DYNAMIC "FORCE-DISPLACEMENT" RELATIONS OF CANTILEVER TEST PIECE APPLIED BY EARTHQUAKE TYPE EXTERNAL FORCE

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1. Preface

Recently, with the wide use of digital computer the aseismic design tends to compute the detailed vibration of structures during an earthquake and to check the safety. In computing the vibration, the stress-strain relation when the structure was subjected to a strong earthquake force is regarded to be a non-linear one with a hysteresis.

At present, though in an aseismic calculation a non-linear vibration is calculated, as the most of the case, the stress-strain relation is assumed as Fig. 1. The assumption is approximately based on the result of static failure-test in the past, and this is inevitable, since at present there is no result when applying a dynamic external force.

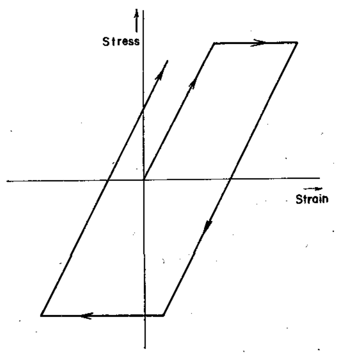


Fig. 1 Idealized Stress - Strain Relation

Thus as some kinds of assumption are considered in the aseismic calculation, we must compare the evaluated results to a practical damage to structures in order to assure the assumption. Observed in the examination of the damages, the structures constructed on the bases of the computation of design is either stronger than computed result or weaker than our expectation as in the Tokachi-oki Earthquake, 1968, of course, except for when the most of damages are owing to the entirely neglected phenomena in design as a liquefaction of sand in the Niigata Earthquake.

It is noticed from the above point of view that the study of dynamical properties and dynamic strength of structural members over a linear range are an important problem of study.

Now, many investigators have ever treated this kind of problem. Limited to the steel-structure, the most general investigation is low cycle fatigue owing to so called alternating load (static loading is applied to exchange its direction).[1) 2)]

Moreover we hear that the experiment of failure by sinusoidal wave begins already. But in the experiments above mentioned the following things are unsatisfactory to investigate the properties of failure of the structures during earthquake.

i) In a failure due to an alternating load, we expect that the failure is smaller in the earthquake type loading than in the experiment, since a loading hour in one cycle is longer in the latter than in the former.

ii) Passing a yield point, it is expected that a force-displacement relation will trace on a hysteresis loop either in the static or in the dynamic. Therefore it is also considered that the energy involved in the test piece consumes to resist against the external force as a viscous reaction in the test piece. Thus we consider that the properties will be different between in the static and dynamic experiments of failure which are different in the ratio of load participation. Moreover, an inertia force due to a mass of test piece may also effect a little to the results of experiment.

iii) In a sinusoidal test of failure, the obtained results cannot be superposed because of a non-linearity in a failure. Therefore it is required to assure the existence of its effects from a failure due to such a complicated wave shape as an earthquake, though there may be no effect in the general situation.

Though a detailed explanation shall be described after the next section, we used a force or displacement with several waves (stationary sinusoidal wave, stationary random wave, transient sinusoidal wave and transient random wave) as external forces, and used a steel piece as structural members.

And we considered about the following relations, the relation between a static strength and dynamic strength, the dynamic "Force-Displacement" relation and the relation between each behavior of stationary and transient external force.

We shall explain the reasons why we used a complicated earthquake type external force. Generally, it is rather easy and inexpensive to make the testing machine which apply such an external force as a sinusoidal wave or a impulse wave than to make that which apply an arbitrary wave force. But the phenomena of failure is non-linear and so the obtained results cannot be superposed. Therefore even if the dynamic behavior to sinusoidal wave external force is known, it is difficult to gain a true answer about complicated earthquake type external force. So it is required to make clear a dynamic behavior to the earthquake type external force. However to the earthquake force used in a testing machine, there is no way than applying the records of earthquake in the past. Since the earthquake wave from to apply to the newly built structures in the future is unknown, some may have a doubt that it is nonsense to make clear a dynamic behavior based on the past records of earthquake.

The following two methods are considered to resolve this doubt. First, a mechanism of dynamic failure of structural materials is made clear from the obtained results to several kinds of complicated wave form force. Then it is possible to know a mechanism of failure even if wave form is complicated. However, at present, our data are not enough to know a fundamental mechanism of dynamic failure and it is doubtful if the general law exists.

Second, a lot of records might be classified stocastically the groups of several type of earthquake wave form. (for example, slowly continued one or impulsive or short-time one). And the properties of dynamic failure will be comparatively resemble each other in a group, and the outline of dynamic behavior will be made clear with the stochastical treatment of the results.

In this paper, we experimented according to the second method.

2. Equipments for Experiment

As follows, the apparatus consists of a testing machine, an apparatus which supports a test piece, an oscillator and a measuring system, and the scheme plan shows as Fig. 2.

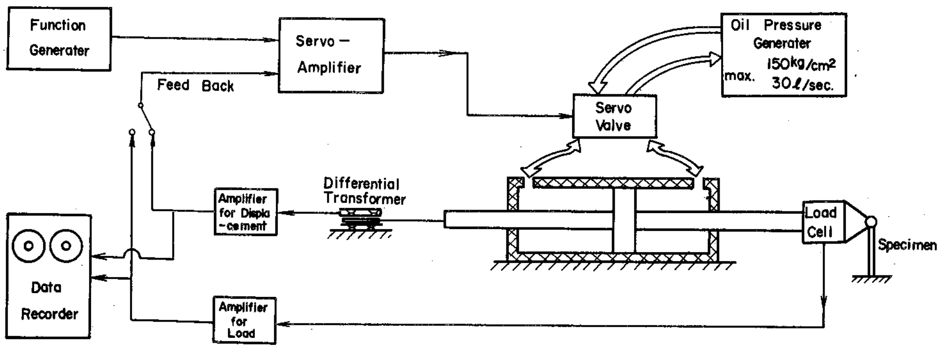


Fig. 2 Block Diagram for Instrumentation

Any wave type dynamic testing machine.

To do a dynamic experiment of failure applying any wave type external force, at the first time, the apparatus is required which apply the force. At present, this apparatus is two kinds,

(i) Electro-magnetic coil type

(ii) Electro-hydraulic type

Principle of type (i) is that a force occurres in an iron core in proportion to the ratio of variability of electric current, and this type has been used in the random wave vibration table. To a testing machine, at present there is little example because of a little output force. Type (ii), an electro-hydraulic type, is an apparatus that a pressure from an oil pressure source is controlled by a servo valve which acts according to external applied voltage. It is used to a fatigue testing machine as well as to a vibration table. Table-1 shows merits and demerits in both type, (i) and (ii).

Table-1

	Merits	Demerits
Electro-hydraulic type	(i) It is easy to produce a great power. (ii) It is low cost by unit power. (iii) Characteristics in a low frequency region is good.	(i) It is difficult to control by an acceleration wave form. (ii) Characteristics drops in such a high frequency as 100 Hz.
Electro-magnetic type	(i) Precision of wave form is good. (ii) Possible to use to a comparatibly high frequency. (iii) Possible to control by acceleration.	(i) It is high cost to produce a great power. (ii) Wave form gets out in a low frequency region.

To a characteristics of our machine, the following matters are required to the maximum for this study.

(1) Power is as large as possible.

(ii) Maximum half amplitude is about 10 cm since a large deformation occurrs in bending failure.

(iii) Not only displacement controlled but also stress controlled is possible.

(iv) To a frequency region, it is possible to use til ten-odd Hz to the maximum when apply an earthquake force.

We developed an electro-hydraulic type testing machine as Table-2, since it is both satisfied the demands above mentioned and low cost.

Test piece	5.0 mm (depth) x 18.0 mm (width) x 240 mm (length), quality : SS41
Support of test piece	Condition of fixed end support is satisfied holding a test piece in a vice settled in the concrete.
Support of vibrated point	An especial support frame so that a support of test piece at vibrated point may be hinge support. (see Fig. 3)

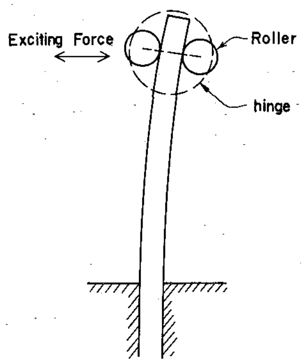


Fig. 3 Supporting of the Specimen

3. Dynamic Failure-Test

Method of experiment

We made the bending failure tests by two kinds of method; one is the way (stress controlled) that we examine the failure applying a prepared wave form force to a test piece, another is the way (displacement controlled) that we break a test piece applying a choosed deformation to it. (Fig. 3)

The former will be the equivalent test to the case when structure is rigid and an earthquake acceleration wave form is equal to an earthquake force applied to a structure, the latter will be equal to the case when a structure is very large and its natural period is longer than a period components of an earthquake and so mainly a structure is destroyed by an earthquake displacement. In experiments, we measured a displacement or a reaction of test piece applying the following wave force or displacement to it. Applied wave is as shown in Table 2.

(a) Stationary sinusoidal wave

1) 1 Hz 2) 3 Hz 3) 10 Hz

(b) Stationary random wave

(c) Transient sinusoidal wave

1) 1 Hz 2) 3 Hz 3) 10 Hz

(d) Transient random wave (pseudo-earthquake wave)

Obtained results show the force-displacement relation corresponding to Fig. 1 in a dynamic external force.

Now, a test piece and others used in the experiment are as follows:

Oscillator

As in a preface, it is a way to apply to a test piece by the same dynamic force as an acceleration of records of earthquake in the past, while it is also a way to treat stocastically the results of the experiment for a lot of external force simulated to earthquake motion. In this study, we selected the latter and used the following wave form f(t) simulated to earthquake

$$f(t) = g(t) \times h(t)$$

where $g(t) = e^{-At} - e^{-Bt}$

h(t) : a white noise cut off over ten-odd Hz.

We developed an oscillator which generates f(t) above mentioned. As shown in Fig. 4, g(t) shows a transient envelop and owing to multiplication of a stationary random wave h(t) with it, a complicated pseudo transient wave form is obtained.

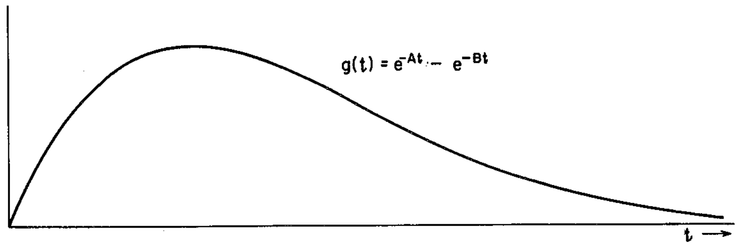


Fig. 4 Transient Envelope

The wave form of this pseudo-earthquake is shown in Fig. 8 etc. To examine the degree of similarity to actual earthquake wave, we did Fourier Analysis about El Centro Earthquake (May 18, 1940, EW component) and an example of this pseudo-earthquake wave. Obtained results are shown in Fig. 5(a), (b). It will be not too much to say that both results are resemble each other for a practical use.

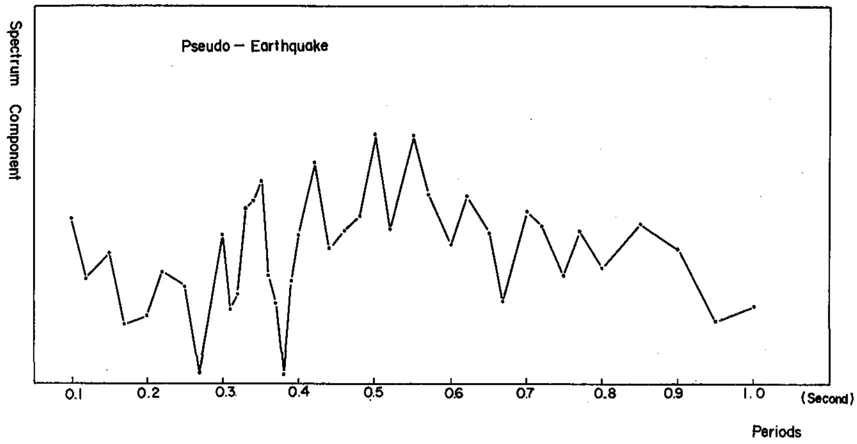


Fig. 5(a) Fourier Analysis of Exciting Force

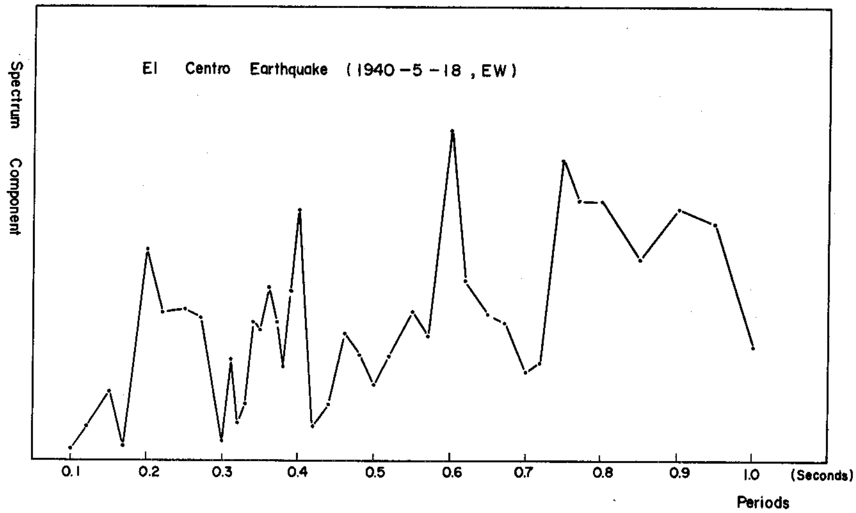


Fig. 5(b) Fourier Analysis of Exciting Force

4. Results of Experiment

Case of stress controlled experiment

The most remarkable feature in results of this experiment is that a plastic deformation remains in a test piece after the load is released to zero.

We made experiments for each of three frequencies, 1, 3, 10 Hz when an external force is sinusoidal. We took a photograph each force-displacement relation on Broun tube. An example is shown in Photo. 1 - 6, Photo. 1 "Force-Displacement" relation for a small external force.

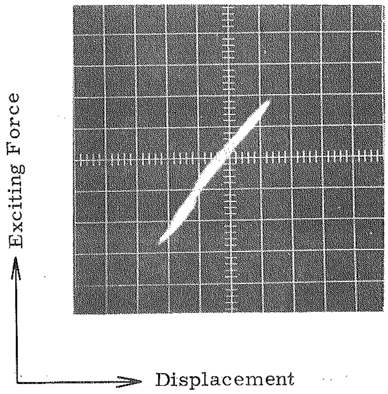


Photo. 1 Exciting Force - Displacement Relation in case of Small Exciting Force, Force Controlled Test

Photo. 2 "Force-Displacement" relation is out of linear and traces a hysteresis. Hysteresis loop stands in a little left side from origin. It may suggest an occurrence of plastic deformation.

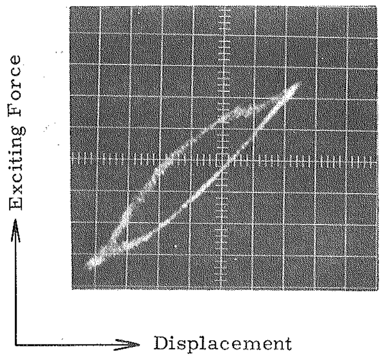


Photo. 2 Hysteretic Relation between Force and Displacement, Exciting Force: Sinusoidal, Force Controlled Test

Photo. 3 Neutral axis moves due to plastic deformation, and the direction of motion is coincident with that in initial step. The reason shall be described unsurrable.

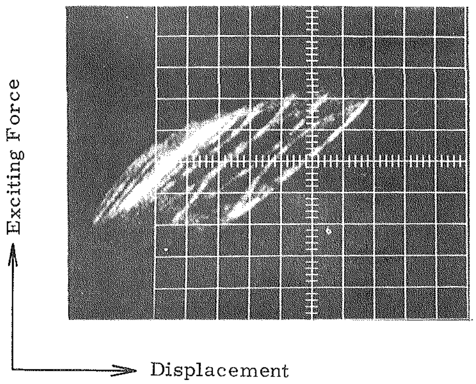


Photo. 3 Incremental Failure,
Exciting Force: Sinusoidal,
Force Controlled Test

Photo. 4 This is a combination figure of a failure test when a test piece was applied a sinusoidal wave force. "Force-Displacement" relation is linear during a small force and begins to trace a hysteresis loop with bigger force. Angle of inclined line remains almost as before. With more bigger force, the vibration force tends to increase the deformation and the area of loop.

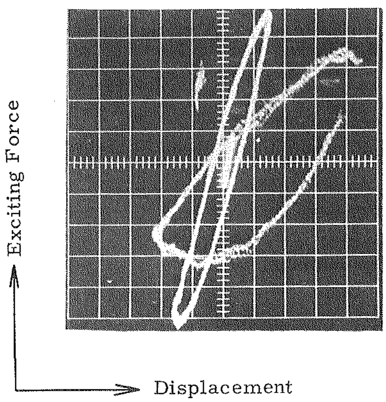


Photo. 4 Combined Photograph of three phases of Dynamic Failure, Force Controlled Test

Photo. 5 When a vibration force is stationary random wave h(t). The envelope is not so much different from in a sinusoidal wave.

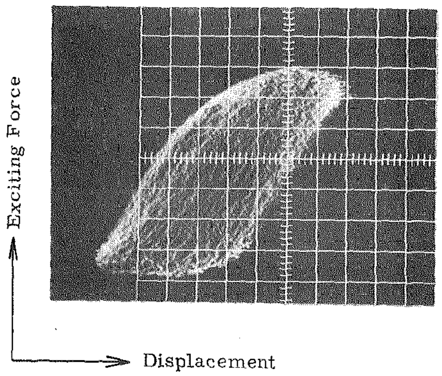


Photo. 5 Force-Displacement Relation due to Stationary Random Force, Force Controlled Test

Photo. 6 When a transient random wave f(t) was pllied a test piece collapses after tracing only few hystereses loops.

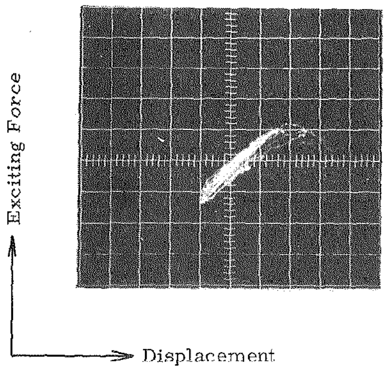


Photo. 6 Hysterisis due to pseudo-Earthquake type Force, Force Controlled Test

Restoring Force Displacement

In the following, we shall show the force-displacement relation with the records of pen-oscillo graph. In this case, the force-displacement relation appears in a time domain.

Fig. 6 Shows an exciting force and a displacement when the force is a transient sinusoidal wave, 10 Hz. Stress is in a linear range.

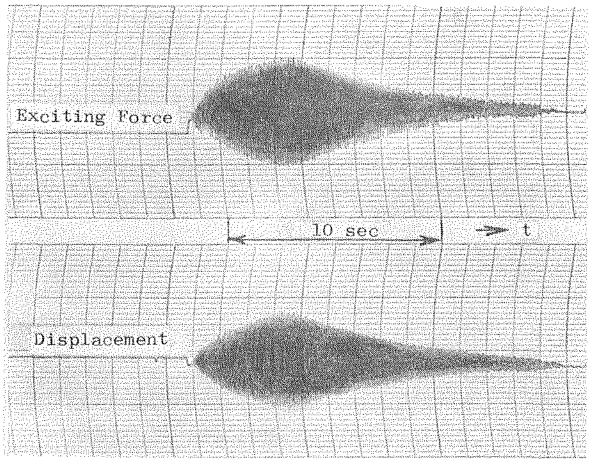
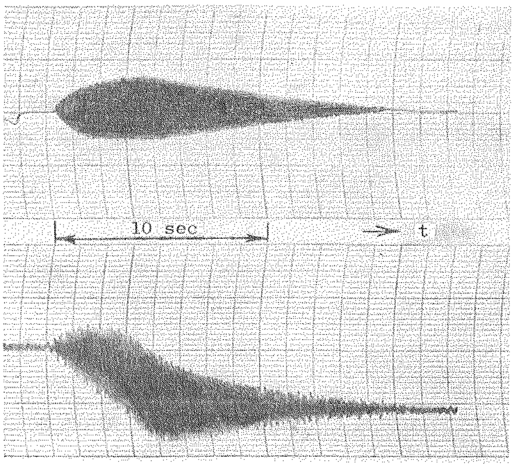


Fig. 6 Exciting Force
(10 Hz, Transient Sinusoidal)

Fig. 7 Input force and frequency are the same as before. A test piece yields since the force is large a little and the neutral axis of record of displacement moves to a certain direction.

Exciting Force



Displacement

Fig. 7 Exciting Force
(10 Hz, Transient Sinusoidal)

Fig. 8 Even when the input force is a pseudo-seismic wave, a displacement vibrates in linear region because of small force.

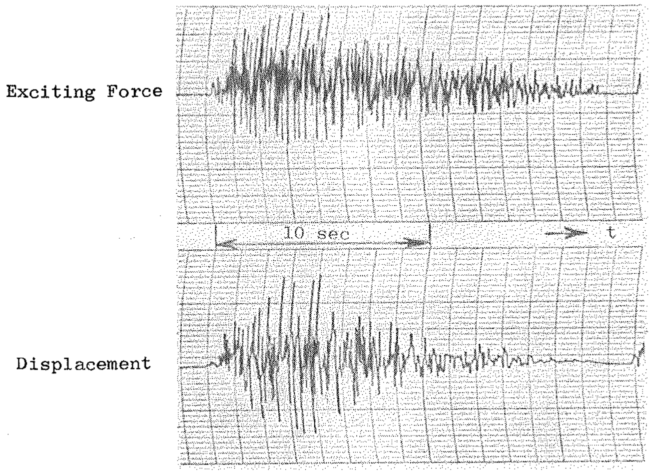


Fig. 8 Exciting Force: Pseudo Earthquake, within linear range

Fig. 9 When input force is larger than before, a neutral axis moves to a certain direction and a displacement vibrates around the moved axis as in sinusoidal wave.

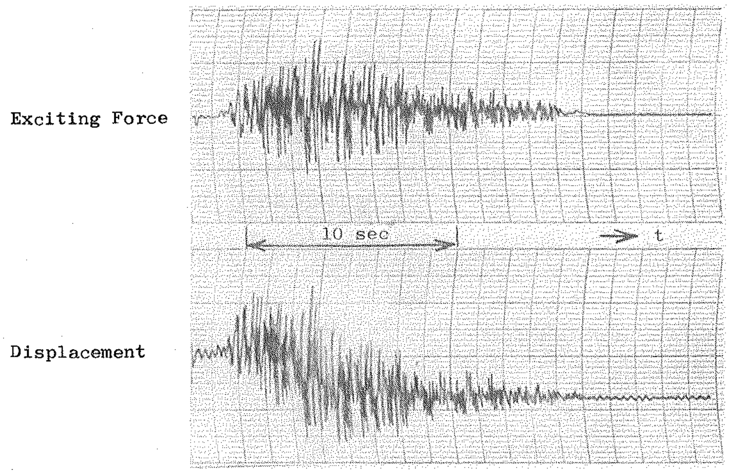


Fig. 9 Exciting Force: Pseudo Earthquake, Permanent Deformation

Fig. 10 A test piece collapses instantaneously without a gradual motion of neutral axis.

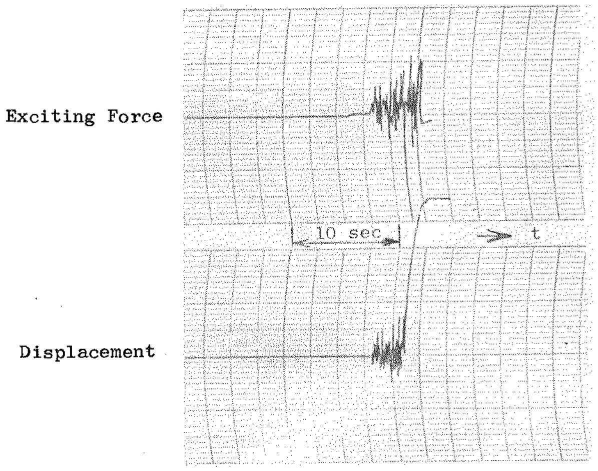


Fig. 10 External Force: Strong Pseudo-Earthquake Force

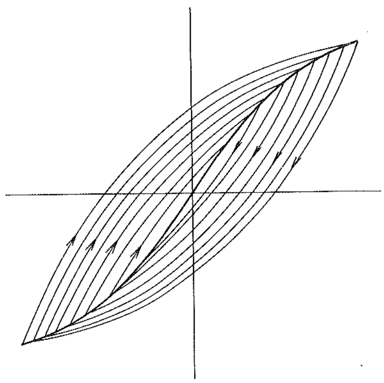


Fig. 11 Soft Spring Type Model

Case of displacement controlled experiment

In this experiment, the reaction occurres in test piece when applying an external force to a top of a test piece.

Force-displacement relation in displacement controlled experiment is obtained from the records of applied displacement and obtained restoring force.

Displacement is recorded by a differential transformer, force by a load cell placed between a shaker and a test piece. In the following, the results of experiment is shown in stress controlled.

Photo. 7 Forced displacement applied a test piece is a stationary sinusoidal wave, force-displacement relation remains in linear because of a small displacement.

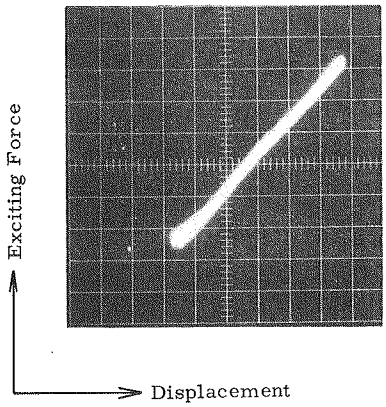


Photo. 7 Displacement - Restoring Force in case of small amplitude, Displacement Controlled Test

Photo. 8 Forced displacement is a stationary sinusoidal wave. One side in a direction of width of test piece may yield, because an amplitude of one side restoring force gradually decrease in spite of applying a forced displacement of constant amplitude.

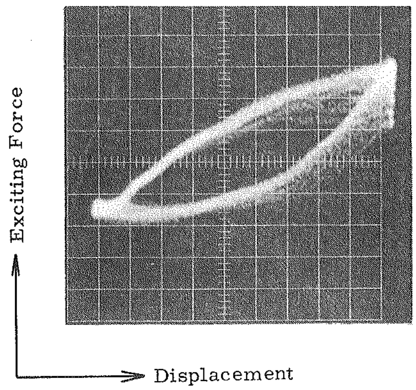


Photo. 8 Decrease of Restoring Force caused from the repetition of the sinusoidal forced displacement, Displacement Controlled Test

Photo. 9 When a decrease of restoring force proceeded than before. Extreme hysteresis loop is almost horizontal as compared with an initial one. This may be an example of Low Cycle Fatigue.

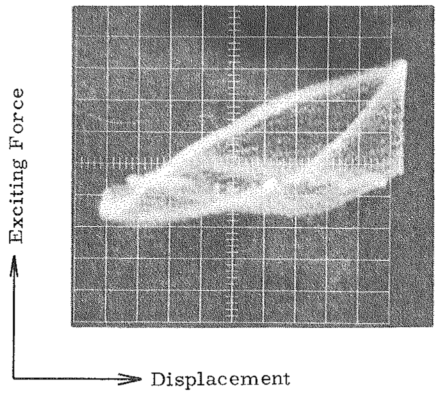


Photo. 9 Plastic Fatigue due to the repetition of the Sinusoidal displacement

Photo. 10 Forced displacement is a transient sinusoidal wave. As shown in Fig. 11, a top of hysteresis loop moves along a so called Soft-Spring shaped curve.

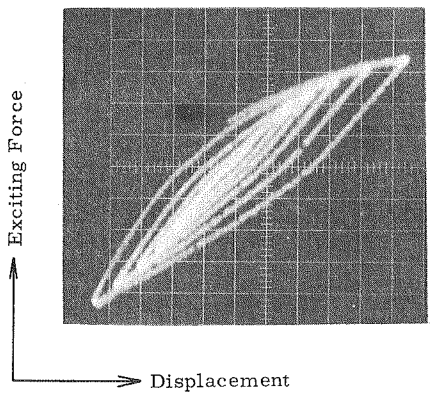


Photo. 10 Displacement - Restoring Force Relation due to Transient Sinusoidal displacement, Displacement Controlled Test

Photo. 11 This shows all steps of a transient sinusoidal forced displacement. It is interesting that a locus of a top point of hysteresis loop is almost corresponding to that in increase and decrease of forced displacement.

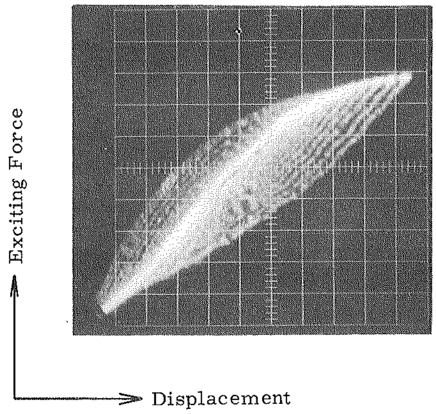


Photo. 11 Due to Transient Sinusoidal Forced Displacement, Displacement Controlled Test

Photo. 12 Forced displacement is a stationary random wave. A test piece is in a little plastic region.

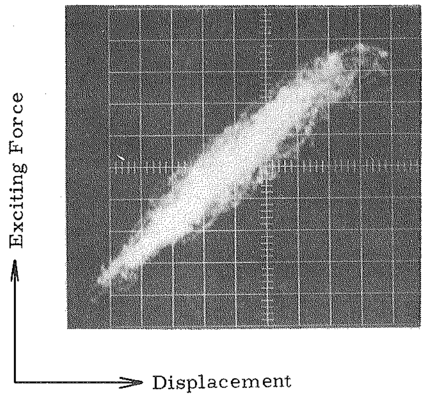


Photo. 12 Displacement - Restoring Force Relation due to comparatively small Stationary Random displacement, Displacement Controlled Test

Photo. 13 A record when a stationary random wave was applied. It is observed that a restoring force does not increase since an amplitude of force displacement is very large. Consequently, we consider that a forced displacement may vibrate randomly within a hysteresis loop for a stationary sinusoidal wave.

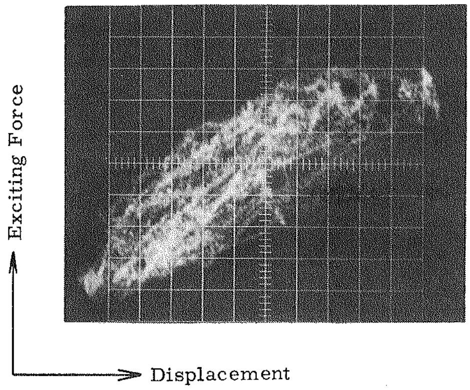


Photo. 13 Displacement - Restoring Force Relation due to Stationary Random Forced Displacement of Large amplitude, Displacement Controlled Test

Fig. 12 Forced displacement is a stationary sinusoidal wave. An amplitude of deformation is gradually increased, and the figure is shown to compare it to an amplitude of restoring force. In a region of initial small amplitude of deformation, displacement-restoring force relation is almost linear, but to increase a deformation, one side amplitude of restoring force decreases (the reason may be a yield of one side of test piece), and an amplitude of force decreases with a time in spite of a constant amplitude of forced displacement.

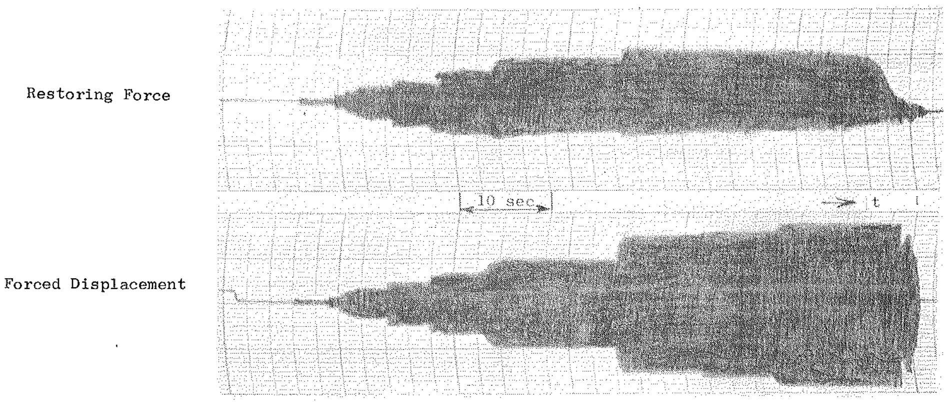


Fig. 12 Forced Displacement (Sinusoidal, 10 Hz)

Fig. 13 Forced displacement is a transient sinusoidal wave. Also in this case, one side amplitude of restoring force decreases not to be in proportion to a displacement and a restoring force is unsymmetric to a neutral axis.

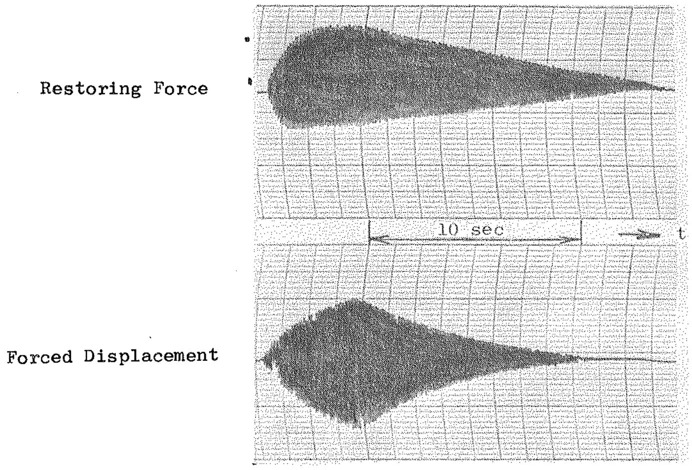


Fig. 13 Forced Displacement
(Transient Sinusoidal, 10 Hz)

On a strength of dynamic failure

What correspond to a strength of dynamic failure in a stress controlled test is obtained for each of the wave form previously mentioned.

In the case of stress controlled experiment, a deformation of material proceeds plastically and rapidly. So we regarded the rapid progress of plastic deformation as a failure of test piece and defined a force immediate before a failure as a force dynamic failure. And we did not regard a failure of material if an external force is vanished even if plastic deformation progresses in some degree. Further, we regarded as a failure only when a material is in the state of failure before mentioned within a few times ten of alternating cycle (without numbering that of external force before a failure). Therefore both a failure in several cycles of alternating load and that in a few times ten cycles of it are identical in data. This may cause promote a change of the consideration of the results. Obtained results are shown in Fig. 14 for a stationary wave input force, in Fig. 15 for a transient one. Both results have much deviation in data. We considered the maximum of one wave before actual failure to be a force of failure. For a stationary sinusoidal wave, static strength and dynamic strength are almost equal independent to input frequency.

For a stationary random wave, dynamic strength is smaller than static strength.

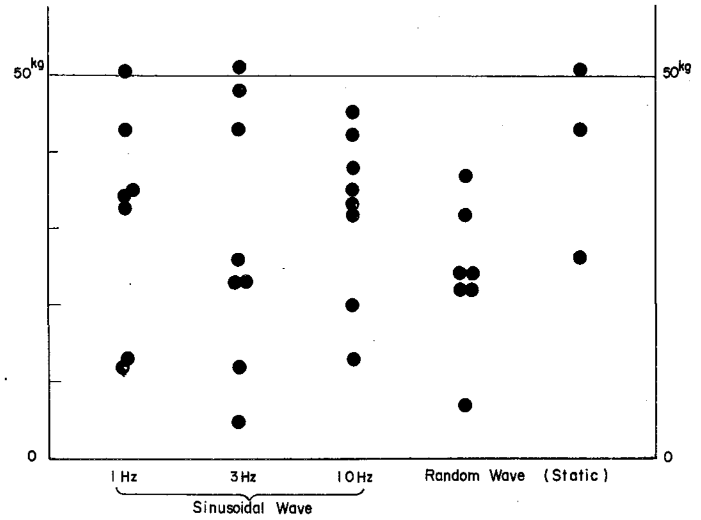


Fig. 14 Dynamic Strength (Stationary External Force)

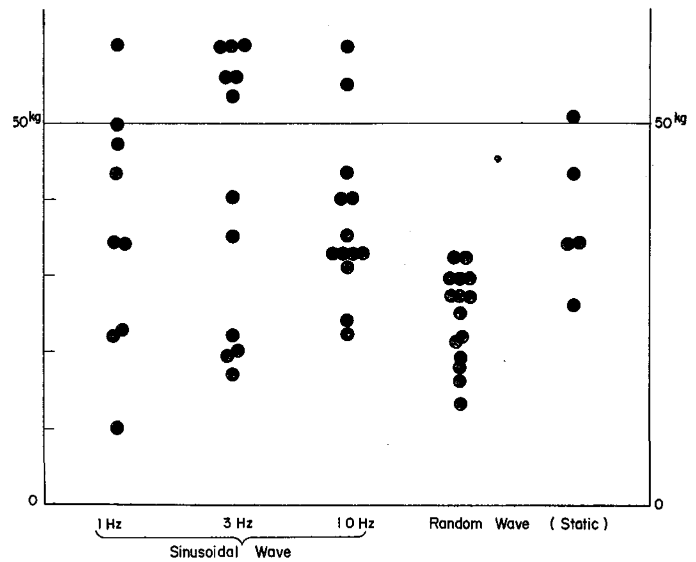


Fig. 15 Dynamic Strength (Transient External Force)

For a transient sinusoidal wave, dynamic strength seems to be independent to input frequency and to be larger a little. For a pseudo-seismic wave, dynamic strength is smaller as for a stationary random wave.

Dynamic strength above mentioned is almost equal to static strength in previously stated definition. This conclusion must be assured from the further more experiments by the following reasons. That is, since dynamic strength defined in this experiment is a peak of input force of one wave before from dynamic failure, it will be confirmed that for a sinusoidal wave, even if it is either stationary or transient, a peak of input force of one wave before from a failure point is almost equal to a force of failure point. However, for a random wave force a peak of wave changes randomly, and so the probability that a peak of wave before a failure point is fairly smaller than a force of failure point must be larger than for a sinusoidal wave.

This may result to decrease fairly a dynamic strength for a random as compared with that for a sinusoidal wave. In experiment, especially for a transient random wave, it was difficult to collapse gradually a test piece tracing a hysteresis loop, that is, for a small input force "Force-Displacement" relation is linear, while a little increase of a input force collapsed abruptly a test piece tracing very few hysteresis loop.

5. Considerations

From the experiment of dynamic bending failure of small steel piece, the following things were known.

(i) When an exciting force was applied to a test piece, force-displacement relation was not so much different to that which has ever been used in a dynamic analysis. But when an external force is a transient random wave (pseudo-seismic wave), a test piece often collapsed not to trace a hysteresis loop.
For a sinusoidal wave and a stationary random wave, it is possible to vibrate a test piece in a state as an amplitude of external force is almost uniformly, while for a transient random wave, when an envelope of amplitude is large as whole, a deformation progresses rapidly from a yield point to a failure region because of a complicated wave.
Therefore though we cannot conclude from this experiment alone, in a dynamic analysis, when we study a vibration of structure during a great earthquake, we consider that it leaves room for consideration to state the following conclusion; assuming a hysteresis loop and evaluating an equivalent viscosity by an equivalent linearization method, an amplitude decreases, and so on. Because an increase of viscous damping may not be expected so much since a deformation for a complicated wave may progress rapidly.

(ii) In a stress controlled experiment, a test piece deforms plastically during vibration and continue to vibrate in centre of neutral axis departed from a initial one, and then the result of test shows that a plastic deformation progresses to the direction of initial step. Though the reason is not still clear, a few reasons are consider as follows.

a) The way to apply a force in a machine is not same in direction.

b) After one side test piece yielded, a mechanical properties of test piece is mechanically weak than before a yield, a plastic deformation progresses only into a weak part.

All test pieces did not collapse plastically into one same direction alone. Thus we do not take a reason a) but a reason b).

(iii) A dynamic strength obtained from stress controlled experiment was not so much different to a static strength for a sinusoidal wave, was smaller for a random wave. Results as above described must be confirmed by more similar experiments.

(iv) Features of failure in case of displacement controled tests is as follows.

a) Plastic deformation don't occur as a deformation is confined even if a material is yielded, and a restoring force decreases with an increasing of a forced displacement.

b) After a yield of material, force-displacement relation traces a hysteresis loop similar to that for stress controlled.

c) As a displacement is forced, a dynamic strength inplies a plastic fatigue failure that a restoring force is almost zero in spite of apply a displacement. This is entirely different to the case of stress controlled.

6. Acknowledgements

We obtained the features of properties of bending failure, especially "Force-Displacement" relation and a dynamic strength, applying several kinds of wave form to a small steel cantilever beam.

Though we cannot conclude quantitatively, we consider that this study will contribute somewhat to an earthquake engineering.