A NEW PHOTOELASTIC METHOD OF THE DYNAMIC STRESS ANALYSIS

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1. Introduction

Recently papers on the stability of structures during an earthquake have been appearing at an increasing rate.

As an approach to dynamic stress analysis, authors carried out dynamic photoelastic experiment of a model made of gel material, in order to measure two- and three-dimensional stresses in a model during vibration.

According to author's experiences, a model made of gel-material is suitable for investigation on dynamic stresses of the structure in vibrating state.

In this report, outlines of the experimental and analytical method adopted in our studies are described.

2. On the Experimental Material and Method

(a) Material

In the case of two-dimensional analysis, gelatin gel[1] is used as photoelastic material. Gelatin gel has very high photoelastic sensitivity and low Young's modulus, so that it is possible to decrease the resonance frequency of a model and to inspect the stresses due to mass force and inertia.

In the case of three-dimensional analysis, the sandwich method is adopted. A sandwich of non-photoelastic sensitive material, with gel with high photoelestic sensitivity as birefringent layer is produced and a fringe pattern in birefringent layer is obtained. This technique is adopted by J.H. Lambe[2], J.W. Dally[3] in the case of hard photoelastic material.

Polyacrylamide gel is used as photo-elastic unactive part of a model, and gelatin gel is used as birefringent layer.

Polyacrylamide gel consists of following material.

Acrylamide added cross-linking agent of any kind as starting mixture (Trade Name : Nitto-SS), ammonium persulfate as initiator and β- dimethylamino - propionitrile as accelerator.
The gel is prepared as follows:

At first, A- and B-liquor are prepared as follows:
In weight

A- liquor Starting mixture: 10, Water: 40, Accelerator: 0.4

B- liquor Water: 50, Initiator: 0.5

Gel is produced in a few minutes by mixing A-, B- liquor and sodium acrylate. Sodium acrylate is used to make gel clear and transparent. It is possible to control Young's modulus of gel, changing the ratio between A- and B- liquor.

The elastic property of this gel is similar to that of gelatin gel. But it is very brittle, so that we have to pay much attention for treatment of the gel.

(b) Experimental Method

A model made of gel-material is set on the shaking table and is passed by polarized light in usual way. As the frequency of the model is comparatively low, it is not necessary to use high-speed camera.

In the experiments, the frequency of excitors to the shaking table is synchronized with the bright and blind of mercury lamp, in order to take a still standing photograph. Therefore, the dynamic photoelastic experiment is able to be carried out in the similar way to ordinary statical one. Furthermore, the phase difference between the movement of the shaking table and the bright and blind of mercury lamp is variable. (Fig.-1)

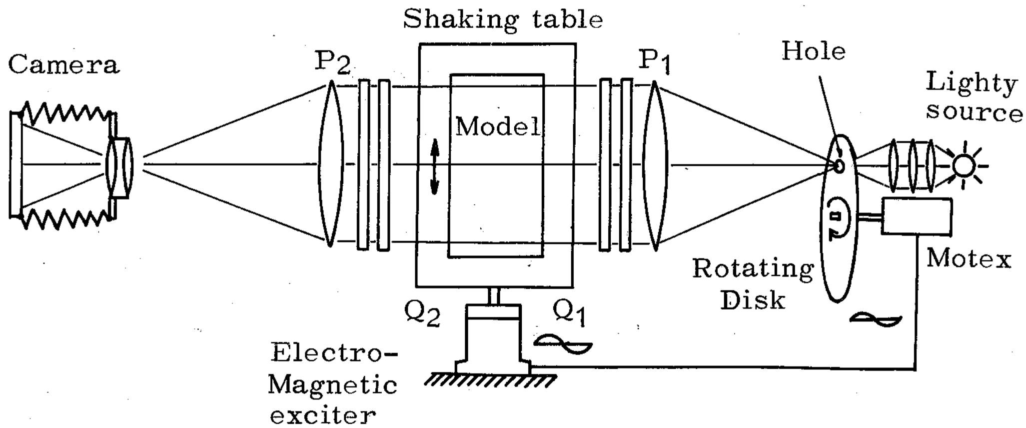


FIG. - 1

3. Calculation of Stresses

When gelatin gel is used as photo-elastic material, stresses produced by its own weight in a model are determined. Furthermore, when a model is vibrating, combined stresses produced by its own weight and vibration are obtained. Therefore, when it is desired to separate stresses only by vibration from combined stresses, it is necessary to halve the difference between stresses at the both maximum displacement or to subtract stresses at the neutral position from the ones at the maximum displacement.

(a) Two-dimensional Analysis

In the case of two-dimensional analysis, free boundary stresses are obtained from isochromatics directly. Photo-1, 2 show the examples of isochromatics and Fig. 2, 3 show the analytical results around a hole.

(i) In General

When equations of motion are used for determination of individual components of stresses, maximum shearing stress obtained from isochromatics, direction of principal stresses and acceleration are necessary.

Eqs. of motion are:

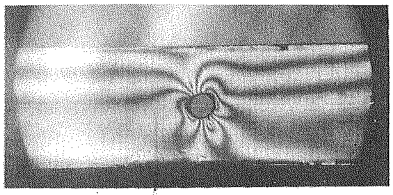
$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2},\quad \frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}=\rho\left(\frac{\partial^2 v}{\partial t^2}-g\right) \quad \cdots\cdots\cdots (1)$$

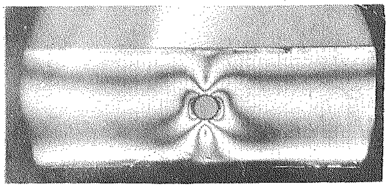
Integrating Eqs. (1),

$$\left.\begin{aligned} \sigma_x &= \sigma_{x0}-\sum_i\frac{\Delta\tau_{xy}}{\Delta y}\Delta x+\rho\sum_i\frac{\partial^2 u}{\partial t^2}\Delta x \\ \sigma_y &= \sigma_{y0}-\sum_i\frac{\Delta\tau_{xy}}{\Delta x}\Delta y+\rho\sum_i\left(\frac{\partial^2 v}{\partial t^2}-g\right)\Delta y \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (2)$$

σ_{x0} and σ_{y0} in Eqs. (2) are respectively σ_x and σ_x at $x=x_0$, $y=y_0$.
And these values are obtained from experimental result, for example, free boundary isochromatics.
τ_{xy} is uniquely determined from maximum shearing stress and direction of principal stress, $\sigma_x-\sigma_y$ is also determined in the same way as in the case of τ_{xy}. So that, if σ_x or σ_y is determined, all individual components of stresses are calculated out.

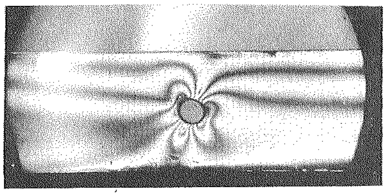
Vibrating State





Still Standing

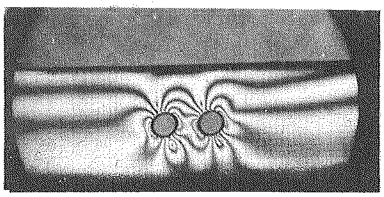
Vibrating State

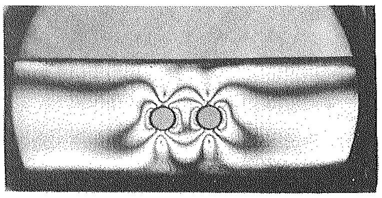


Isochromatics around holes during vibration

PHOTO - 1

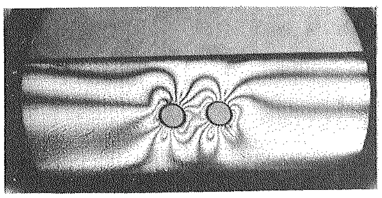
Vibrating State





Still Standing

Vibrating State



Isochromatics around a hole during vibration

PHOTO - 2

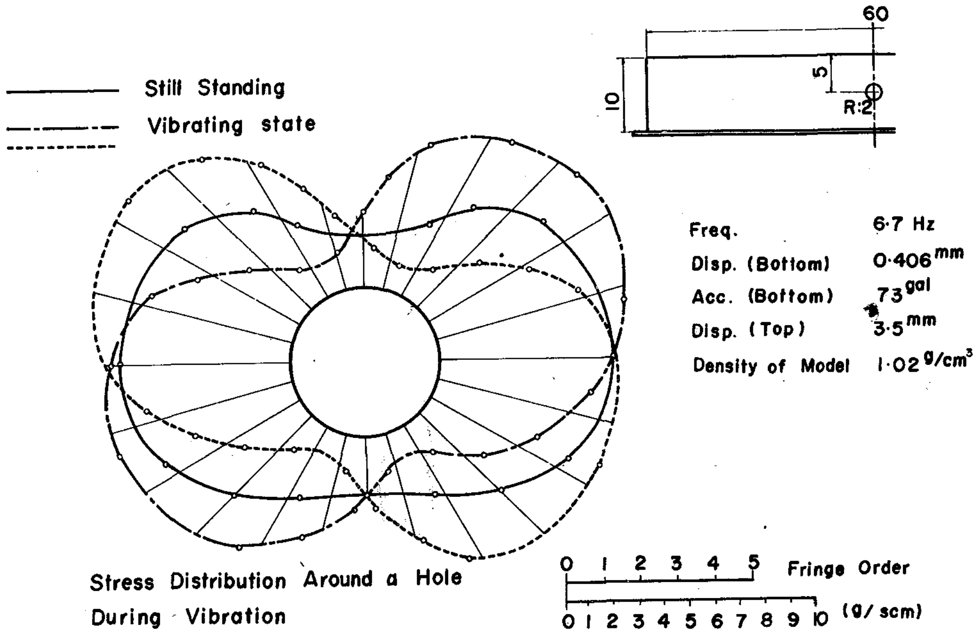

60
10
5
R:2
Still Standing
Vibrating state
Freq. 6·7 Hz
Disp. (Bottom) 0·406 mm
Acc. (Bottom) 73 gal
Disp. (Top) 3·5 mm
Density of Model 1·02 g/cm³
0 1 2 3 4 5 Fringe Order
0 1 2 3 4 5 6 7 8 9 10 (g/scm)
Stress Distribution Around a Hole
During Vibration

FIG. - 2

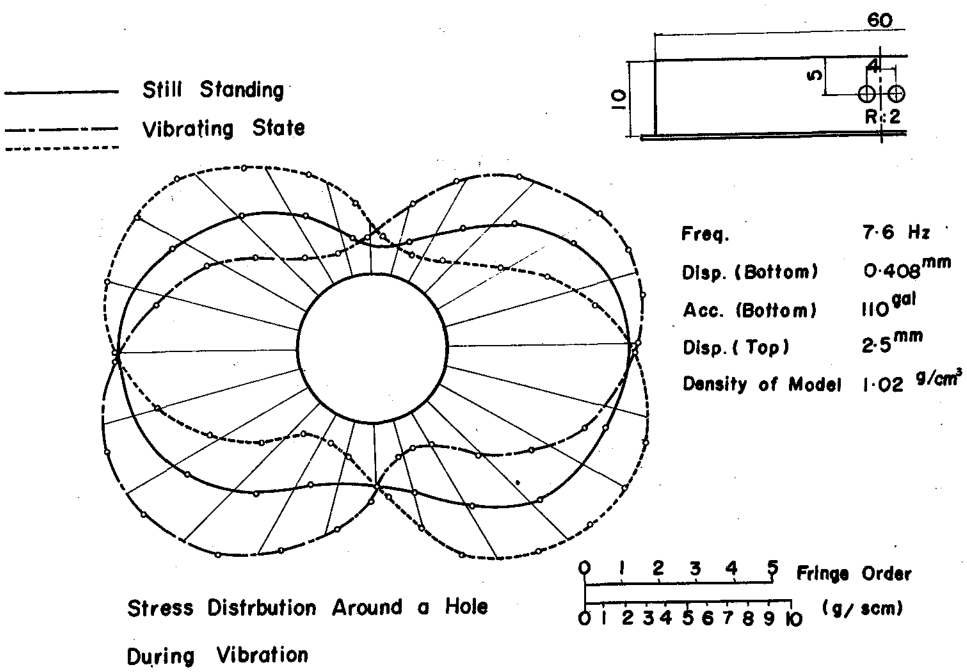

60
10
5
4
R:2
Still Standing
Vibrating State
Freq. 7·6 Hz
Disp. (Bottom) 0·408 mm
Acc. (Bottom) 110 gal
Disp. (Top) 2·5 mm
Density of Model 1·02 g/cm³
0 1 2 3 4 5 Fringe Order
0 1 2 3 4 5 6 7 8 9 10 (g/scm)
Stress Distrbution Around a Hole
During Vibration

FIG. - 3

(ii) Harmonic Vibration

The following relationship is obtained from compatibility condition and equations of motion,

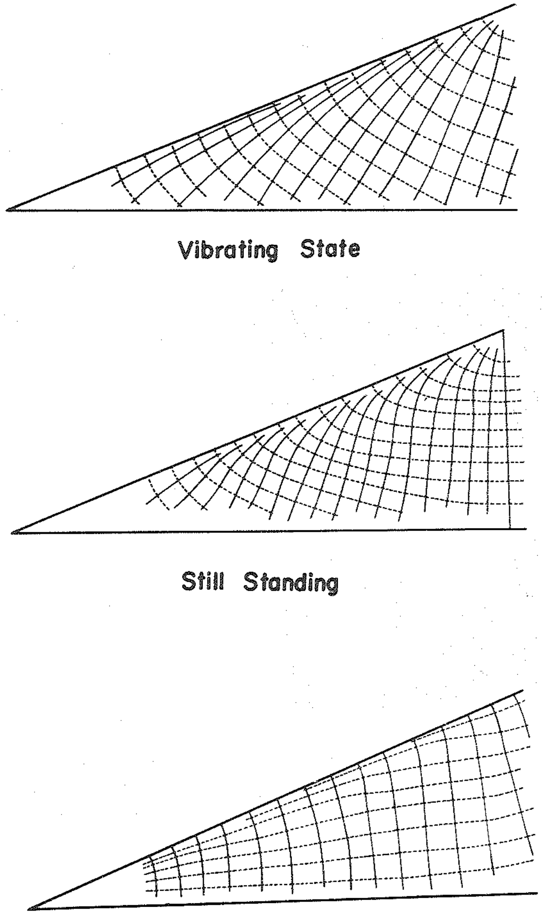
$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) (\sigma_x - \sigma_y) + 4 \frac{\partial^2 \tau_{xy}}{\partial x \partial y}$$

$$= - \frac{\rho w^2 (1-\nu)^2}{E} (\sigma_x + \sigma_y) \qquad \dots\dots\dots\dots (3)$$

τ_{xy} , $\sigma_x - \sigma_y$ are determined from maximum shearing stress and direction of principal stress. All components of stresses are decided, if the value of $\sigma_x + \sigma_y$ is determined from Eq. (3). Acceleration mode in a model is not needed in this method, differing from (i). However, the second order differentiation of τ_{xy}, $\sigma_x - \sigma_y$ obtained from experimental date includes normally errors, so other techniques have to be adopted to obtain high amount of accuracy of the experiment.

(iii) Determination of Direction of Principal Stresses and Acceleration

Determination of direction of principal stresses are described subsequently. In usual, photoelastic experiment with hard material, directions of principal stresses are obtained from isoclinics. However, according to author's experiences, it seems to be difficult to determine directions of principal stresses by isoclinics to a model in which gradient of stresses is smooth. Because isoclinics in such a model is vague. Therefore, some other technique has to be deviced. The authors tried following technique in order to determine the direction of principal stresses. Very small circular holes are driven in a model, and directions of principal stresses are found photoelastically from stress concentration around a hole. When circular holes of 2.0^{mm} in diameter are driven in an earth dam model of which height and thickness are 10^{cm} respectively, isochromatics as shown in Photo-3 are obtained. And the principal stress lines determined by the experiment are shown in Fig.-4. Fig.-4 seems to show that this method is suitable for analysis, so authors are making an effort to improve this method, especially to increase accuracy. The characteristics of this method consist in application of stress concentration around a hole. Hence, it is not proper to apply this method to stress condition such as strict stress concentration. In such a stress condition, it is possible to obtain isoclinics in ordinary way without applying this technique.

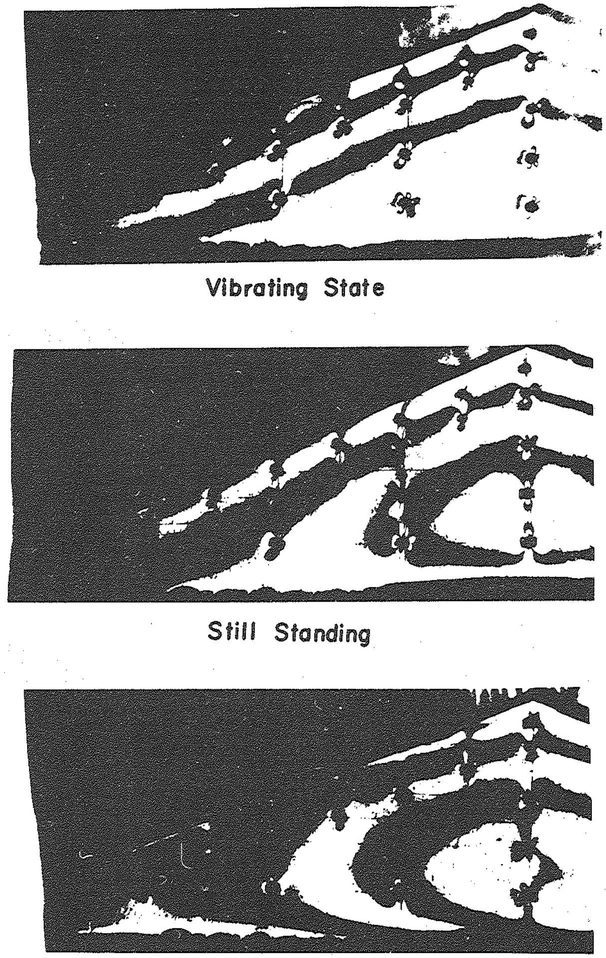


Vibrating State

Still Standing

Vibrating State

FIG. - 4



Vibrating State

Still Standing

Vibrating State

PHOTO - 3

Subsequently the measurement of acceleration is described. It seems to be difficult to measure directly acceleration on each point of a model during vibration. However, in the case of harmonic vibration, it is possible to measure acceleration on each point of a model undirectly, by measuring displacement. As one of example, mode of a model is obtained as shown in Photo-4. However, when vibration mode are generally complicated, acceleration measurement can not always be made by this method.

Moiré method is useful for measurement of displacement. Photo-5 shows an example of Moiré fringe pattern of earth dam during vibration. And Fig.-5 shows the analytical results of horizontal displacement of earth dam. This result indicates that an accurate displacement is obtained by this method. And the method is now under studies, expecting measurement of strain.

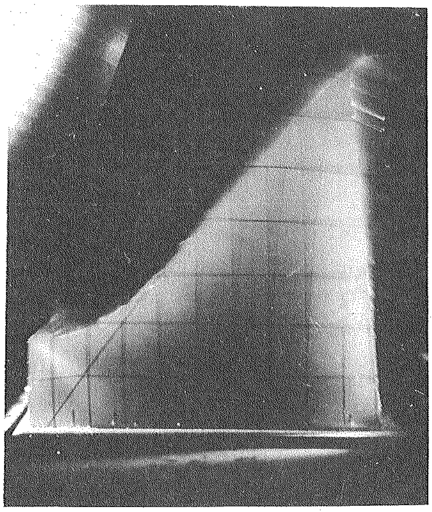


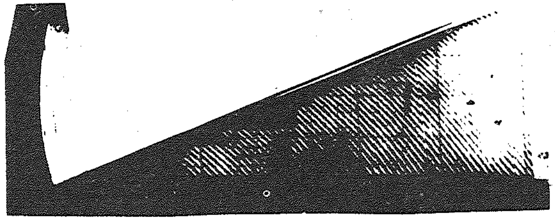
PHOTO - 4

(b) Three-dimensional Analysis

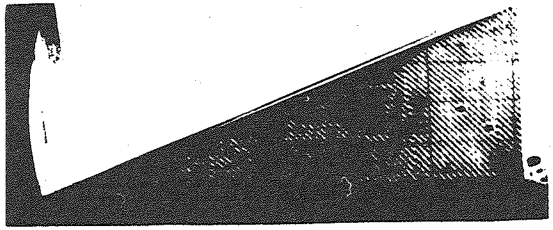
In a model of Polyacrylamide gel, isoclinics are obtained. Hence, it is impossible to determine shearing stresses, using isoclinics.

The oblique incident method (4) is applied, in order to measure shearing stress. Now let's pay attention to a plane S parallel to x- , z- axis in a body. Assuming that the polarized light parallel to y- , z- plane passes with inclination θ to y- axis (Fig.-6).

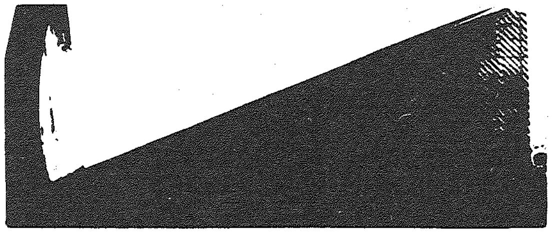
In such a case, at S-plane we can obtain isochromatics which is proportional to difference between principal stresses in the



Vibrating State



Still Standing



Vibrating State

PHOTO - 5

Horizontal Displacement of Earth Dam

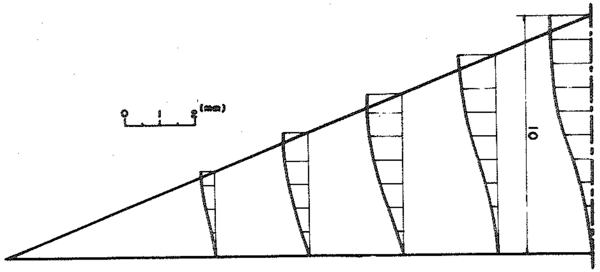


FIG. - 5

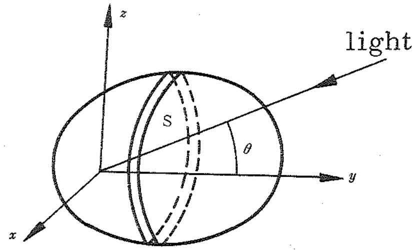


FIG. - 6

plane perpendicular to the polarized light. Now representing the difference between principal stresses in the plane perpendicular to polarized light by σ_θ

$$\sigma_\theta^2 = [\,(\sigma_z - \sigma_x) + (\sigma_y - \sigma_z)\, sin^2\theta - \tau_{xy}\, sin 2\theta]^2$$

$$+4\,[\tau_{xz}\, cop\theta - \tau_{yz}\, sin\,\theta]^2 \qquad \ldots\ldots\ldots\ldots (4)$$

When σ_θ are measured in the five different θ , absolute values of τ_{xy}, τ_{yz}, τ_{xz}, $\sigma_z - \sigma_x$, $\sigma_y - \sigma_z$ are to be determined by Eq. (4).

Assuming that shearing stresses and accelerations on each point of a model are obtained, it is possible to obtain all individual components of shearing stresses, by use of the shear difference method.

Photo-6 shows three-dimensional isochromatics resulting from rocking of circular column based on semi-infinite body.

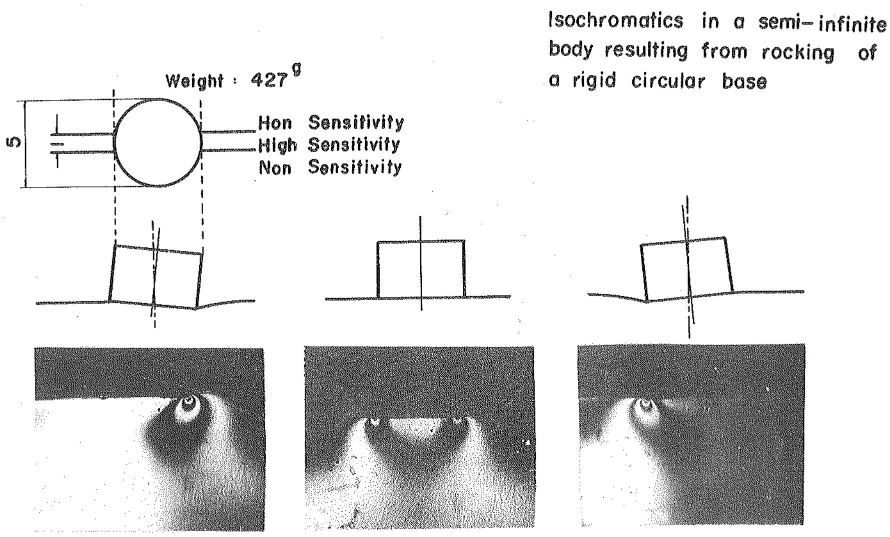


PHOTO - 6

4. Conclusion

As mentioned above, outlines and unsolved problems of dynamic photoelastic experiment with a model made of gel-material are described. In the case of two-dimensional analysis, it seems to be possible to determine individual components of stresses with practical enough accuracy in many cases. However, in the case of three-dimensional analysis, there are many difficulties.

This method is available for dynamic stress analysis of foundations, for example, caisson, pile, group action piles and underground structures, such as tunnel, subway etc. Furthermore, it is applicable to problems on rock-mechanics.

REFERENCES

1) Yamamoto, M and Morichi, S. "Two-dimensional photoelastic experiment made by gelatine gel" Trans of J.S.C.E., No.144, Aug, 1967.
2) Lambe, J.H. and Bayomi S.E.A., "A room temperature photoelastic technique for three-dimensional problems" Proc. Inst. Mech Engrs. B, Vol. 12, 1952-1953.
3) Dally, J.W. and Rilly, W.F., "Initial studies in three-dimensional dynamic photoelasticity" presented at the 5th National U.S. Congress of Applied Mechanics.
4) Photoelasticity (In Japanese) by Tuzi, J. (Editor)

APPENDIX

Notations

x , y , z	:	Rectangular Coordinates
σ_x, σ_y, σ_z	:	Normal components of stresses parallel to x-, y-, z- axis.
τ_{xy}, τ_{yz}, τ_{xz}	:	Shearing stresses
u , v	:	Components of displacements parallel to x-, y- axis.
t	:	time
ρ	:	density
g	:	Gravitational acceralation
E	:	Young's modulus
ν	:	Poisson's ratio
ω	:	Angular frequency