A SUGGESTION OF GENERATING A PHASE RESTRICTED PSEUDO-EARTHQUAKE MOTION.

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This paper is summariezed from the paper which was presented to the 2nd meeting of the U.S. - Japan Joint Seminar in Applied Stochastics. This seminar had been held in Washington, D. C. from September 19 to 25 this year under the sponsorships of National Science Foundation and Japan Science Promotion Society. The anthors would like to express their grate thanks to the very hearty discussions of Professor Kozin, Professor Isobe and other participants. The part of this paper was changed as the result of their discussions.

1. Introduction

In the previous report(1) for the 1st joint seminar, 1966, the authors commented that the possibility of using the step response of a multi-degree-of-freedom system as a pseudo-earthquake motion.

There are several reports(2, 3, 4) on the statistical analysis of complicated systems. But in this paper the authors will describe the procedure to estimate the next step response of the system, of which characteristics is complicated and fluctuating, through the observation and analysis on the group of the previous responses. The continuous rod or layer of which cross-section is not uniform but almost uniform along its axial co-ordinate, so-called a disordered system is used as a theoretical model. The block diagram of the authors procedure is shown in Fig. 1.

It is very difficult that we obtain the many records of the strong earthquake motion at the specific point. But if we obtain several of them, we could estimate the properties of their pathes by assuming the pattern of their statistical distributions. Analyzing the wave form of each sample (record) and expressing its characteristics with its eigen-frequency distribution, we could say the average and variance of their cross-sectional parameters in a certain accuracy according to the number of samples. This is the first step. And the second step is estimating the nature of the step response of the system sampled randomly from the population, of which characteristics have been decided statistically at the first step. To evaluate the statistical natures of these samples is also the job of the second step.

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This report starts the theoretical analysis of a one-dimensional wave equation possessing randomly distributed parameter for the second step block in Fig. 1.

2. Fundamental analysis

The authors introduced a one-dimensional wave equation for their fundamental analysis of this subject, and used the disordered rod system subjected by longitudinal vibration for experimental analysis.

The basic notations are as follows:

$u(x, t)$: displacement solution along the axis x

$u_o(x, t)$: $u(x, t)$ of the corresponding uniform (undisturbed) rod

x : co-ordinate of longitudinal axis of rod

$r(x)$: radius of rod

$s(x)$: cross-sectional area of rod

a : velocity of longitudinal wave

r_o : mean radius

t : time

ℓ : length of rod

ρ : density of rod material

E : elastic modulus of rod material

$$\frac{\partial^2 U(X,T)}{\partial X^2} + \frac{d}{dX}(\log S(X))\frac{\partial U(X,T)}{\partial X} = \frac{\partial^2 U(X,T)}{\partial T^2} \tag{1}$$

Here a capital letter denotes a dimensionless value of the variable which is denoted by a corresponding small letter, for example,

$$X = \frac{x}{\ell}, \quad T = \frac{t}{\ell / a} \qquad U(X,T) = U(x,t)/\ell$$

and so on. In the case of the rod infinite long, ℓ might be introduced as a standard length.

2.1 Step-response

We assumed $S(X)$. is known, then $P(X)$ dimensionless cross-sectional parameter becomes given. If the change of cross-sectional parameters is caused by only that of cross-sectional area,

$$r(x) = r_o(1 + P(x)) \tag{2}$$

Hereafter, we assume

$$P(X)=O(\varepsilon)\ ,\qquad \frac{dP(X)}{dX}=O(\varepsilon)$$

for $1>|\varepsilon|$.

$$\frac{d(\log S(X))}{dX}=2\frac{d\,P(X)}{dX}(1-P(X)+O(\varepsilon^2)) \tag{3}$$

then we put

$$Q(X)=\frac{dP(X)}{dX}(1-P(X)+O(\varepsilon^2)), \tag{4}$$

the physical meaning of $Q(X)$ is going to be described later.
Here, two types of differential operators will be introduced.

$$L_0=\frac{\partial^2}{\partial X^2}-\frac{\partial^2}{\partial T^2}\ ,\qquad \varepsilon L_1=2\,Q(X)\frac{\partial}{\partial X} \tag{5}$$

then eq. (1) becomes

$$(L_0+\varepsilon L_1)\cdot U(X,T)=O. \tag{6}$$

Then

$$U(X,T)=U_0(X,T)-L_0^{-1}\left\{2\,Q(X)\frac{\partial U_0(X,T)}{\partial X}\right\} \tag{7}$$

could be led, assuming $L_1U(X,T)$ to be $L_1U_0(X,T)$. This means the neglects of the terms of which orders are higher than ε^2.

The solution of the undisturbed rod, U_0, $\frac{\partial U_0}{\partial X}$ are already known, and if $Q(X)$ is known, eq. (7) can be easily solved.

The reflective rate α of strain wave at the point, where the cross-
At the farther points than lines $\overline{OA}$ and $\overline{OB}$ in Fig. 2,

$$U=U_0-\frac{F}{Es_0}\left\{\int_0^X Q(\xi)(-X+T)d\xi+\int_X^{\frac{X+T}{2}}Q(\xi)(-2\xi+X+T)d\xi+\int_{\frac{X-T}{2}}^{o}Q(\xi)(2\xi-X+T)d\xi\right\} \tag{8}$$

can be obtained, by using Green's function approach over the shaded area in Fig. 2.

The reflective rate α of strain wave at the point, where the corss-sectional area suddenly changes from s_1 to s_2, is

$$\alpha=\frac{s_2-s_1}{s_2+s_1} \tag{9}$$

If the cross-sectional area changes gradually as

$$s(x)=\pi r_0^2(1+P(x))^2, \quad \alpha=\frac{ds(x)}{dx}dx\,/\,2s(x)=\frac{dP(X)}{dX}dX\,/\,(1+P(X)), \tag{10}$$

therefore

$$\alpha \fallingdotseq \frac{dP(X)}{dX}(1-P(X))dX=Q(X)dX. \tag{11}$$

Through the analyses of this section, the terms of which order is higher than ε^2 are neglected. This corresponds to take the only one reflection at the mid-point for all pathes into account.

The step response in an each section in Fig. 3 is written as follows.

For the section $2i$

$$\begin{aligned}\frac{\partial U}{\partial X}=-\frac{F}{Es_0}\Bigg\{&\left.\frac{\partial U}{\partial X_{2i-1}}\right|_{T=X+2i-2}+b_0^{i-1}\,b_1^{i-1}\left(1-\int_0^X Q(\xi)d\xi\right)\\&+b_0^{i-1}\,b_1^{i-1}\,i\int_X^{\frac{-X+T-(2i-2)}{2}}Q(\xi)d\xi-b_0^{i-2}\,b_1^i(i-1)\int_{\frac{-X-T+2i}{2}}^{1-X}Q(\xi)d\xi\\&+b_0^i\cdot b_1^{i-1}\,i\int_0^{\frac{-X+T-(2i-2)}{2}}Q(\xi)d\xi-b_0^{i-1}\cdot b_1^i(i-1)\int_{\frac{X-T+2i}{2}}^{1}Q(\xi)d\xi\Bigg\},\end{aligned}$$

and for the section $2i+1$

$$\begin{aligned}\frac{\partial U}{\partial X}=-\frac{F}{Es_0}\Bigg\{&\left.\frac{\partial U}{\partial X_{2i}}\right|_{T=-X+2i}+b_0^{i-1}\cdot b_1^i\cdot\left(1-\int_0^X Q(\xi)d\xi\right)\\&+b_0^i\cdot b_1^{i-1}\,i\int_{1-X}^{\frac{-X+T-(2i-2)}{2}}Q(\xi)d\xi-b_0^{i-1}\cdot b_1^i\cdot i\int_{\frac{X-T+2i}{2}}^{X}Q(\xi)d\xi\\&+b_0^i\cdot b_1^i\,i\int_0^{\frac{X+T-2i}{2}}Q(\xi)d\xi-b_0^{i-1}\cdot b_1^{i+1}\cdot i\int_{\frac{-X-T+2i+2}{2}}^{1}Q(\xi)d\xi\Bigg\},\end{aligned} \tag{12}$$

here b_0 and b_1 are the parameters expressed the boundary condition at the both ends, for the sections 1 and 2, it becomes the simpler form.

2.2 Statistical expression of the step response

If the sectional parameter or $P(X)$ is known completely, the time history of the step response can be written with the eq. (12). But if we know only the tendency of the cross-sectional parameter, then we can only say the tendency of the step response.

$P(X)$ and $\frac{dP(X)}{dX}$ are assumed to be described two statistical density functions $p(P)$ and $q\left(\frac{dP(X)}{dX}\right)$. Here we assume that there two density functions would be independent each other.

Hereafter in this section the authors refer only to the result of the rod which is fixed at an end and is free at the other end where step force input is given.

Variable m and σ^2 denotes space average (in the sense of time average) and variance respectively. And suffix refers to those of it, for example $m_{p'}$

denotes the average of $\frac{dP}{dX}$. Every terms of eq. (12) can be developed as

$$\int_0^1 Q(\xi)d\xi = \int_0^1 \frac{dP(\xi)}{d\xi}(1-P(\xi))d\xi = m_{p'} - m_{pp'} \tag{13}$$

For an integral like $I = \int_0^\eta Q(\xi)d\xi$, we can treat it, in the relation to eq. (13), as the sample for (0 , η) drawn from the population (0, 1). If we draw the samples whose number is N from the population whose number is n , the average of samples converges to the average of the population and their variance converges to $\frac{N-n}{n(N-1)}\sigma^2$.

This relation can be extended to our case as follows:

$$\begin{aligned} \langle I \rangle &= \eta(m_{p'} - m_{pp'}) \\ \langle I^2 \rangle &= (1-\eta)\,\eta(\sigma_{p'}^2 + \sigma_{pp'}^2), \end{aligned} \tag{14}$$

here $I \equiv \int_0^\eta Q(\xi)d\xi$.

Then those of strain for the period $2i+1 \geqq T \geqq 2i-1$

$$\begin{aligned} \langle \partial U/\partial X \rangle &= -\frac{F}{Es_0}\,[1+(-1)^{i+1} + 2\{(-1)^{i+1}(i-1)-1 \\ &\quad + (-1)^i 2i\,(T-2i+1)\,(m_{p'}-m_{pp'})\}] \\ \langle \{\partial U/\partial X - \partial U_0/\partial X\}^2 \rangle &= 4\left(\frac{F}{Es_0}\right)^2 i^2 (\sigma_{p'}^2 + \sigma_{pp'}^2)(-T+2i \\ &\quad +1)(T-2i+1)(T-2i+1), \end{aligned} \tag{15}$$

here we should note that $\langle \cdot \rangle$ and $\langle (\cdot)^2 \rangle$ refer to *ensemble*, but $m_{p'}$, $m_{pp'}$, $\sigma_{p'}^2$ and $\sigma_{pp'}^2$ refer to space, and $\langle \partial U/\partial X \rangle$ is equal to the average of the corresponding undisturbed rod $\langle \{\partial U/\partial X - \partial U_0/\partial X\}^2 \rangle$ is the function of time shown in Fig. 4. At T = 1, 3, 5, ----- the uncertainty vanishes completely, but between such moments. the peaks of the uncertainty of response increase quadraticaly.

We assume that P and $\frac{dP}{dX}$ would have normal distribution independent each other, that is

$$\begin{cases} p(P) = \dfrac{1}{\sqrt{2\pi}\sigma_p} e^{-\frac{P^2}{2\sigma_p^2}} \\ q(P') = \dfrac{1}{\sqrt{2\pi}\,\sigma_{p'}} e^{-\frac{P'^2}{2\sigma_{p'}^2}} \end{cases} \tag{16}$$

Then the density function of $P \cdot P'$ would be obtained through the following integral,

$$s(P\cdot P') = \frac{\partial}{\partial X}\iint_{P\cdot P' \leqq X} p(P)\cdot q(P')dP\cdot dP' . \tag{17}$$

If P and P' are independent each other, the following relation should be held $m_{pp'} = m_p m_{p'}$, $\sigma_{pp'}^2 = \sigma_p^2\,\sigma_{p'}^2$, so the average and variance of strain response in this case are so the average and variance of strain response in this case are

$$\begin{cases} \langle \partial U/\partial X \rangle = -\dfrac{F}{Es_0}\{1+(-1)^{i+1}\} \\ \langle \{\partial U/\partial X - \partial U_0/\partial X\}^2 \rangle = 4\left(\dfrac{F}{Es_0}\right)^2 i^2(-T+2i+1)(T-2i+1)\sigma_{p'}^2(1+\sigma_p^2). \end{cases} \tag{18}$$

3. Analysis of eigen-frequencies

In this section, the authors analyze the eigen-frequency distribution of the model expressed in eq. (1). The statistical behavior of eigen-frequencies can be estimated by the similar procedure that was described in the section 2.

3.1 Solution for deterministic system

By substituting $U=\bar{U}\cdot e^{ipt}$ into eq. (1), then

$$\frac{d^2\bar{U}}{dX^2}+\frac{d}{dX}(\log S(X))\frac{d\bar{U}}{dX}+\frac{\ell^2}{a^2}p^2\bar{U}=0 \tag{19}$$

is obtained. The relation,

$$\frac{d(\log S(X))}{dX}=2\frac{d(\log(1+P(X)))}{dX}=2\frac{1}{1+P(X)}\cdot\frac{dP(X)}{dX} \tag{20}$$

could be approximated by putting $2\cdot\dfrac{dP(X)}{dX}$ instead of

$$2\frac{dP(X)}{dX}(1-P(X)).$$

Here P and P_0 are, respectively, eigh-frequencies of a disordered rod and those of the corresponding undisturbed rod, and we assume the following relation to be held

$$p^2=p_0^2+\varepsilon p_1^2. \tag{21}$$

By using the similar type differential operators which are used in the section 2, eq. (19) can be written as

$$(M_0+\varepsilon M_1)\bar{U}=0, \tag{22}$$

here

$$\begin{cases} M_0=\dfrac{d^2}{dX^2}+\dfrac{\ell^2}{a^2}p_0^2 \\ M_1=2\cdot\dfrac{dP}{dX}\cdot\dfrac{d}{dX}+\dfrac{\varepsilon\ell^2}{a^2}p_1^2 \end{cases} \tag{23}$$

Through such a procedure we obtain

$$\bar{U}=\bar{U}_0-\varepsilon M_0^{-1}\left[2\frac{dP}{dX}\frac{d\bar{U}_0}{dX}+\frac{\ell\varepsilon^2}{a^2}p_1^2\bar{U}_0\right], \tag{24}$$

by substituting $\bar{U}_0$ instead of $\bar{U}$ in the left hand side of the equation.

It is assumed that the radius of a rod could be expanded with Fourier series as

$$r(X)=r_0\left[1+\varepsilon\left\{\sum_{j=1}^{\infty}(a_j\cos 2\pi jX+b_j\sin 2\pi jX)\right\}\right]. \quad (25)$$

By inversing eq. (24), the solution becomes

$$\begin{aligned}
\bar{U}= &(A\cos\lambda_0\ell X+B\sin\lambda_0\ell X)/\ell-\frac{4\varepsilon\pi\lambda_0}{\ell^2}\Bigg[\frac{A}{2}\\
&\times\Bigg[\sum_{\substack{j=1\\(j\neq k)}}^{\infty}ja_j\left\{\frac{-\cos(\lambda_j+\lambda_0)\ell X}{Q_j}+\frac{\cos(\lambda_j-\lambda_0)\ell X}{R_j}\right\}+ka_k\left\{\frac{-\cos(\lambda_j+\lambda_0)\ell X}{Q_j}\right.\\
&\left.+\frac{1}{2\lambda_0}\ell X\sin\lambda_0\ell X\right\}_{j=k}-\sum_{\substack{j=1\\(j\neq k)}}^{\infty}jb_j\left\{\frac{\sin(\lambda_j+\lambda_0)\ell X}{Q_j}-\frac{\sin(\lambda_j-\lambda_0)\ell X}{R_j}\right\}\\
&-kb_k\left\{\frac{\sin(\lambda_j+\lambda_0)\ell X}{Q_j}+\frac{1}{2\lambda_0}\ell X\cos\lambda_0\ell X\right\}_{j=k}\Bigg]+\frac{B}{2}\\
&\times\Bigg[-\sum_{\substack{j=1\\(j\neq k)}}^{\infty}ja_j\left\{\frac{\sin(\lambda_j+\lambda_0)\ell X}{Q_j}+\frac{\sin(\lambda_j-\lambda_0)\ell X}{R_j}\right\}-ka_k\frac{\sin(\lambda_j+\lambda_0)\ell X}{Q_j}\\
&-\frac{1}{2\lambda_0}\ell X\cos\lambda_0\ell X\}_{j=k}+\sum_{\substack{j=1\\(j\neq k)}}^{\infty}jb_j\left\{\frac{\cos(\lambda_j+\lambda_0)\ell X}{Q_j}+\frac{\cos(\lambda_j-\lambda_0)\ell X}{R_j}\right\}\\
&+kb_k\left\{\frac{\cos(\lambda_j+\lambda_0)\ell X}{Q_j}+\frac{1}{2\lambda_0}\ell X\sin\lambda_0\ell X\right\}_{j=k}\Bigg]\Bigg]\\
&-\varepsilon\lambda_1^2\left\{\frac{A}{2\lambda_0}X\sin\lambda_0\ell X-\frac{B}{2\lambda_0}X\cos\lambda_0\ell X\right\},
\end{aligned}$$

here

$$\lambda_j=\frac{2j\pi}{\ell}, \quad \lambda_0=\frac{k\pi}{\ell},$$

$$Q_j=-(\lambda_j+\lambda_0)^2+\lambda_0^2, \quad R_j=-(\lambda_j-\lambda_0)^2+\lambda_0^2$$

and we differentiate eq. (26) by X and fit it to the boundary conditions, free-free,

$$\left.\frac{d\bar{U}}{dX}\right]_{X=0,\ X=1}=0. \quad (27)$$

Then the characteristic equations can be obtained as follows:

$$\begin{cases} \left[\dfrac{2\varepsilon\pi\lambda_0}{\ell} \left\{ \sum_{j\neq k} jb_j \dfrac{2\lambda_0}{\lambda_j^2-4\lambda_0^2} + \dfrac{kb_k}{8\lambda_0} \right\} \right] A \\ + \left[\lambda_0 + \dfrac{2\varepsilon\pi\lambda_0}{\ell} \left\{ \sum_{j\neq k} ja_j \dfrac{-2(\lambda_j^2-2\lambda_0^2)}{\lambda_j(\lambda_j^2-4\lambda_0^2)} - \dfrac{7ka_j}{8\lambda_0} \right\} + \dfrac{\varepsilon p_1^2}{2\lambda_0 a^2} \right] B = 0 \\ \left[\dfrac{2\varepsilon\pi\lambda_0}{\ell} \left\{ -\dfrac{ka_k\ell}{2} + \sum_{j\neq k} jb_j \dfrac{2\lambda_0}{\lambda_j^2-4\lambda_0^2} + \dfrac{kb_k}{8\lambda_0} \right\} - \dfrac{\varepsilon p_1^2\ell}{2a^2} \right] A \\ + \left[\lambda_0 + \dfrac{2\varepsilon\pi\lambda_0}{\ell} \left\{ \sum_{j=k} ja_j \dfrac{-2(\lambda_j^2-2\lambda_0^2)}{\lambda_j(\lambda_j^2-4\lambda_0^2)} - \dfrac{7ka_k}{8\lambda_0} + \dfrac{kb_k\ell}{2} \right\} + \dfrac{\varepsilon p_1^2}{2\lambda_0 a^2} \right] B = 0 \end{cases} \tag{28}$$

Again by neglecting the terms of which orders are higher than ε^2 eigen-frequencies for a free-free rod are obtained as

$$p_k^2 = p_{0k}^2 (1-2\varepsilon a_k), \tag{29}$$

for a fixed-free rod

$$p_k^2 = p_{0k}^2 \left\{ 1 + \frac{16\varepsilon}{\pi} \sum_{j=1}^{\infty} \frac{jb_j}{4j^2-(2k-1)^2} \right\} \tag{29'}$$

According to its boundary conditions, only sine terms or cosine terms of eq. (29) are effective. For some condition, only one correspondint term is so. Some results of numerical calculation are shown in Fig. 5.

3.2 Statistical evaluation of eigen-frequencies of disordered rod population

We assume that there are many rods and that we would have the statistical information about their radii $r(x)$ (Fig. 6).

Here $p(r(x))$ is the density function of $r(x)$ at the specific point x for the disordered rod population. We should assume that *ensemble* mean of radii, $\langle r(x)\rangle$ and that of square radii, $\langle r^2(x)\rangle$ would be constant at all points, and that the radius $r(x)$ of the specific section would not corelated to that of other sections, that is,

$$\langle r(x)\cdot r(y)\rangle = 0. \tag{30}$$

These two assumptions coincide with the assumption in the section 2, that $P(X)$ and $P'(X)$ are independet each other and described by two-dimensional Gausian distribution function.

Again we consider the case in which the boundary conditions of rods are fixed-free. So the authors start their analysis from eq. (29'). At first b_j should be estimated from the information on $r(x)$.

$$b_j = \frac{1}{r_0\varepsilon\ell}\int_0^\ell r(x)\sin\frac{2j\pi x}{\ell}\,dx \tag{31}$$

for one of the disordered rods, then by taking their *ensemble* average eq. (31) can be rewritten as

$$\langle b_j \rangle = \frac{1}{r_0 \varepsilon \ell} \int_0^{\ell} \langle r(x) \rangle \sin \frac{2j\pi x}{\ell} dx, \tag{32}$$

and

$$\langle b_j \cdot b_k \rangle = \frac{1}{r_0^2 \varepsilon^2 \ell^2} \int_0^{\ell} \int_0^{\ell} \langle r(x) \cdot r(y) \rangle \sin \frac{2j\pi x}{\ell} \sin \frac{2k\pi y}{\ell} dx \cdot dy. \tag{33}$$

From eq. (29') the estimated mean value of eigen-frequencies can be written

$$\langle p_k^2 \rangle = p_{0k}^2 \left\{ 1 + \frac{16\varepsilon}{\pi} \sum_{j=1}^{\infty} \frac{j \langle b_j \rangle}{4j^2 - (2k-1)^2} \right\} \tag{34}$$

and by substituting $\langle b_j \rangle$ with eq. (32)

$$\langle p_k^2 \rangle = p_{0k}^2 \left[1 + \frac{16}{r_0 \pi \ell} \sum_{j=1}^{\infty} \frac{j}{4j^2 - (2k-1)^2} \int_0^{\ell} \langle r(x) \rangle \sin \frac{2j\pi x}{\ell} dx \right] \tag{35}$$

The variances of $p_k^2 - p_{0k}^2$ are

$$\langle \{ p_k^2 - p_{0k}^2 \}^2 \rangle = \frac{256 p_{0k}^4}{r_0^2 \pi^2 \ell^2} \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{jn}{\{ 4j^2 - (2k-1)^2 \} \{ 4n^2 - (2k-1)^2 \}} \times \int_0^{\ell} \int_0^{\ell} \langle r(x) r(y) \rangle \sin \frac{2j\pi x}{\ell} \sin \frac{2n\pi y}{\ell} dx\, dy \right] \tag{36}$$

According to the assumption described in eq. (30) and others, they become

$$\langle p_k^2 \rangle = p_{0k}^2, \tag{35'}$$

$$\langle \{ p_k^2 - p_{0k}^2 \}^2 \rangle = \frac{128 p_{0k}^4 \langle r^2(x) \rangle}{r_0^2 \pi^2} \sum_{j=1}^{\infty} \frac{j^2}{[4j^2 - (2k-1)^2]} \tag{36'}$$

4. Experiment

As the authors mentioned it already, several steel rods are used as a model system. A rod is supported by piano wires and rubber pads at an end. To measure the strain semi-conductor type strain gauges are used with cathode ray tube oscilloscope. Stepwise force is given by the hammer which has the similar diameter and its round head.

A disordered rod R of which the inclination of radius $\frac{dr}{dx}$ and the length of each section are decided independently from a random number table. Both values are chosen from the table of nine-step value accordint to the digits of a random number table (Table 1). Rod R is 3 m long.

Several rods U, T and others are used to compair the result to the exact solution, so rod U is a uniform undisturbed rod and rod T is a uniform tapered rod. The lengths of both rods are 1 m.

The velocity of a longitudinal wave in these models is 5120 m/s, and the time required for going up and down of the wave through rod R is approximate 1.2 ms.

Some examples of wave patterns are shown in Figs. 7 and 8.
In the case of numerical experiment the authors neglect the effect of the internal dampings.

Although the acceleration wave obtained by theoretical analysis of the continuous model contains all components of frequency, the wave pattern of the experimental model is observed to be more oscillatory than that of the continuous model because of the stronger effect of damping to the higher frequency component.

The varying range of the cross-sectional parameter of the model is not enough to simulate earthquake waves, because in this model the parasitic vibration caused by multi-reflections at discontinuous points is weaker than the fundamental vibration.

The authors tried to estimate the eigen-frequency distribution by analyzing such response waves. Scuh waves are deterministic and periodic and not satisfy the conditon of power spectral density analysis. But we often applied the PSD technique really without knowing the exact nature of the waves which we want to analyze, and as a result we should conclude that they might be nearly periodic. The earthquake analysis is one these examples. In Figs.9 the estimated power spectral density of the step response of rod R is shown. The arrows in this figure show the eigen-frequencies which were obtained through forced vibration experiment. In Table 2 and Fig. 10 also the relative values of the eigen-frequecnies of rod R are compaired. Tendencies of these curves seem to coincide rather well, but the deviations from unity are sometimes opposite. And this fact gives very poor result for estimating the cross-sectional parameters of a rod as the authors refer it later.

5. Estimation of cross-sectional parameter

As the authors mentioned above, we can estimate cross-sectional parameter of a rod or an earthlayer along the wave path.

But, we should assume that a system would be almost uniform and the deviation of the cross-sectional parameters are small enough. Through such analysis we could know the eigen-frequency distribution as the deviations from those of the corresponding undisturbed system. So, if we do not know about the eigen-frequency distribution of the undisturbed rod, we should substitute the *ensemble* average of the eigen-frequencies of their samples for them.

In the case of free-fixed boundary condition, from eq. (29') we can obtain the n elements linear equations,

$$\frac{16\varepsilon}{\pi}\sum_{j=1}^{n}\frac{j\cdot p_{0k}^2}{4j^2-(2k-1)^2}\,b_j = p_k^2 - p_{0k}^2. \qquad k=1,---,n \qquad (37)$$

So we can determine b_j exactly, if we know $p_k^2 - p_{0k}^2$ $(k=1,---,n)$ deterministically.

Using the eigen-frequency distribution the estimated co-efficients $\hat{a}_k r_0$ are obtained as the figures in the right-hand side columns of Table 2. The original values of $a_k r_0$ are also shown. The result is very poor in this case.

6. Conclusion

Through this article the authors described an idea of simulating strong earthquake motions. Even though there are several weak-points, the authors could show the detail on each blocks in Fig. 1.

To apply this method to the real earthquake record, we need the farther studies. The authors assumed an almost uniform rod or layer which involved only small fluctuation of parameters inside. But real systems usually consist of at least several segments of such almost uniform rod or layer. It is not clear that the system which consists of only one almost uniform segment could be substituted for such a real system. It seems to be not possible, because the density of the eigen-frequencies of a real system are more dense than those of the simplified system. Then the next problem is how to separate the train of eigen-frequencies to those of each segments.

They also assumed that an independent step input is applied on the system, but in a real system it might be a train of several inputs. They do not know the nature of such a train, but we could assume the nature through a statistical approach. This kind of problems was already discussed by Professor Ang.(4) Estimating the response of the almost uniform system to the input train which is described only statistically is another new problem.

Acknowledgement

The authors are very grateful to Professors A. Watari, S. Fujii and H. Sato for their kind guidances and discussions. And they also wish to thank the member of the Technical Committee on Random Fatigue and Vibration in the Japan Society of Mechanical Engineers for giving us the chance to discuss this subject at their meeting.

A part of this paper is summarized from the thesis for the master's degree to Mr. Miyamoto, one of the authors.

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Table 1. Dimension of Rod. R

Unit: mm

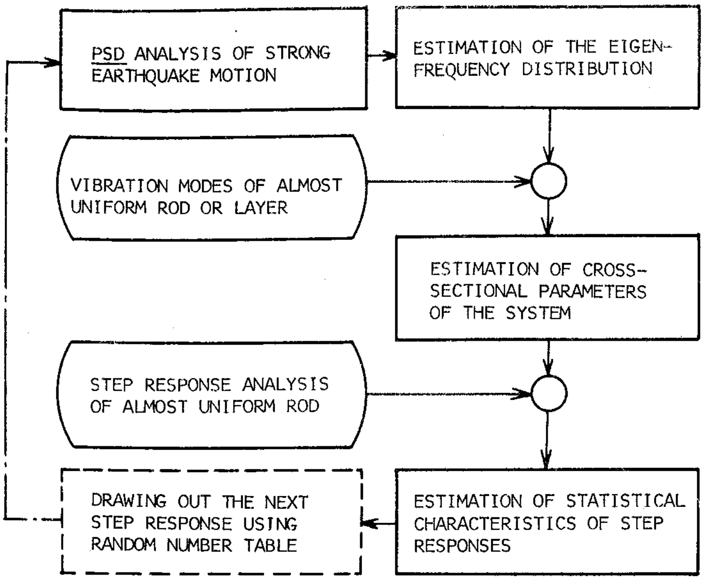
i	0	1	2	3	4	5	6	7	8
Diameter d_i	38.000	39.000	43.800	42.333	38.333	38.333	36.333	37.267	42.600
Location of section ℓ_i	0	150	330	550	750	940	1090	1230	1430

i	9	10	11	12	13	14	15	16
Diameter d_i	39.667	35.867	41.200	44.800	39.200	40.667	37.467	40.000
Location of section ℓ_i	1650	1840	2040	2220	2430	2650	2810	3000

Table 2. Eigen-frequencies of Rod R

Number of mode k	Dimension-less eigen-frequency p_k / p_{ok}			Eigen-frequency Rad/sec x 10^4	Cross-sectional parameter $a_k r_0$	
	Theoretical	Forced vibration	PSD analysis		Estimated	Original
1	0.985	0.987	1.035	0.536	− 0.0748	− 0.03042
2	1.016	0.998	1.038	1.07	− 0.0797	− 0.03234
3	1.037	1.021	1.038	1.61	− 0.0797	− 0.07480
4	0.982	0.984	1.000	2.14	− 0	− 0.03620
5	1.028	1.028	1.052	2.68	− 0.1095	− 0.05667
6	0.995	0.996	1.010	3.22	− 0.0200	− 0.00993
7	0.992	0.988	1.016	3.75	− 0.0349	− 0.01619
8	0.995	0.995	1.016	4.29	− 0.0349	− 0.01010
9	0.993	0.990	1.018	4.83	− 0.0378	− 0.01442
10	0.999	1.007	1.019	5.36	− 0.0395	− 0.0200

Fig. 1. Flow chart of simulating strong earthquake motion



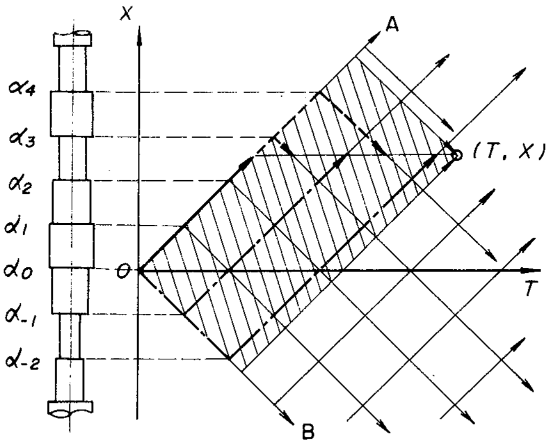


Fig. 2. Wave propagation pattern of infinite long rod

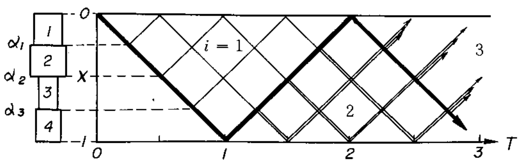


Fig. 3. Wave propagation pattern of finite rod

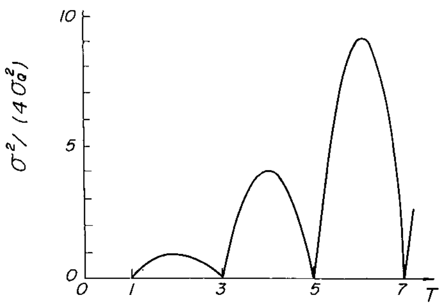


Fig. 4. Time vs. change of variance of strain at fixed end

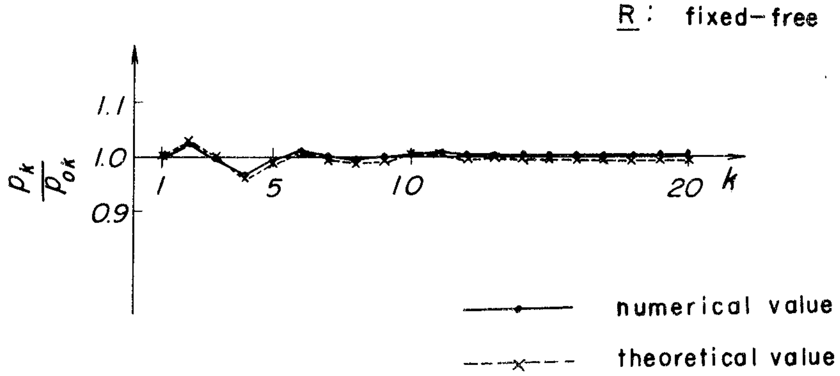


Fig. 5. Effects of rod shapes on eigen-frequencies

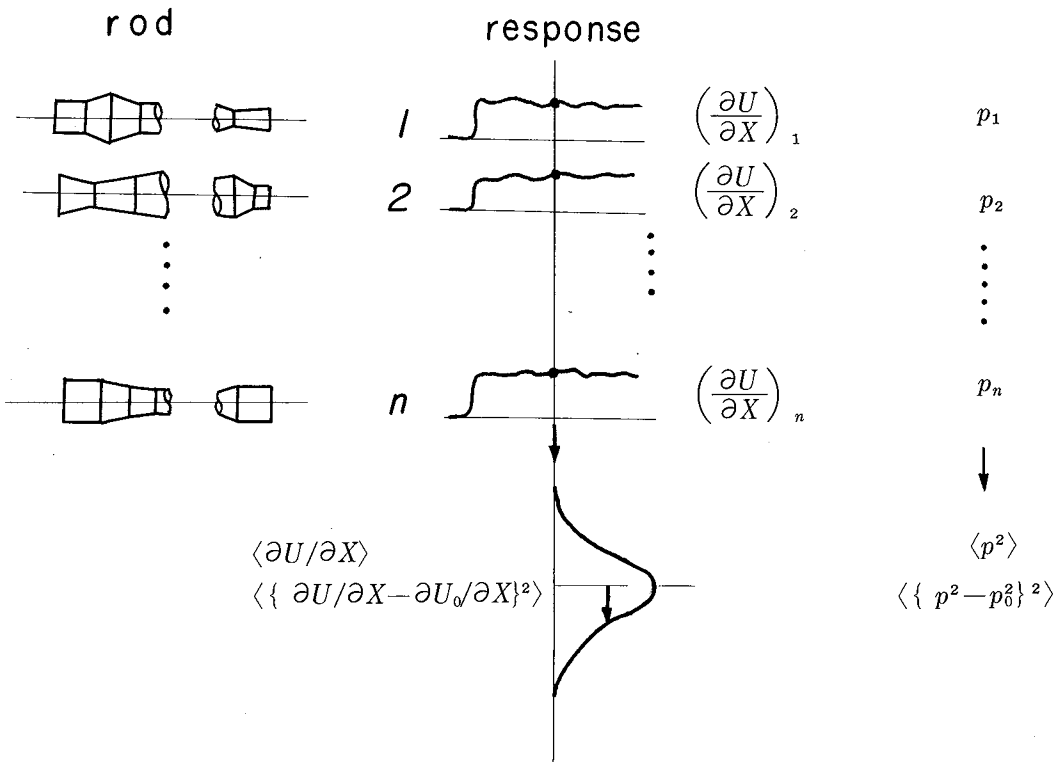


Fig. 6. Samples from rod population

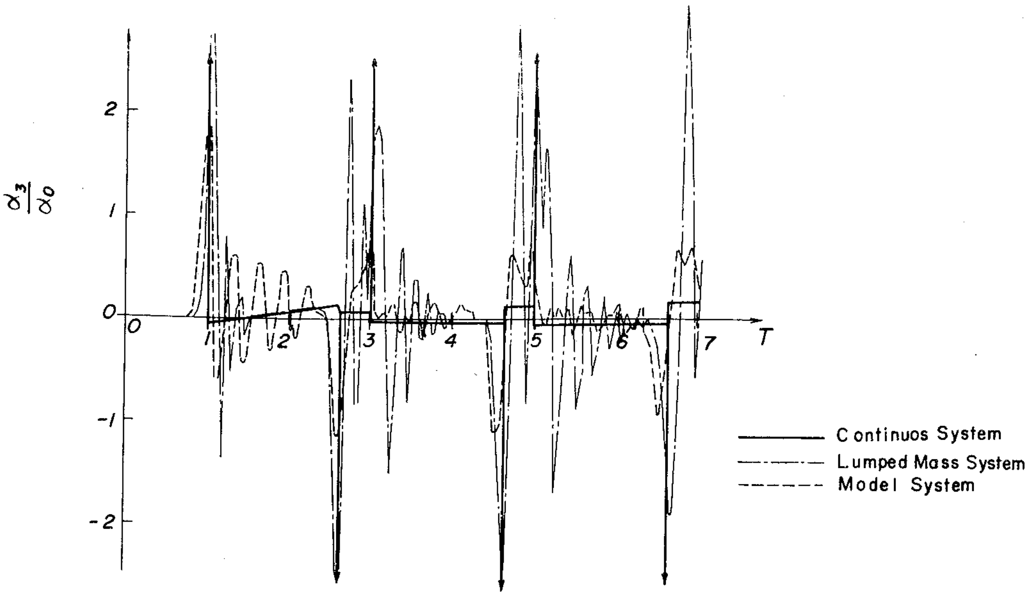


Fig. 7. Step response of fixed-free rod R in acceleration

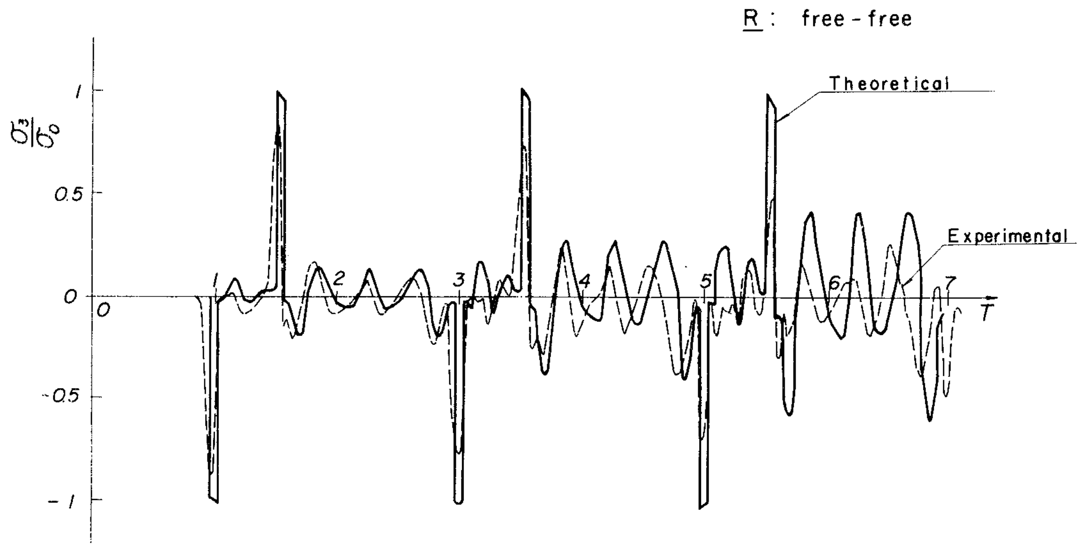


Fig. 8. Step response of free-free rod R in stain

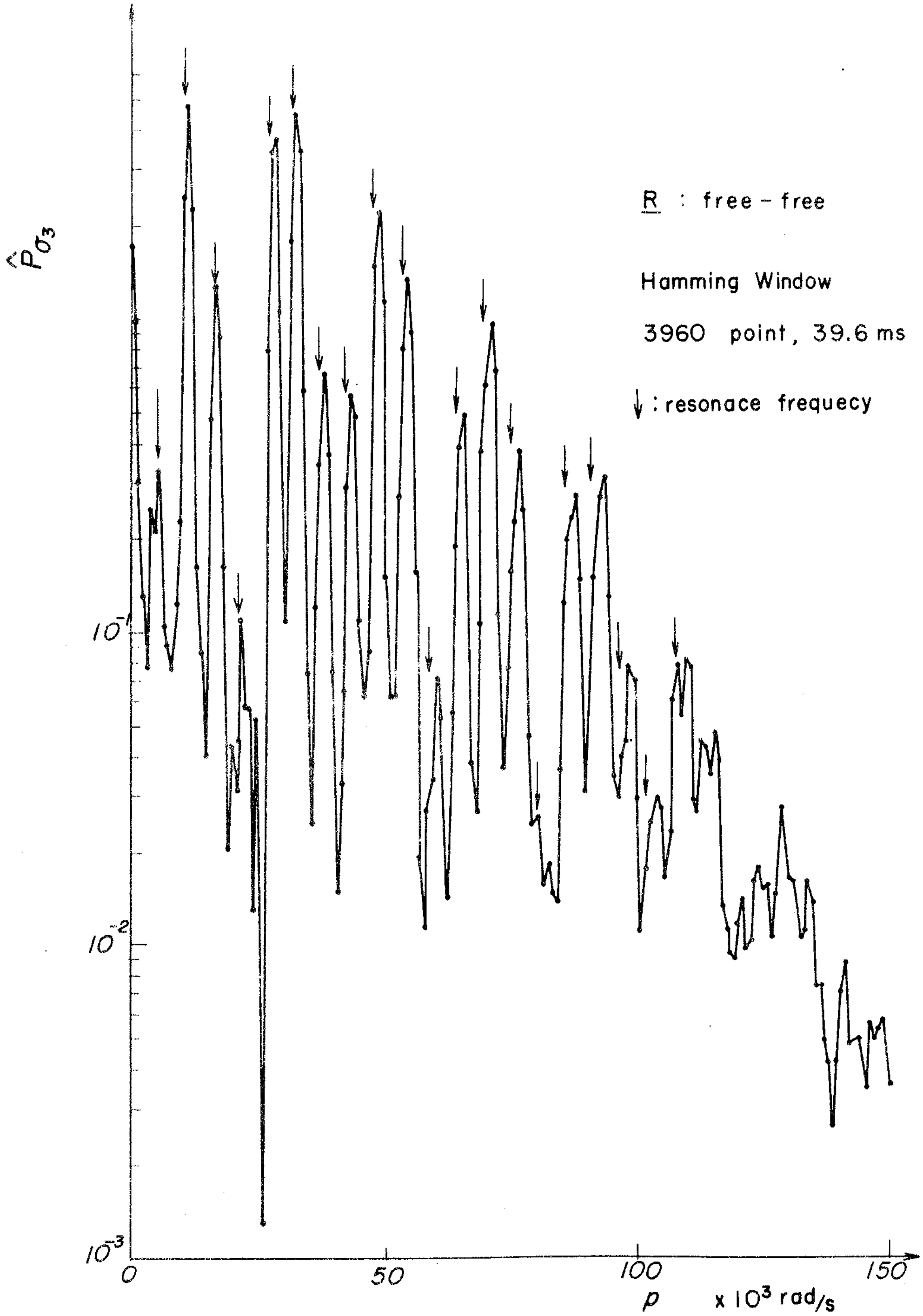


Fig. 9. Power spectral density of strain response wave

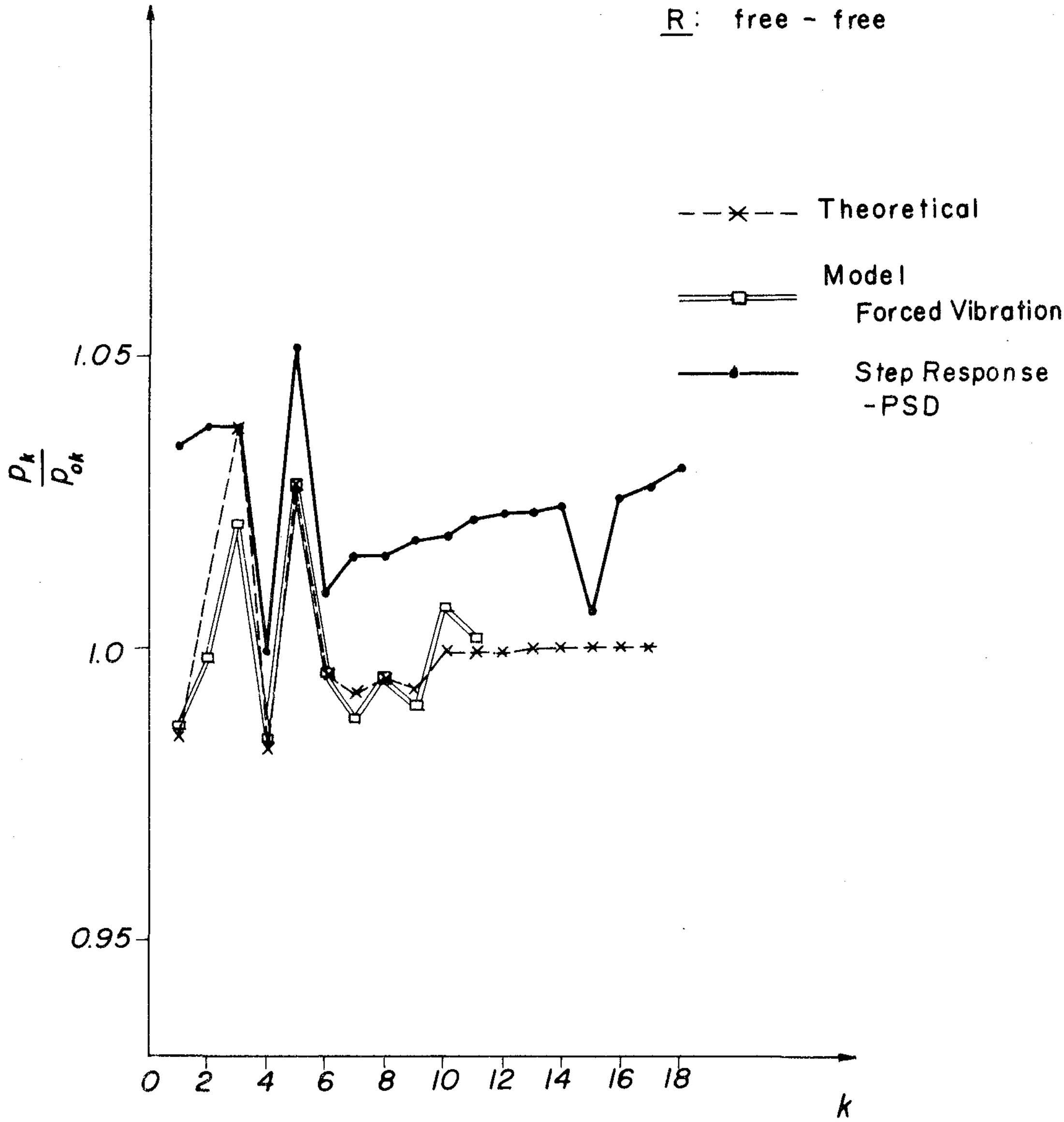


Fig. 10. Comparison of the eigen-frequency distribution of free-free rod R