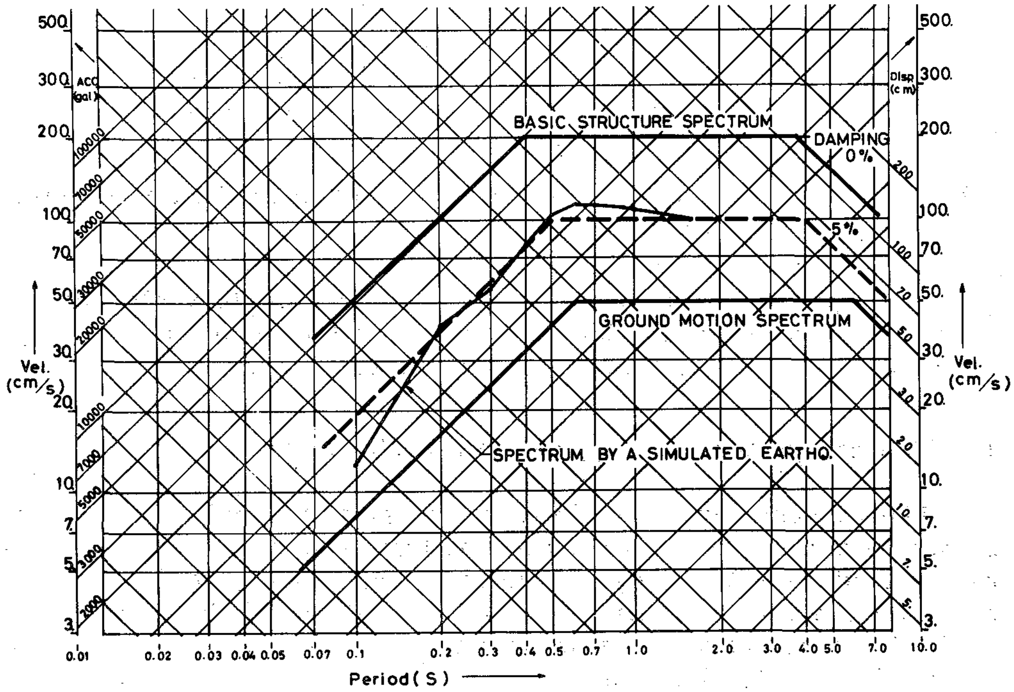


Fig. 7. Comparison of the spectrum with a proposed design spectrum

The power spectrum of El Centro earthquake and that of the artificial earthquake are compared in Fig. 8. λ is set as 1.0 for the analytical model. Frequency where the spectrum reaches the maximum and the maximum value itself are made equal for both. The actual earthquake motion has more dominant power than the analytical model around low predominant frequency. It should be said that the difference is quite large for

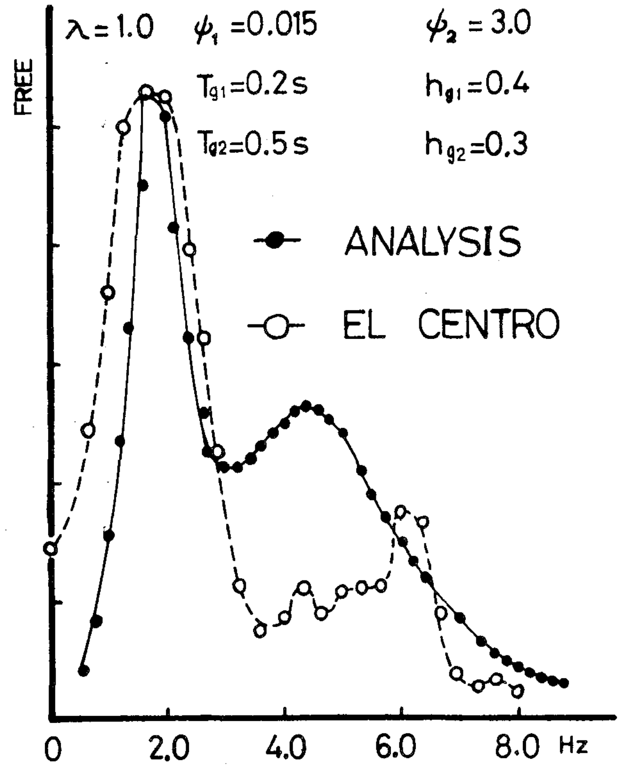


Fig. 8. Comparison of the power spectrum for El Centro with that for the artificial earthquake

the higher predominant frequency. It would be worth while noticing that such correspondence as the magnitude and the shape of the response spectrum in Fig. 2 and Fig. 4 can be obtained in spite of this discrepancy.

4. The Analysis for the Appendage Structure System

The response analysis of the appendage in the building-appendage structure system for the artificial earthquake with two ground predominant periods is extensively conducted. Fig. 9 shows the response spectra of the acceleration amplification factor for the artificial earthquake and El Centro earthquake. The parameters of the building system are all same, that is, T_b = 0.2s, h_b = 0.07, h_m = 0.02 and λ = 0. The natural period of the appendage T_m is varied. The parameters of the ground model system for the simulation, ψ_1 = 0.008, ψ_2 = = 3.0, T_{g1} = 0.2s, T_{g2} = 0.5s, h_{g1} = 0.4 and h_{g2} = 0.3. λ = 0 and 1.0 are taken for these. The former is the case that the ground predominant period is single. The magnitude of the spectrum at T_m = 0.2s for the artificial earthquake is much larger than that for the earthquake motion. On the other hand it is demonstrated that very good coincidence is obtained by the simulation as λ = 1.0.

The growth of the second ground predominant period T_{g2} = 0.5s pushes the spectrum at T_m = 0.5s up and the magnitude of the spectrum

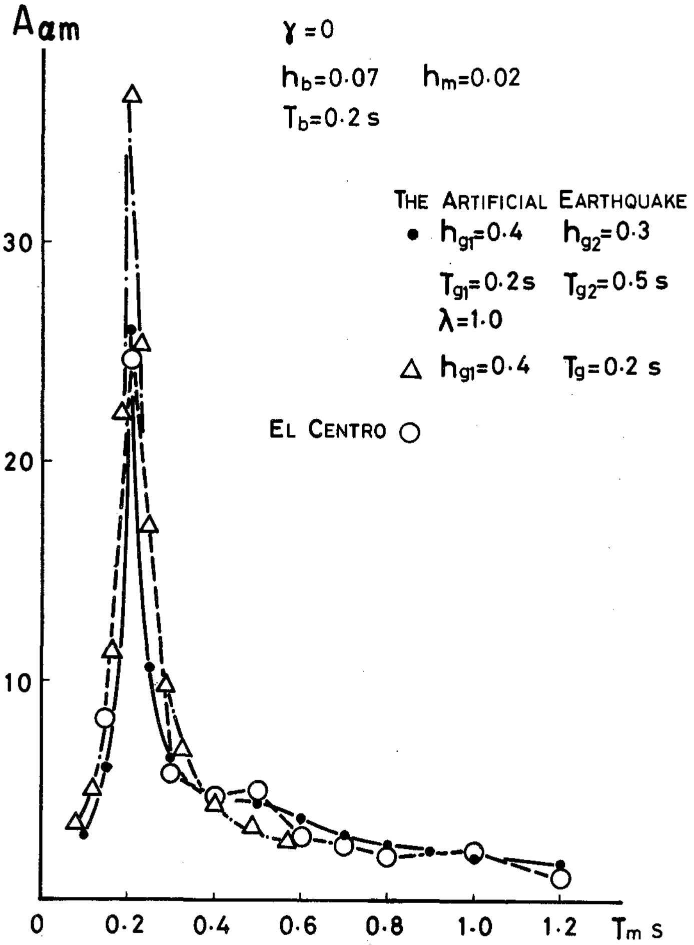


Fig. 9. Comparison of the acceleration amplification factor spectrum for the appendage system

for $\lambda = 1.0$ is larger than that for $\lambda = 0$ at $T_m = 0.5$s. So the spectra for $\lambda = 0$ and 1.0 cross each other at about $T_m = 0.4$s The spectrum for the artificial earthquake of $\lambda = 1.0$ again agrees well with that for El Centro. Not only in the magnitude of the spectrum at $T_m = 0.2$s, but in the whole shape of the spectrum especially around $T_m = 0.5$s the simulated spectrum shows surprisingly good coincidence with that for the earthquake motion. The analysis clarifies that the simulation approach by two ground predominant periods is effective for the investigation of the response of the appendage system. It is worthy to notice that the suppression of the acceleration amplification factor at $T_m = 0.2$s is caused by introducing the component of the second ground predominant period. This trend was found in case of building system, although the amount of suppression was not so large.

Fig. 10 compares the both spectra for the artificial earthquake and for El Centro as for $T_b = 0.5$s. $\lambda = 1.0$ is taken for the former. Looking at the spectrum of the peak, the result for El Centro is smaller than that of the simulation in spite that the comparison as for the spectrum of $T_b = 0.2s$ showed good agreement in its magnitude and shape. This is probably caused by the fact that the duration of component of $T_b = 0.5s$ is not comparably long enough. Then it would be necessary to take the nonstationarity into account to find better fit. However, the whole shape of the spectrum agrees well with each other The existence of $T_{g1} = 0.2s$ does not affect the spectrum such as found at $T_m = 0.5s$ in the spectrum of $T_b = 0.2s$, where the effect of $T_{g1} = 0.5s$ caused slight raise of the spectrum.

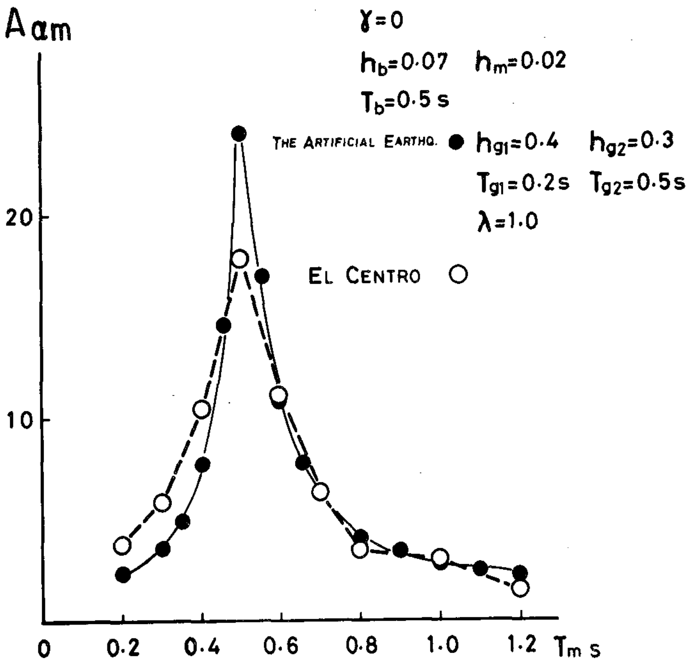


Fig. 10. **Comparison** of the acceleration amplification factor spectrum **for the** appendage system

Fig. 11 **depicts** $T_b - T_m$ response spectrum which is obtained by combining **the** spectra at $T_b = T_m$ keeping other parameters constant. Two sorts of h_m are chosen for the two ground predominant periods. These are compared with the spectra for El Centro. In addition to these the spectrum for the simulation with one ground predominant period is given. The last one cannot be compared with the spectrum for El Centro from the view point of the marked difference of the maximum value at $T_b = T_m = 0.2s$. The coincidence found at $T_b = T_m = 0.5s$ and 0.6s occurs by chance. Whole shape of the spectrum for El Centro suggests that it should be compared with the theoretical one with two ground predominant periods in spite that there exists some differences. These differences seen at = =0.15s, 0.3s, 0.5s and so on would be caused by that the theoretical earthquake cannot completely simulate the power spectrum and the nonstationarity of the actual earthquake motion.

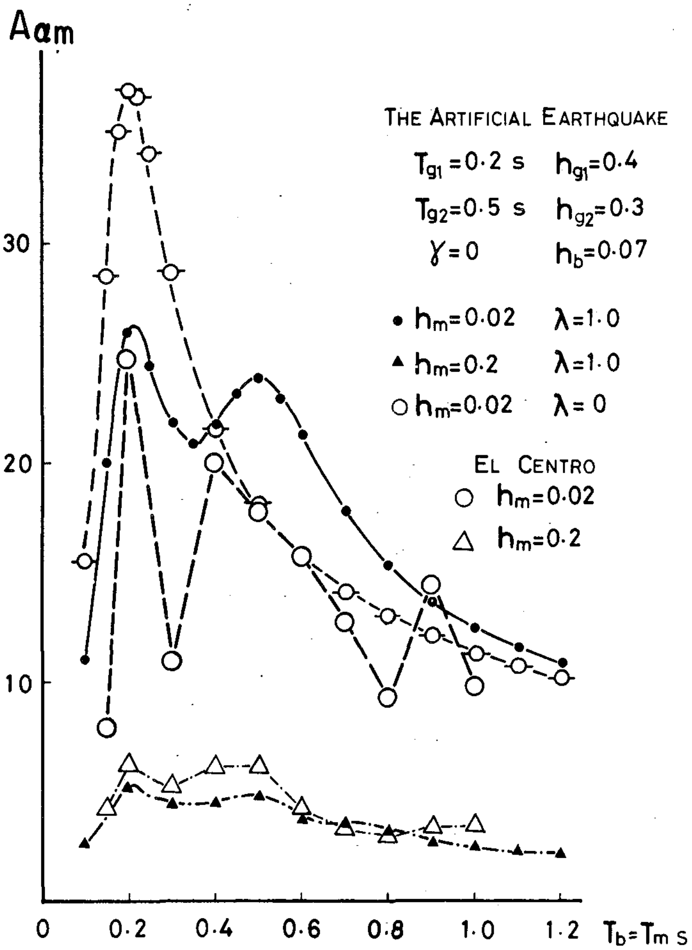


Fig. 11. Comparison of T_b—T_m acceleration amplification factor spectrum

Then the spectrum for El Centro shows deep trough at $T_b = T_m = 0.3$s, however such small amplification factor cannot be used for the practical design. It is to be considered that the theoretical result gives a safer aspect of the factor. As for the results of $T_b = T_m = 0.5$s $\sim$ 0.8s same comments can be made. The spectrum for El Centro is only a sample from those for the earthquake motions furnished with same sort characteristics. Taking this into account, the difference seen for these periods is not so large.

The spectrum for another case $h_m = 0.2$, better similarity of the shape can be found between the spectra for the earthquake motion and the artificial earthquake. It should be noted that the spectrum for the earthquake motion covers the theoretical one. This tendency is contrary to the case for small h_m. This would be caused by the difference between the assumption of the stationarity for the theoretical treatment and the nonstationarity characteristic for the actual earthquake motion. It should also be remembered that the ground model is persistently the simulation from the engineering view point.

Fig. 12 depicts the relation between γ and the normalized amplification factor at $T_b=T_m=0.2s$ taking h_m as parameter. The characteristic for $\lambda=0$ and 1.0 is described by a same curve and cannot be distinguished for the respective h_m. The amount that the factor decreases as γ increases is larger for larger h_m. Fig. 13 gives the relation between h_m and the acceleration amplification factor taking γ as the

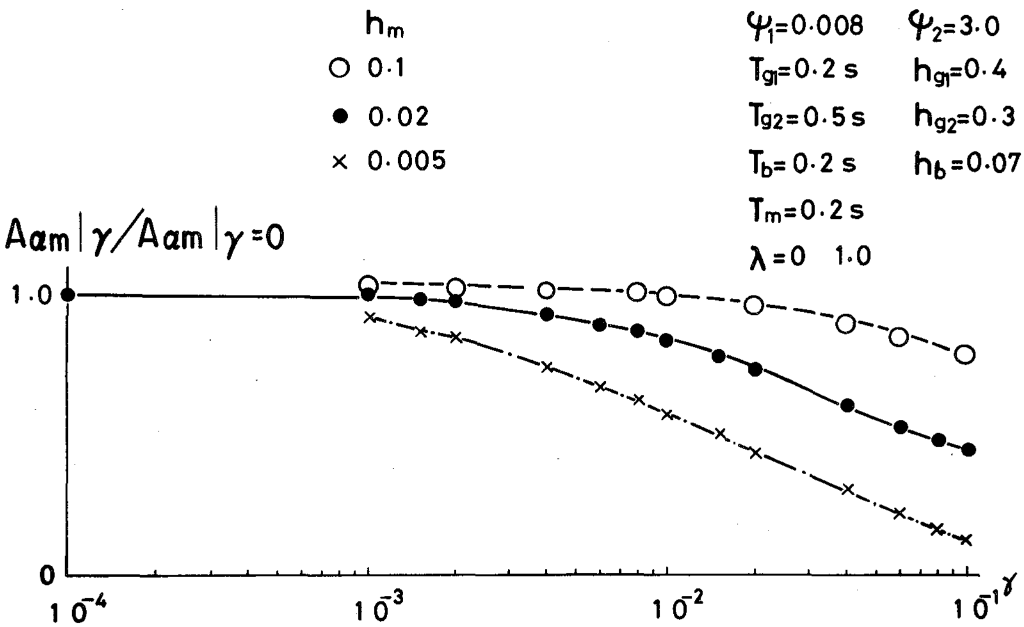


Fig. 12. The relation between the mass ratio and the normalized amplification factor at a peak

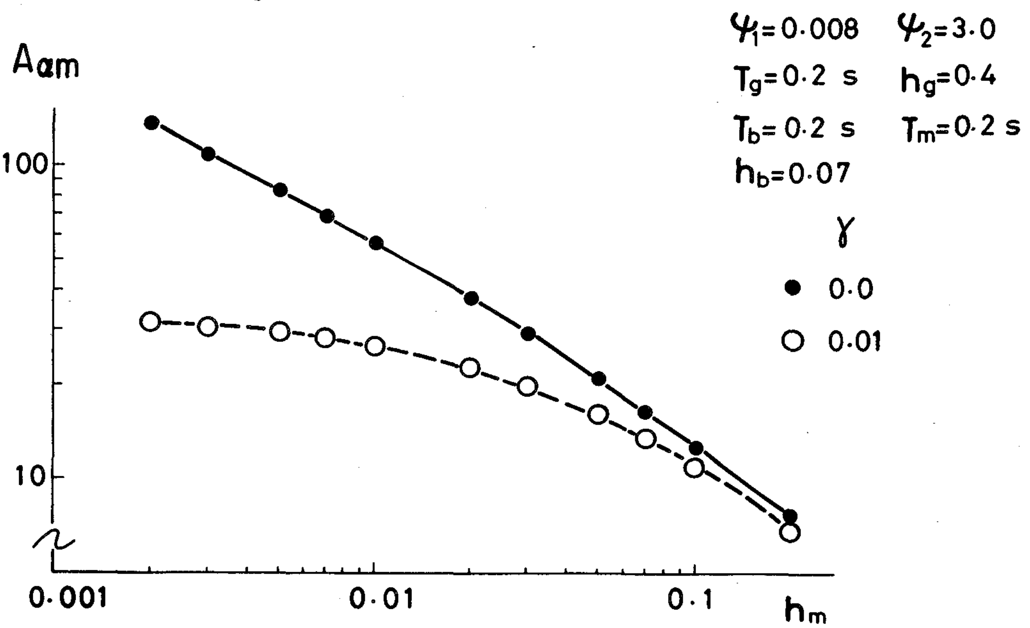


Fig. 13. The relation between the damping ratio of the appendage system and the acceleration amplification factor

parameter. The line for $\gamma = 0.0$ decreases almost linearly as γ increases. The curve for $\gamma = 0.0$ is an asymptote for the curve $\gamma = 0.01$ in large h_m .

Although the results shown in Fig. 12 and Fig. 13 are depicted from the spectrum for the artificial earthquake, these are well comparable with the results for earthquake motions as was seen in Fig. 9 and Fig. 11. The response spectrum analysis by the artificial earthquake makes it possible to describe the effect of each parameter to the spectrum definitely. This would be difficult from the response spectrum analysis for the actual earthquake motions because the statistical characteristic of the earthquake motion is furnished with too much ambiguity.

5. Conclusions and Acknowledgement

The response spectrum of the building structure which is represented by one-degree-of-freedom system and the appendage structure which is attached to the building as another one-degree-of-freedom system with small mass ratio is investigated for the artificial earthquake with two ground predominant periods. The artificial earthquake is simulated by stationary random vibration with Gaussian distribution and in the estimation of the maximum of the random vibration the probability density of the extreme is made use of. It is demonstrated that the response spectrum for the earthquake motions can be well simulated by that for the artificial earthquake. Several marked results are derived as follows.

1) Assuming two ground predominant periods for the artificial earthquake, the acceleration response spectrum for it can well simulate that for the earthquake motion in its shape and the magnitude. This is applicable for both the building and the appendage systems.

2) As for the response spectrum of the building system the addition of the second longer ground predominant period makes the acceleration amplification factor corresponding to the original predominant period decrease, and makes the displacement response in the period longer than the second ground predominant period increase.

3) As for the response spectrum of the acceleration amplification factor for the building system the magnitude of either peak for the two ground predominant periods is smaller than that for the one ground predominant period and is larger than that at which the peak height of the spectrum is same. In practical aseismic design it is required that the acceleration amplification factor should be taken larger than the latter, and may be assumed smaller than the former.

4) As for the response spectrum of the building it is shown that the spectrum with the effect of the two ground predominant periods agrees with the recommended spectrum on tripartite diagram for certain range of periods, which is based on San Fernando earthquake and aims at contribution to practical design.

5) The acceleration amplification factor of the appendage system for the earthquake motion and the artificial earthquake with the two ground predominant periods shows good agreement at the peak as for the change of the mass ratio and the daming ratio. The role of such system parameters to the spectrum is investigated, which was difficult to find out only from the spectrum for the earthquake motion because its ambiguous characteristics.

These studies make it possible for us to apply the simulated spectrum to the practical design in stead of using the spectrum for some particular earthquake motions, if the system parameters can be predicted. However, further investigation seems to be necessary for the accurate prediction of the system parameters as T_{g1}, T_{g2}, λ and so on for a specific ground.

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