PROPOSAL FOR DESIGN FORMULAE OF H-BEAM TO BOX-COLUMN CONNECTIONS

by Hisashi TANAKA and
Tsuneo SHIGENOBU

1. Introduction

When severe lateral forces such as destructive earthquake loading act on steel frames, strong shearing stresses occur in webs of beam-column connections and the webs yield in shear more prematurely than the other parts of frames. Therefore, the strength of frames under lateral forces depends upon the thickness of connection webs. The authors have already investigated the yield strength of H-beam to box-column connections, theoretically and experimentally (1), and they propose, in this paper, more simplified formulae to obtain the required thickness of connection-webs for facilities of design practice.

2. Design Method

The connections are, in general, subjected to the forces shown in Fig. 1 to Fig. 4. In the design procedure, however, these forces are replaced by the equivalent ones which are the groups of symmetric or anti-symmetric forces.

2.1 Cruciform and ├ type of connections

The cruciform connection shown in Fig. 1 may be designed by the following steps;

(a) Calculate the effective thickness t of column plates:

$$t = \frac{1}{2}(t_1 + t_2) \qquad (2.1)$$

where t_1 and t_2 are the plate thickness of upper and lower columns, respectively.

(b) Calculate the effective moment M and axial force N shown in Fig. 5 :

$$M = \frac{1}{2}(M_2 + M_4) - \frac{1}{4}(V_1 + V_3) H$$

$$= \frac{1}{2}(M_1 + M_3) - \frac{1}{4}(V_2 + V_4) D \qquad (2.2)$$

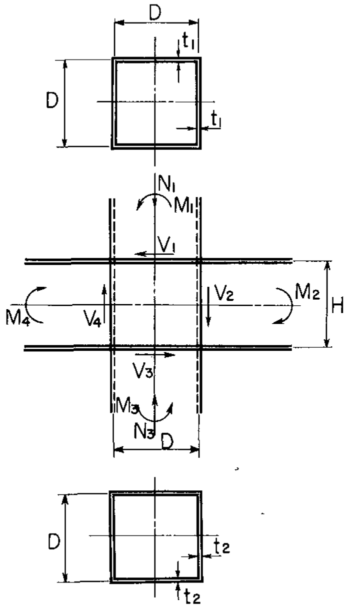

Fig. 1 Cruciform Connection

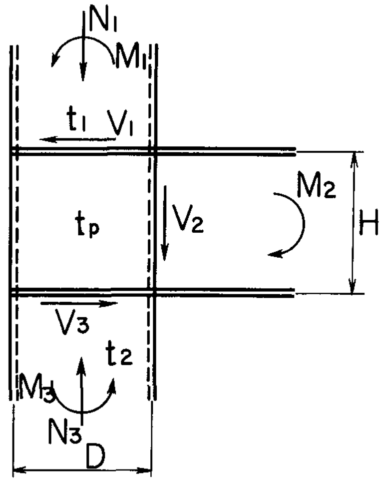

Fig. 2 ├ -type Connection

$$N = \frac{1}{2}(N_1 + N_3) \tag{2.3}$$

where M_i, V_i (i = 1, 2, 3, 4) and N_i (i = 1, 3) are the applied moments, shearing forces and axial forces, as shown in Fig. 1.

(c) Calculate the required thickness t_p of connection-web using the formulae shown in Table 1, where σ_o denotes the yield strength of material.

For the ├ type of connection shown in Fig. 2, let M_4 and V_4 be zero in the above procedure.

Table 1 Design Formulae

Conditions		Required thickness of connection-web : t_p
$M \leqq D^2 t \sigma_0$	$\frac{N}{2} + \frac{M}{D} \leqq D t \sigma_0$	$t_p \geqq \frac{\sqrt{3}\,M}{H D \sigma_0}$
	$\frac{N}{2} + \frac{M}{D} \geqq D t \sigma_0$	$t_p \geqq \sqrt{3\left(\frac{M}{H D \sigma_0}\right)^2 + \left(\frac{N}{2 D \sigma_0} + \frac{M}{D^2 \sigma_0} - t\right)^2}$
$M \geqq D^2 t \sigma_0$		$t_p \geqq \sqrt{3\left(\frac{Dt}{H}\right)^2 (2-\xi)^2 + \left(\frac{N}{2 \xi D \sigma_0}\right)^2}$ $\xi = \sqrt{3 - 2M/D^2 t \sigma_0}$

2.2 T and L type of connection

The T type connection shown in Fig. 3 may be treated as follows:

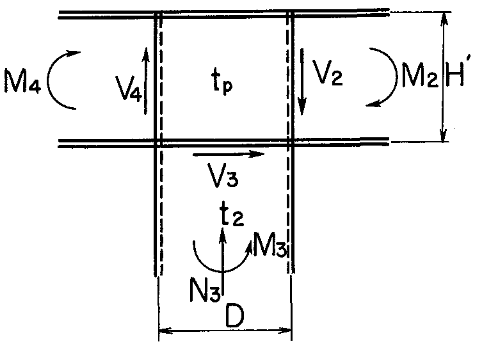

Fig. 3 T-type Connection

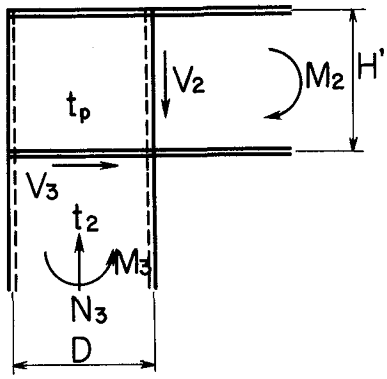

Fig. 4 L-type Connection

(a) Take the thickness of column plates and the effective depth of beam as

$$t = t_2, \quad H = 2H' \tag{2.4}$$

(b) Calculate the effective moment M and axial force N :

$$M = M_2 + M_4 - V_3 H'$$

$$= M_3 - \frac{1}{2}(V_2 + V_4)D \tag{2.5}$$

$$N = N_3 \tag{2.6}$$

where M_i, V_i (i = 2, 3, 4) and N_3 are the applied loads as shown in Fig. 3.

(c) Calculate the required thickness t_p using the formulae shown in Table 1.

For the L type of connection shown in Fig. 4, let M_4 and V_4 be zero in the procedure for T type of connection.

3. Derivation of Design Formulae

For the cruciform connection shown in Fig. 5, analyses are classified into three cases shown in Table 2, and for each case the stress fields are divided into several regions shown in Fig. 6 and Fig. 7. In each region the stress distributions are assumed as shown in Tables 3, 4 and 5.

Applying the von Mises' yield condition

$$\sigma_y^2 + 3\tau^2 \leqq \sigma_0^2 \tag{3.1}$$

to the stress in the region (4), the required thickness of connection-web can be obtained as follows:

Case A :

$$t_p \geqq \frac{\sqrt{3}\,M}{HD\sigma_0} \tag{3.2a}$$

Case B :

$$t_p \geqq \sqrt{3\left(\frac{M}{HD\sigma_0}\right)^2 + \left(\frac{N}{2D\sigma_0} + \frac{M}{D^2\sigma_0} - t\right)^2} \tag{3.2b}$$

Case C:

$$t_p \geqq \sqrt{3\left(\frac{Dt}{H}\right)^2 (2-\xi)^2 + \left(\frac{N}{2\xi D\sigma_0}\right)^2} \tag{3.2c}$$

where

$$\xi = \sqrt{3 - 2M/D^2 t\sigma_0} \tag{3.3}$$

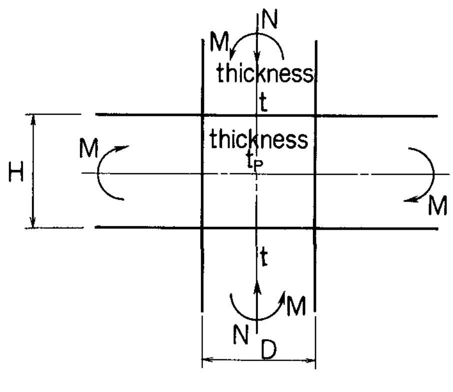

Fig. 5 Idealized Connection

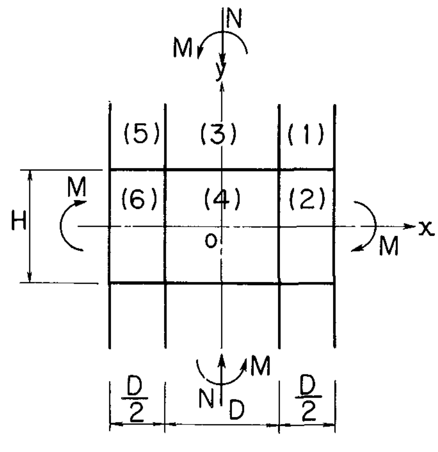

Fig. 6 Stress Field in Case A and B

Table 2 Classification of Analysis

Conditions	$M \leqq D^2 t \sigma_0$	$M \geqq D^2 t \sigma_0$
$\frac{N}{2} + \frac{M}{D} \leqq D t \sigma_0$	Case A	Case C
$\frac{N}{2} + \frac{M}{D} \geqq D t \sigma_0$	Case B	

Table 3 Stress Distribution for Case A

region \ stress	σ_y	τ
(1)	$\frac{M}{D^2 t} - \frac{N}{2Dt}$	0
(2)	$\frac{M}{D^2 tp} \frac{2y}{H} - \frac{N}{2Dtp}$	$\frac{2M}{HDtp}(1 - \frac{x}{D})$
(3)	0	0
(4)	0	$\frac{M}{HDtp}$
(5)	$-\frac{M}{D^2 t} - \frac{N}{2Dt}$	0
(6)	$-\frac{M}{D^2 tp} \frac{2y}{H} - \frac{N}{2Dtp}$	$\frac{2M}{HDtp}(1 + \frac{x}{D})$

Table 4 Stress Distribution for Case B

stress / region	σ_y	τ
(1)	$\frac{2M}{D^2 t} - \sigma_0$	0
(2)	$\frac{M}{D^2 t_p}\left(1+\frac{2y}{H}\right) - \frac{t}{t_p}\sigma_0$	$\frac{2M}{HDt_p}\left(1-\frac{x}{D}\right)$
(3)	$-\frac{N}{2Dt} - \frac{M}{D^2 t} + \sigma_0$	0
(4)	$-\frac{N}{2Dt_p} - \frac{M}{D^2 t} + \frac{t}{t_p}\sigma_0$	$\frac{M}{HDt_p}$
(5)	$-\sigma_0$	0
(6)	$\frac{M}{D^2 t_p}\left(1-\frac{2y}{H}\right) - \frac{t}{t_p}\sigma_0$	$\frac{2M}{HDt_p}\left(1+\frac{x}{D}\right)$

Table 5 Stress Distribution for Case C

stress / region	σ_y	τ
(1)	σ_0	0
(2)	$\frac{t\sigma_0}{t_p} \cdot \frac{2y}{H}$	$\frac{2Dt\sigma_0}{Ht_p}\left(1-\frac{x}{D}\right)$
(3)	$-\frac{N}{2\xi Dt}$	0
(4)	$-\frac{N}{2\xi Dt_p}$	$\frac{Dt\sigma_0}{Ht_p}(2-\xi)$
(5)	$-\sigma_0$	0
(6)	$-\frac{t\sigma_0}{t_p} \cdot \frac{2y}{H}$	$\frac{2tD\sigma_0}{Ht_p}\left(1+\frac{x}{D}\right)$

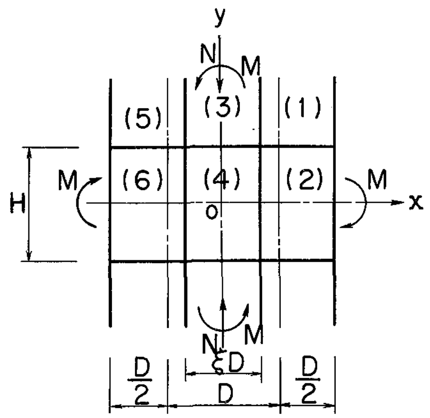

Fig. 7 Stress Field in Case C

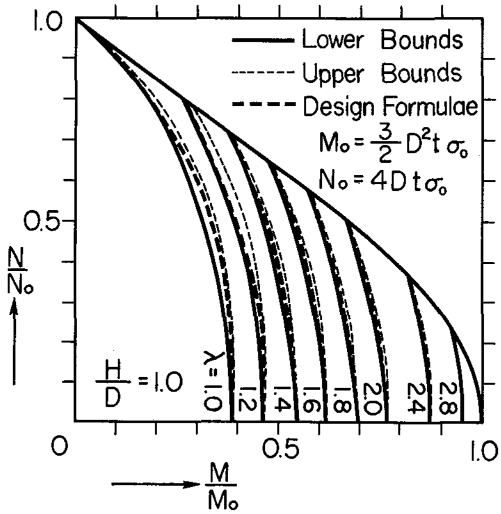

Fig. 8(a) Comparison of Design Formulae and Limit Analysis

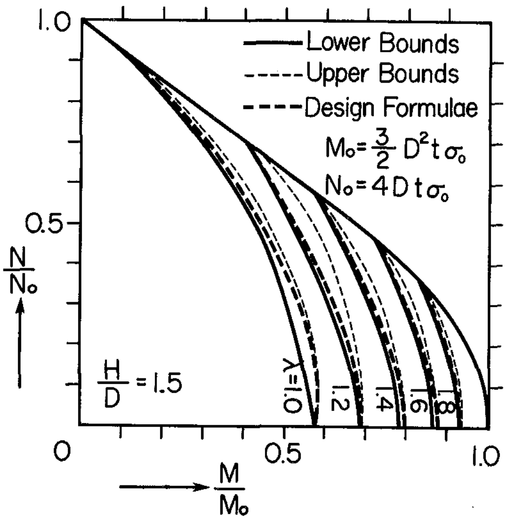

Fig. 8(b) Comparison of Design Formulae and Limit Analysis

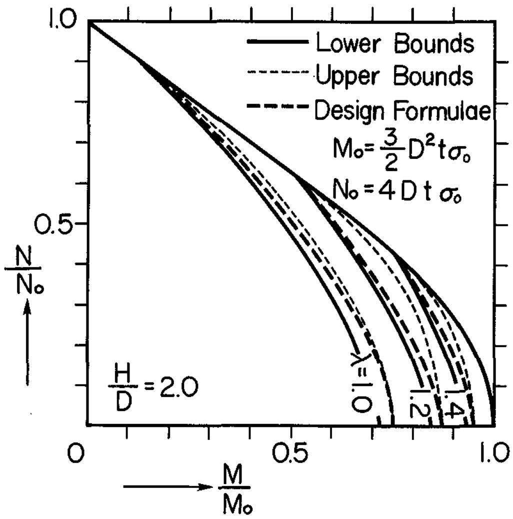

Fig. 8(c) Comparison of Design Formulae and Limit Analysis

4. Accuracy of the Design Formulae

Figs. 8(a), (b) and (c) show the comparisons between the proposed design formulae and the theoretical results.[1] As seen in the figures, good agreement of both results assures that the proposed design formulae are acceptable in design practice.

Reference :

(1) H. TANAKA et al.; Limit Analysis of Beam-Column Connections, (V) Connections of Box-Column to Wide-Flange Beam Subjected to Anti-symmetrical Bending, Trans. A.I.J. No.163, Sept., 1969 (VII) Connections of Wide-Flange Beam to Box-Column Subjected to Bending and thrust, Trans. A.I.J. No.165, Nov. and No.166, Dec., 1969 (IX) Experimental Verification, Trans. A.I.J. No.169, March, 1970